# Mutual Coupling Calibration in Super-Resolution Direction Finding for Wideband Signals

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**Abstract.** Most super-resolution direction finding methods need to know the array manifold exactly, but there is usually mutual coupling error in application, which directly leads to the performance degradation, and even failure. The paper proposed a novel calibration method in super-resolution direction finding for wideband signals based on spatial domain sparse optimization when mutual coupling exists in the array. First, the optimization functions are founded by the signals of every frequency bin, then the functions are optimized iteratively, after that the information of all frequencies is integrated for the calibration, finally, the actual directions of arrival (DOA) can be acquired, the performance of the method has been proved by simulations.

**Keywords:** Super-resolution direction finding · Array calibration · Mutual coupling · Wideband signals

## 1 Introduction

Super-resolution direction finding is one of the major research contents in array signal processing, it is widely used in radio monitoring [1–3], internet of things [4, 5] and electronic countermeasure [6, 7]. At present, most direction finding methods are based on knowing the accurate array manifold, but there are often high frequency oscillation and amplifiers in practical systems, which lead to the mutual coupling in the array, it causes the performance of the direction finding methods deteriorated, and even failure, so they are necessary to be calibrated.

For the presence of mutual coupling, the early methods are usually based on the electromagnetic measurement [8], or calculating the mutual coupling coefficient by matrix measure, then compensate and calibrate the corresponding parameters [9–11]. These methods can often not meet the calibration accuracy in the actual projects,

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especially for the radar, sonar and other fields of array signal processing, their electromagnetic environments are very complex, whether measure or electromagnetic computation are not suitable for the array calibration. With the development of the research, more and more scholars are keen to parameterize the coefficient by special structure of the mutual coupling matrix, this kind of methods not only have a high precision but also adapt to the various circumstances and electromagnetic parameters. Some classic algorithms are proposed successively: Wang et al. proposed a calibration method for mutual coupling in 2003 [12], it only needs one dimensional searching and is easy to be implemented; Dai et al. [13] eliminated the unknown mutual coupling in the uniform array by inherent mechanism; Xie et al. [14] achieved localization of mixed far-field and near-field sources under unknown mutual coupling. Liu et al. [15] proposed a DOA estimation method based on fourth-order cumulants along with mutual coupling, it can be applied when non-Gaussian signals coexist with unknown colored Gaussian noise. Elbir and Tuncer [16] estimated DOA and mutual coupling coefficient for arbitrary array structures with single and multiple snapshots. But all of these methods are just appropriate for narrowband signals, so far there are few public literatures about mutual coupling among sensors in super-resolution direction finding for wideband signals.

The paper proposed a novel mutual coupling calibration method in super-resolution direction finding for wideband signals based on spatial domain sparse optimization, when mutual coupling exists in the array, the corresponding optimization functions are founded by the signals of every frequency bin, then the functions are optimized iteratively, at last, the information of all frequencies is integrated for the calibration, consequently the actual DOA can be obtained.

## 2 Signal Model

### 2.1 Ideal Signal Model

It is seen from Fig. 1, suppose there are $K$ far-field wideband signals $s_k(t)$ $(k = 1, 2, \cdots, K)$ impinging on the uniform linear array composed of $M$ omnidirectional sensors, the space of them is $d$, it is equal to half of the wavelength of the center frequency, DOAs of them are $\boldsymbol{\alpha} = [\alpha_1, \cdots, \alpha_k, \cdots, \alpha_K]$, the first sensor is defined as the reference, then output of the $m$th sensor can be written as

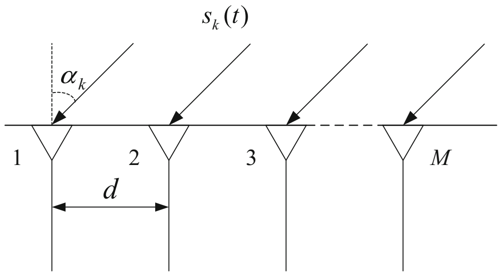


**Fig. 1.** Array signal model

$$x_m(t) = \sum_{k=1}^{K} s_k(t - \tau_m(\alpha_k)) + n_m(t), m = 1, 2, \cdots, M \tag{1}$$

Where $\tau_m(\alpha_k) = (m-1)\frac{d}{c}\sin\alpha_k$ is the propagation delay for the $k$th signal arriving at the $m$th sensor with respect to the reference of the array, $c$ is the propagating speed of the signal, $n_m(t)$ is the Gaussian white noise on the $m$th sensor.

