An Efficient Nonparametric Belief Propagation-Based Cooperative Localization Scheme for Mobile Ad Hoc Networks

Chaojie Xu(✉), Hui Yu, and Ming Yang

Department of Electronic Engineering, Shanghai Jiao Tong University, Shanghai, People's Republic of China
{xuchaojie,yuhui}@sjtu.edu.cn, msangel0706@foxmail.com

Abstract. In mobile ad hoc networks, nonparametric belief propagation (NBP) algorithm is a promising cooperative localization scheme because of high accuracy, applicability to non-Gaussian uncertainty. However, the high computational cost limits the application of NBP. To solve the problem, an efficient and practical NBP-based cooperative localization scheme is proposed. In the scheme, the issues of anchor node selection, node mobility and non-Gaussian uncertainty are considered. Firstly, anchor nodes are selected based on a distributively clustered network. Then the cooperative localization process is performed, in which a practical ranging error model is employed. Moreover, to mitigate the influence of node mobility, the re-selection process of anchor nodes is conducted when necessary. The simulation results demonstrate the efficiency of the proposed scheme in improving the positioning accuracy and reducing the computational cost compared with the conventional NBP method.

Keywords: Mobile ad hoc networks · NBP · Anchor node selection · Ranging error model

1 Introduction

In mobile ad hoc networks (MANETs), accurate positioning information is crucial since it enables a wide variety of applications, such as emergency services, first responders operations and factory automation [1,2]. In typical localization schemes, nodes in a MANET can be divided into anchor nodes, which have known positions and account for a small proportion in the nodes, and agent nodes that need to be located by utilizing the information from anchor nodes.

Generally, the existing range-based localization schemes can be classified into non-cooperative schemes and cooperative schemes. In the non-cooperative schemes [1], an agent node is located only depending on the measured distances with neighboring anchor nodes. For the cooperative schemes, by contrast, agent nodes estimate their positions through ranging and exchanging information with neighboring nodes, including anchor nodes and other agent nodes. Cooperation

Q. Chen et al. (Eds.): ChinaCom 2016, Part II, LNICST 210, pp. 519–528, 2018.
DOI: 10.1007/978-3-319-66628-0_49

among the agent nodes is highly beneficial for improving the performance of localization processes on accuracy and coverage [1]. Lately, extensive works have been focused on cooperative localization [3–7], and most of these schemes are based on belief propagation (BP) algorithm and its extension algorithms [3,6] for high accuracy and distributed implementation. BP is an efficient message-passing method of estimating the a posterior marginal probability density function (PDF) for the positions of the agent nodes in the network, but the inability in resolving non-Gaussian uncertainty, which is a common occurrence in practical localization scenarios, limits the application of BP. As an extension of BP, nonparametric belief propagation (NBP) algorithm [3] is sample-based and can be applied in the positioning systems with non-Gaussian uncertainties. However, considerable complexity and communication overhead are associated with the employment of NBP in MANETs. Many works have been proposed to reduce the complexity of NBP. In [4], a minimum spanning tree approach is proposed to mitigate the influence of loops in message passing process. In [5], the communication overhead and the computational cost are reduced by passing approximate beliefs represented by Gaussian distributions. In addition, a space-time hierarchical-graph model is proposed in [6] that messages propagates by layers to achieve decrease in computational complexity.

However, many of the proposed NBP-based schemes are validated with simplified assumptions such as static network [4] and Gaussian uncertainty [7]. In MANETs, where nodes are all mobile and randomly deployed, achieving a reasonable distribution of anchor nodes is advantageous in enhancing the performance of localization processes [8] and should be performed in distributed way. Moreover, considering a practical ranging error model is necessary and of great importance when designing an efficient and practical localization scheme [5].

To address the problems mentioned, an efficient and practical NBP-based cooperative localization scheme is proposed. In the scheme, anchor node selection is considered firstly, nodes in the network are aggregated into clusters in a distributed manner and anchor nodes are selected based on the established clusters. Then the cooperative localization process is performed. Considering the influence of node mobility, the re-selection process of anchor nodes is conducted when needed. Furthermore, a practical ranging error model is employed in the scheme with the propose of enhancing the performance of the localization process. The results of the simulations verify that the proposed scheme can significantly improve the positioning accuracy and evidently reduce the computational cost of the localization process in comparison with the conventional NBP algorithm.

2 Preliminaries

2.1 Ranging Error Model

In range-based positioning systems, ranging errors usually obey some kind of distribution. Reasonable modeling of the ranging errors can be beneficial in mitigating the influence of ranging noises. However, many existing localization schemes are validated using simulations based on simplified ranging error models such as the Gaussian ranging error model. In [5], based on the collected ranging data from a real positioning system, a range-dependent asymmetric double exponential model is proposed. And in our previous experimental work, we observed a kind of similar distribution, which is strictly non-Gaussian and long tailed on the right hand side, when using an ultra-wideband based positioning system. In this paper, the ranging error model proposed in [5] is adopted.

2.2 Network Model

We consider a two-dimensional MANET composing of N mobile nodes (N_a anchor nodes and N_t agent nodes), which are randomly deployed. Initial position of each node is known and the position of node n_u is denoted as $\mathbf{x}_u$=$[x_u, y_u]^T$.

Considering the transmission radius R of each node and the actual distance between nodes, $P_o(\mathbf{x}_u, \mathbf{x}_{u'})$ can be achieved to denote the probability of whether n_u and $n_{u'}$ can detect each other or not

$$P_o(\mathbf{x}_u, \mathbf{x}_{u'}) = \exp\left(-||\mathbf{x}_u - \mathbf{x}_{u'}||^2/2R^2\right), \tag{1}$$

where $||\mathbf{x}_u - \mathbf{x}_{u'}||$ is the Euclidean distance between n_u and $n_{u'}$. We use a binary variable $o_{uu'} = 1$ to denote the situation that n_u and $n_{u'}$ can detect each other and are neighbor nodes, then a noisy distance measurement can be obtained

$$d_{uu'} = ||\mathbf{x}_u - \mathbf{x}_{u'}|| + v_{uu'}, \tag{2}$$

where $v_{uu'}$ is the range error. Following [3], potential functions are used to represent the joint posterior PDF for the locations of all the nodes. For n_u, the single potential function $\psi_u(\mathbf{x}_u)$ is defined as corresponding a prior distribution $p(\mathbf{x}_u)$. If $o_{uu'} = 1$, the pairwise potential function defined over n_u and $n_{u'}$ is given by

$$\psi_{uu'}(\mathbf{x}_u, \mathbf{x}_{u'}) = P_o(\mathbf{x}_u, \mathbf{x}_{u'}) p_v(d_{uu'}, d_{uu'} - ||\mathbf{x}_u - \mathbf{x}_{