Assume that the range of the frequency band of all signals is $[f_{\mathrm{Low}}, f_{\mathrm{High}}]$, before the processing, we divide the output vector into $J$ nonoverlapping components, Discrete Fourier Transform (DFT) is performed on (1) and the array outputs of $J$ frequencies can be represented as

$$\boldsymbol{X}(f_i) = \boldsymbol{A}(f_i, \boldsymbol{\alpha})\boldsymbol{S}(f_i) + \boldsymbol{N}(f_i) \quad i = 1, 2, \cdots, J \tag{2}$$

Where $f_{\mathrm{Low}} \le f_i \le f_{\mathrm{High}}$ $(i = 1, 2, \cdots, J)$, $KP$ snapshots are collected at every frequency, then we have

$$\boldsymbol{X}(f_i) = [\boldsymbol{X}_1(f_i), \cdots, \boldsymbol{X}_m(f_i), \cdots, \boldsymbol{X}_M(f_i)]^{\mathrm{T}} \tag{3}$$

Where

$$\boldsymbol{X}_m(f_i) = [X_m(f_i, 1), \cdots, X_m(f_i, kp), \cdots, X_m(f_i, KP)] \tag{4}$$

$\boldsymbol{A}(f_i, \boldsymbol{\alpha})$ is a $M \times K$ dimensional steering vector

$$\boldsymbol{A}(f_i, \boldsymbol{\alpha}) = [\boldsymbol{a}(f_i, \alpha_1), \cdots, \boldsymbol{a}(f_i, \alpha_k), \cdots, \boldsymbol{a}(f_i, \alpha_K)] \tag{5}$$

$$\boldsymbol{a}(f_i, \alpha_k) = \left[1, \exp(-\mathrm{j}2\pi f_i \frac{d}{c}\sin\alpha_k), \cdots, \exp\left(-\mathrm{j}(M-1)2\pi f_i \frac{d}{c}\sin\alpha_k\right)\right]^{\mathrm{T}} \tag{6}$$

And

$$\boldsymbol{S}(f_i) = [\boldsymbol{S}_1(f_i), \cdots, \boldsymbol{S}_k(f_i), \cdots, \boldsymbol{S}_K(f_i)]^{\mathrm{T}} \tag{7}$$

is the signal vector matrix after DFT to $s_k(t)$ $(k = 1, 2, \cdots, K)$, where

$$\boldsymbol{S}_k(f_i) = [S_k(f_i, 1), \cdots S_k(f_i, kp), \cdots, S_k(f_i, KP)] \tag{8}$$

Here, $S_k(f_i, kp)$ is the $kp$th snapshots of the $k$th signal at $f_i$, then

$$\boldsymbol{N}(f_i) = [\mathbf{N}_1(f_i), \cdots, \boldsymbol{N}_m(f_i), \cdots, \boldsymbol{N}_M(f_i)]^{\mathrm{T}} \tag{9}$$

$$\boldsymbol{N}_m(f_i) = [N_m(f_i, 1), \cdots, N_m(f_i, kp), \cdots, N_m(f_i, KP)] \tag{10}$$

is the noise vector after performing DFT on $n_m(t)$ $(m = 1, 2, \cdots, M)$ with mean 0 and variance $\mu^2(f_i)$.

### 2.2 Array Error Model

For convenience, we only discuss the information at frequency $f_i$ for the moment, the degree of mutual coupling is closely related to signal frequency, when there is only mutual coupling among sensors, perturbation matrix can be expressed by $\boldsymbol{W}(f_i)$, we itemize $Q$ corresponding the freedom degree of the array, according to the property of uniform linear array, we know $\boldsymbol{W}(f_i)$ can be expressed as:

$$\boldsymbol{W}(f_i) = \begin{bmatrix} 1 & c_1(f_i) & \cdots & c_Q(f_i) & & & \\ c_1(f_i) & 1 & c_1(f_i) & & \ddots & & \\ & c_1(f_i) & & & & c_Q(f_i) & \\ \vdots & & \ddots & \ddots & \ddots & & \\ c_Q(f_i) & & & & & & \\ & \ddots & & & & 1 & c_1(f_i) \\ & & c_Q(f_i) & & c_1(f_i) & & 1 \end{bmatrix} \quad (11)$$

Where $c_q(f_i)$ $(q = 1, 2, \cdots, Q)$ is the mutual coupling coefficient, when the distance between two sensor is $q$, signal frequency is $f_i$, the steering vector of the array can be revised to

$$\boldsymbol{a}'(f_i, \alpha_k) = \boldsymbol{W}(f_i)\boldsymbol{a}(f_i, \alpha_k) \quad (k = 1,\ 2, \cdots, K) \quad (12)$$

Corresponding array manifold is

$$\boldsymbol{A}'(f_i, \boldsymbol{\alpha}) = [\boldsymbol{a}'(f_i, \alpha_1), \cdots, \boldsymbol{a}'(f_i, \alpha_k), \cdots, \boldsymbol{a}'(f_i, \alpha_K)] = \boldsymbol{W}(f_i)\boldsymbol{A}(f_i, \boldsymbol{\alpha}) \quad (13)$$

For the sake of simplicity, we define the mutual coupling perturbation vector between sensors as $\boldsymbol{w}\,(f_i) = [c_1(f_i), \cdots, c_Q(f_i)]^{\mathrm{T}}$. So the output of the array at frequency $f_i$ can be expressed as

$$\begin{aligned} \boldsymbol{X}'(f_i) = \boldsymbol{A}'(f_i, \boldsymbol{\alpha})\boldsymbol{S}(f_i) + \boldsymbol{N}(f_i) &= \boldsymbol{W}(f_i)\boldsymbol{A}(f_i, \boldsymbol{\alpha})\boldsymbol{S}(f_i) + \boldsymbol{N}(f_i) \\ &= \boldsymbol{A}(f_i, \boldsymbol{\alpha})\boldsymbol{N}(f_i) + \boldsymbol{\Lambda}(f_i)\boldsymbol{w}(f_i) + \boldsymbol{N}(f_i) \end{aligned} \quad (14)$$

Where $\boldsymbol{\Lambda}\,(f_i)$ is the coefficient vector related to the mutual coupling.

## 3 Estimation Theory

The searching space can be divided into several discrete angle grids $\boldsymbol{\Omega} = [\bar{\alpha}_1, \cdots, \bar{\alpha}_l, \cdots, \bar{\alpha}_L]$, and $K << L$, take it into (2), we have

$$\bar{\boldsymbol{X}}'(f_i) = \boldsymbol{A}'(f_i, \boldsymbol{\Omega})\bar{\boldsymbol{S}}(f_i) + \boldsymbol{N}(f_i) \quad (i = 1, 2, \cdots, J) \quad (15)$$

Then corresponding covariance matrix of $\bar{\boldsymbol{X}}'(f_i)$ is

$$\bar{\boldsymbol{R}}'(f_i) = E\left\{\bar{\boldsymbol{X}}'(f_i)(\bar{\boldsymbol{X}}'(f_i))^{\mathrm{H}}\right\} \quad i = (1, 2, \cdots, J) \tag{16}$$

In (15), $\bar{\boldsymbol{S}}(f_i) = [\bar{\boldsymbol{S}}(f_i, 1), \cdots, \bar{\boldsymbol{S}}(f_i, kp), \cdots, \bar{\boldsymbol{S}}(f_i, KP)]$, where $\bar{\mathbf{S}}(f_i, kp) = [\bar{S}_1(f_i, kp), \cdots \bar{S}_l(f_i, kp), \cdots, \bar{S}_L(f_i, kp)]^{\mathrm{T}}$ is a sparse matrix, it only contains $K$ non-zero elements, they are non-zero if and only if $\bar{\alpha}_l = \alpha_k$ and $\bar{S}_l(f_i, kp) = S_k(f_i, kp)$, $(l = 1, 2, \cdots, L;\ \ k = 1, 2, \cdots, K)$. Define $\boldsymbol{\delta}(f_i) = [\delta_1(f_i), \cdots, \delta_l(f_i), \cdots, \delta_L(f_i)]^{\mathrm{T}}$ as the vector formed by variances of the elements in $\bar{\boldsymbol{S}}(f_i)$, it reflects the energy of the signal, that is

$$\bar{\boldsymbol{S}}(f_i) \sim N(0, \boldsymbol{\Sigma}(f_i)) \tag{17}$$

Where $\boldsymbol{\Sigma}(f_i) = \mathrm{diag}(\boldsymbol{\delta}(f_i))$, as $\bar{\boldsymbol{S}}(f_i)$ is $\boldsymbol{S}(f_i)$ jointed many zero elements, $\boldsymbol{\delta}(f_i)$ contains $K$ non-zero elements too.

It can be seen from (15) and (17), probability density of the output signal at $f_i$ along with the error simultaneously is

$$\begin{aligned} P\left(\bar{\boldsymbol{X}}'(f_i)\middle|\bar{\boldsymbol{S}}(f_i); \boldsymbol{w}(f_i), \mu^2(f_i)\right) &= \left|\boldsymbol{\pi}\mu^2(f_i)\boldsymbol{I}_M\right|^{-KP} \exp\left\{-\mu^2(f_i)\left\|\bar{\boldsymbol{X}}'(f_i) - \boldsymbol{A}'(f_i, \boldsymbol{\Omega})\bar{\boldsymbol{S}}(f_i)\right\|_2^2\right\} \\ &= \left|\boldsymbol{\pi}\mu^2(f_i)\boldsymbol{I}_M\right|^{-KP} \exp\left\{-\mu^2(f_i)\times\left\|\bar{\boldsymbol{X}}'(f_i) - \boldsymbol{W}(f_i)\boldsymbol{A}(f_i, \boldsymbol{\Omega})\bar{\boldsymbol{S}}(f_i)\right\|_2^2\right\} \end{aligned} \tag{18}$$

Combining (15), (17) and (18), probability density of $\bar{\boldsymbol{X}}'(f_i)$ is

$$\begin{aligned} P\left(\bar{\boldsymbol{X}}'(f_i); \boldsymbol{\delta}(f_i), \boldsymbol{w}(f_i), \mu^2(f_i)\right) &= \int P\left(\bar{\boldsymbol{X}}'(f_i)\middle|\bar{\boldsymbol{S}}(f_i); \boldsymbol{w}(f_i), \mu^2(f_i)\right) P(\bar{\boldsymbol{S}}(f_i); \boldsymbol{\delta}(f_i))\mathrm{d}\bar{\boldsymbol{S}}(f_i) \\ &= \left|\pi\left(\mu^2(f_i)\boldsymbol{I}_M + \boldsymbol{A}'(f_i, \boldsymbol{\Omega})\boldsymbol{\Sigma}(f_i)(\boldsymbol{A}'(f_i, \boldsymbol{\Omega}))^{\mathrm{H}}\right)\right|^{-KP} \\ &\times \exp\left\{-KP \times \mathrm{tr}\left(\left(\mu^2(f_i)\boldsymbol{I}_M + \boldsymbol{A}'(f_i, \boldsymbol{\Omega})\boldsymbol{\Sigma}(f_i)(\mathbf{A}'(f_i, \boldsymbol{\Omega}))^{\mathrm{H}}\right)^{-1}\bar{\boldsymbol{R}}'(f_i)\right)\right\} \end{aligned} \tag{19}$$

Then we can employ Expectation Maximization (EM) method [17] to iteratively estimate each unknown parameters, compute distribution function of $P(\bar{\boldsymbol{X}}'(f_i), \bar{\boldsymbol{S}}(f_i); \boldsymbol{\delta}(f_i), \boldsymbol{w}(f_i), \mu^2(f_i))$, in the E-step:

$$\begin{aligned} & F(\bar{\boldsymbol{X}}'(f_i), \bar{\boldsymbol{S}}(f_i); \boldsymbol{\delta}(f_i), \boldsymbol{w}(f_i), \mu^2(f_i)) \\ &= \left\langle \mathrm{In}P(\bar{\boldsymbol{X}}'(f_i), \bar{\boldsymbol{S}}(f_i); \boldsymbol{\delta}(f_i), \boldsymbol{w}(f_i), \mu^2(f_i)) \right\rangle \\ &= \left\langle \mathrm{In}P\left(\bar{\boldsymbol{X}}'(f_i)\middle|\bar{\boldsymbol{S}}(f_i); \boldsymbol{w}(f_i), \mu^2(f_i)\right) + \mathrm{In}P(\bar{\boldsymbol{S}}(f_i); \boldsymbol{\delta}(f_i)) \right\rangle \\ &= \left\langle -M \times KP \times \mathrm{In}\mu^2(f_i) - \mu^{-2}(f_i)\left\|\bar{\boldsymbol{X}}'(f_i) - \boldsymbol{A}'(f_i, \boldsymbol{\Omega})\bar{\boldsymbol{S}}(f_i)\right\|_2^2 - \sum_{l=1}^{L}\left(KP \times \mathrm{In}\delta_l(f_i) + \frac{\left(\sum_{kp=1}^{KP}|\bar{S}_l(f_i, kp)|^2\right)}{\delta_l(f_i)}\right)\right\rangle \\ &= \left\langle -M \times KP \times \mathrm{In}\mu^2(f_i) - \mu^{-2}(f_i)\|\bar{\boldsymbol{X}}'(f_i) - \boldsymbol{W}(f_i)\boldsymbol{A}(f_i, \boldsymbol{\Omega})\bar{\boldsymbol{S}}(f_i)\|_2^2 - \sum_{l=1}^{L}\left(KP \times \mathrm{In}\delta_l(f_i) + \frac{\left(\sum_{kp=1}^{KP}|\bar{S}_l(f_i, kp)|^2\right)}{\delta_l(f_i)}\right)\right\rangle \end{aligned} \tag{20}$$

In the M-step, solve derivatives of $F(\bar{\boldsymbol{X}}'(f_i), \bar{\boldsymbol{S}}(f_i); \boldsymbol{\delta}(f_i), \boldsymbol{w}(f_i), \mu^2(f_i))$ for each parameter, that is

$$\begin{aligned}&\frac{\partial F\left(\bar{\boldsymbol{X}}'(f_i),\bar{\boldsymbol{S}}(f_i);\boldsymbol{\delta}(f_i),\boldsymbol{w}(f_i),\mu^2(f_i)\right)}{\partial \boldsymbol{w}(f_i)}\\ &= -2\mu^{-2}(f_i)\left[\left\langle \boldsymbol{\Lambda}^{\mathrm{H}}(f_i)\boldsymbol{\Lambda}(f_i)\right\rangle \boldsymbol{w}(f_i) - \left\langle \boldsymbol{\Lambda}^{\mathrm{H}}(f_i)(\bar{\boldsymbol{X}}'(f_i) - \boldsymbol{A}(f_i,\boldsymbol{\Omega})\bar{\boldsymbol{S}}(f_i))\right\rangle\right]\end{aligned} \tag{21}$$

$$\frac{\partial F(\bar{\boldsymbol{X}}'(f_i),\bar{\boldsymbol{S}}(f_i);\boldsymbol{\delta}(f_i),\boldsymbol{w}(f_i),\mu^2(f_i))}{\partial \mu^2(f_i)} = -\frac{M\times KP}{\mu^2(f_i)} + \frac{1}{(\mu^2(f_i))^2}\left\langle \left\|\bar{\boldsymbol{X}}'(f_i) - \boldsymbol{A}'(f_i,\boldsymbol{\Omega})\bar{\boldsymbol{S}}(f_i)\right\|_2^2\right\rangle \tag{22}$$

$$\frac{\partial F(\bar{\boldsymbol{X}}'(f_i),\bar{\boldsymbol{S}}(f_i);\boldsymbol{\delta}(f_i),\boldsymbol{w}(f_i),\mu^2(f_i))}{\partial \delta_l(f_i)} = -\frac{KP}{\delta_l(f_i)} + \frac{1}{\delta_l^2(f_i)}\left\langle \sum_{kp=1}^{KP} \left|\bar{S}_l(f_i,kp)\right|^2\right\rangle \tag{23}$$

Set them to be 0 respectively, then estimation values of every parameter of the $p$th iteration can be solved

$$\boldsymbol{w}^{(p)}(f_i) = \left\langle \boldsymbol{\Lambda}^{\mathrm{H}}(f_i)\boldsymbol{\Lambda}(f_i)\right\rangle^{-1}\left\langle \boldsymbol{\Lambda}^{\mathrm{H}}(f_i)(\bar{\boldsymbol{X}}'(f_i) - \boldsymbol{A}(f_i,\boldsymbol{\Omega})\bar{\boldsymbol{S}}(f_i))\right\rangle \tag{24}$$

$$\left(\mu^2(f_i)\right)^{(p)} = \frac{1}{M\times KP}\left\langle \left\|\bar{\boldsymbol{X}}'(f_i) - (\boldsymbol{A}'(f_i,\boldsymbol{\Omega}))^{(p)}\bar{\boldsymbol{S}}(f_i)\right\|_2^2\right\rangle \tag{25}$$

$$\delta_l^{(p)}(f_i) = \frac{1}{KP}\left\langle \sum_{kp=1}^{KP} \left|\bar{S}_l(f_i,kp)\right|^2\right\rangle \tag{26}$$

Where $(p)$ denotes number of iterations, after several times, the variations of $\boldsymbol{w}(f_i)$, $\mu^2(f_i)$ and $\delta_l(f_i)$ tend to be zero, then they are deemed to be convergent, we can acquire their final estimation results: $\hat{\boldsymbol{w}}(f_i)$, $\hat{\mu}^2(f_i)$ and $\hat{\delta}_l(f_i)$, combining $\hat{\boldsymbol{\delta}}(f_i) = [\hat{\delta}_1(f_i), \cdots, \hat{\delta}_l(f_i), \cdots, \hat{\delta}_L(f_i)]^{\mathrm{T}}$ and $\hat{\boldsymbol{\Sigma}}(f_i) = \mathrm{diag}\left(\hat{\boldsymbol{\delta}}(f_i)\right)$. They can be used for array calibration, define $\boldsymbol{X}$ as the vector composed by sum of signal of all frequencies, as the signal of every frequency is independent of one another, the joint probability density of $\boldsymbol{X}$ is

$$\begin{aligned}P(\boldsymbol{X}) &= \prod_{i=1}^{J} P\left(\bar{\boldsymbol{X}}'(f_i);\hat{\boldsymbol{\delta}}(f_i),\hat{\boldsymbol{w}}(f_i),\hat{\mu}^2(f_i)\right)\\ &= |\pi|^{-J\times KP}\prod_{i=1}^{J}\left|\left(\hat{\mu}^2(f_i)\boldsymbol{I}_M + \boldsymbol{A}'(f_i,\boldsymbol{\Omega})\hat{\boldsymbol{\Sigma}}(f_i)(\boldsymbol{A}'(f_i,\boldsymbol{\Omega}))^{\mathrm{H}}\right)\right|^{-KP}\\ &\times \exp\left\{-KP\times\sum_{i=1}^{J}\mathrm{tr}\left(\begin{pmatrix}\hat{\mu}^2(f_i)\boldsymbol{I}_M + \boldsymbol{A}'(f_i,\boldsymbol{\Omega})\times\\ \hat{\Sigma}(f_i)(\boldsymbol{A}'(f_i,\boldsymbol{\Omega}))^{H}\end{pmatrix}^{-1}\bar{\boldsymbol{R}}'(f_i)\right)\right\}\end{aligned} \tag{27}$$

Perform logarithm operation on both sides of the (27), we have

$$\begin{aligned}\mathrm{In}(P(\boldsymbol{X})) = {} & -J \times KP \times \mathrm{In}\pi - KP \times \left( \sum_{i=1}^{J} \mathrm{In}\left| \hat{\mu}^2(f_i)\boldsymbol{I}_M + \boldsymbol{A}'(f_i, \boldsymbol{\Omega})\hat{\boldsymbol{\Sigma}}(f_i)(\boldsymbol{A}'(f_i, \boldsymbol{\Omega}))^{\mathrm{H}} \right| \right) \\ & - KP \times \sum_{i=1}^{J} \mathrm{tr}\left( \begin{pmatrix} \hat{\mu}^2(f_i)\mathbf{I}_M + \mathbf{A}'(f_i, \boldsymbol{\Omega}) \times \\ \hat{\Sigma}(f_i)(\mathbf{A}'(f_i, \boldsymbol{\Omega}))^{\mathrm{H}} \end{pmatrix}^{-1} \bar{\mathbf{R}}'(f_i) \right) \end{aligned} \tag{28}$$

Maximize (28), that is

$$\frac{\partial \mathrm{In}(P(\boldsymbol{X}))}{\partial \boldsymbol{\alpha}} = 0 \tag{29}$$

Take (28) into (29) and we can infer

$$\hat{\alpha}_k = \arg\max_{\alpha_k} \left| \mathrm{Re} \left[ \begin{array}{l} \sum_{i=1}^{J} \left[ \left( (\boldsymbol{a}'(f_i, \alpha_k))^{\mathrm{H}} \times \begin{pmatrix} \hat{\mu}^2(f_i)\boldsymbol{I}_M + \boldsymbol{A}'(f_i, \boldsymbol{\Omega}_{-k}) \times \\ \hat{\boldsymbol{\Sigma}}_{-k}(f_i)(\boldsymbol{A}'(f_i, \boldsymbol{\Omega}_{-k}))^{\mathrm{H}} \end{pmatrix}^{-1} \right) \right] \\ \times \left[ \sum_{i=1}^{J} \begin{pmatrix} \boldsymbol{a}'(f_i, \alpha_k)(\boldsymbol{a}'(f_i, \alpha_k))^{\mathrm{H}} \times \\ \left( \begin{pmatrix} \hat{\mu}^2(f_i)\boldsymbol{I}_M + \boldsymbol{A}'(f_i, \boldsymbol{\Omega}_{-k}) \times \\ \hat{\boldsymbol{\Sigma}}_{-k}(f_i)(\boldsymbol{A}'(f_i, \boldsymbol{\Omega}_{-k}))^{\mathrm{H}} \end{pmatrix}^{-1} \bar{\boldsymbol{R}}'(f_i) \right) \end{pmatrix} - \sum_{i=1}^{J} \left( \bar{\boldsymbol{R}}'(f_i) \left( \begin{array}{l} \begin{pmatrix} \hat{\mu}^2(f_i)\boldsymbol{I}_M + \\ \boldsymbol{A}'(f_i, \boldsymbol{\Omega}_{-k})\hat{\boldsymbol{\Sigma}}_{-k}(f_i) \times \\ (\boldsymbol{A}'(f_i, \boldsymbol{\Omega}_{-k}))^{\mathrm{H}} \end{pmatrix}^{-1} \times \\ \boldsymbol{a}'(f_i, \alpha_k)(\boldsymbol{a}'(f_i, \alpha_k))^{\mathrm{H}} \end{array} \right) \right) \right] \\ \times \sum_{i=1}^{J} \left[ \begin{pmatrix} \hat{\mu}^2(f_i)\boldsymbol{I}_M + \boldsymbol{A}'(f_i, \boldsymbol{\Omega}_{-k}) \times \\ \hat{\boldsymbol{\Sigma}}_{-k}(f_i)(\boldsymbol{A}'(f_i, \boldsymbol{\Omega}_{-k}))^{\mathrm{H}} \end{pmatrix}^{-1} \times \frac{\partial \boldsymbol{a}'(f_i, \alpha_k)}{\partial \alpha_k} \right] \end{array} \right] \right|^{-1} \tag{30}$$

Then final result of DOA can be estimated.

We can get $c_1(f_i), \cdots, c_Q(f_i)$ according to $\hat{\boldsymbol{w}}(f_i)$, then $\boldsymbol{W}(f_i)$ can be acquired by (11), then $\boldsymbol{a}'(f_i, \alpha_k)$ and $\boldsymbol{A}'(f_i, \boldsymbol{\Omega}_{-k})$ can be acquired, we will get the accurate estimation of the DOA based on (30) and the parameters above.

The method adapts to wideband signal, and has employed spatial domain sparse optimization for mutual coupling, so we can call it WSM for short.

## 4 Simulations

In order to verify the effective of the method, some simulations are presented with matlab below, consider some wideband chirp signals impinge on a uniform linear array with 8 omnidirectional sensors from directions (5°, 15°, 25°), the center frequency of the signals is 3 GHz, width of the band is 20% of the center frequency, the band is divided into 10 frequencies, and spacing $d$ between adjacent sensors is equal to half of the wavelength of the center frequency, the array errors are related to the signal frequency and very complicated, it is difficult to establish accurate function, therefore we will simplify the process in the simulations, suppose there is mutual coupling error in the array, and it is subject to zero mean Gaussian distribution, the freedom degree among sensors $Q = 2$, mutual coupling perturbation vector $\boldsymbol{w}(f_i) = [a + b\mathrm{j}, c + d\mathrm{j}]^{\mathrm{T}}$, $a$, $b$ is selected between −1 – +1 randomly and $c$, $d$ is selected between −0.5 – +0.5 randomly.

### 4.1 Mutual Coupling Estimation

Suppose SNR is 16 dB, the number of snapshots at every frequency is 40, WSM is employed for estimating the error, 300 Monte-Carlo simulations are repeated, their average values are deemed as the final results, the mutual coupling estimation is shown in Table 1.

**Table 1.** Mutual coupling estimation

| | $c_1$ | $c_2$ |
|---|---|---|
| Actual value of $f_1$ | 0.726 + j0.527 | 0.213 + j0.132 |
| Estimated value of $f_1$ | 0.702 + j0.509 | 0.187 + j0.151 |
| Actual value of $f_2$ | 0.647−j0.234 | 0.286 + j0.172 |
| Estimated value of $f_2$ | 0.667−j0.255 | 0.309 + j0.193 |
| Actual value of $f_3$ | 0.645 + j0.257 | 0.288 + j0.176 |
| Estimated value of $f_3$ | 0.617 + j0.240 | 0.271 + j0.194 |
| Actual value of $f_4$ | 0.742 + j0.218 | 0.385−j0.183 |
| Estimated value of $f_4$ | 0.727 + j0.202 | 0.401−j0.199 |
| Actual value of $f_5$ | 0.969 + j0.316 | 0.380 + j0.188 |
| Estimated value of $f_5$ | 0.977 + j0.322 | 0.374 + j0.197 |
| Actual value of $f_6$ | 0.916 + j0.792 | 0.495 + j0.257 |
| Estimated value of $f_6$ | 0.920 + j0.801 | 0.488 + j0.264 |
| Actual value of $f_7$ | 0.836 + j0.491 | 0.434 + j0.279 |
| Estimated value of $f_7$ | 0.850 + j0.506 | 0.417 + j0.288 |
| Actual value of $f_8$ | 0.758 + j0.343 | 0.392 + j0.156 |
| Estimated value of $f_8$ | 0.777 + j0.331 | 0.408 + j0.140 |
| Actual value of $f_9$ | 0.772 + j0.306 | −0.318−j0.277 |
| Estimated value of $f_9$ | 0.791 + j0.323 | −0.302−j0.299 |
| Actual value of $f_{10}$ | 0.562 + j0.297 | 0.235 + j0.148 |
| Estimated value of $f_{10}$ | 0.540 + j0.321 | 0.252 + j0.175 |

It can be seen from Table 1, the method can effectively estimate the mutual coupling vectors, especially when the frequency is near to the center point, we can use these results to calibrate the array and acquire the actual DOA of the wideband signal.

### 4.2 DOA Estimation

First, traditional two-sided correlation transformation (TCT) [18] and WSM are employed for estimating DOA of wideband signals along with the mutual coupling, here, TCT is performed without correction, 300 Monte-Carlo simulations are repeated, their average values are deemed as the final results. Suppose snapshots is 40, other conditions are the same with 4.1, the root mean square error (RMSE) versus SNR are shown in Fig. 2; then suppose SNR is 12 dB, the RMSE versus number of snapshots are shown in Fig. 3.

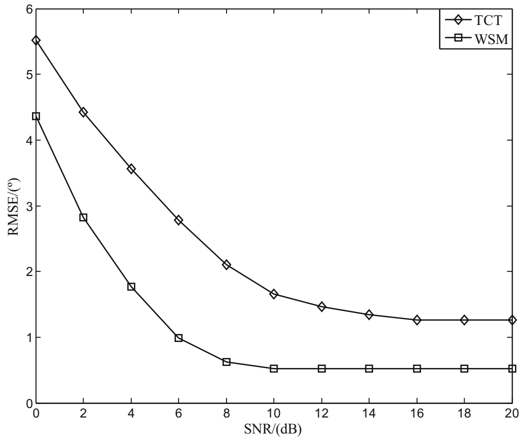


**Fig. 2.** Calibration accuracy versus SNR

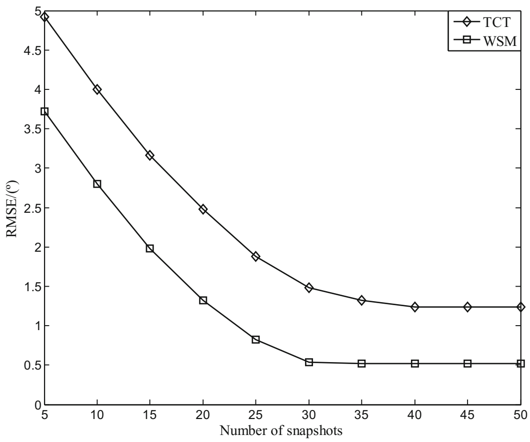


**Fig. 3.** Calibration accuracy versus number of snapshots

It can be seen from Figs. 2 and 3, WSM method can effectively estimate the DOA of wideband signals along with the mutual coupling existing in the array, when the SNR or number of snapshots increase to some threshold, the estimation error approximately converges to 0.52°, but that of the traditional TCT method without correction converges to 1.2° under the same condition.

## 5 Conclusion

The paper proposed a novel array calibration method in super-resolution direction finding for wideband signals based on spatial domain sparse optimization to the mutual coupling existing in the array, it can calibrate the array and estimate the DOA relatively accurately.

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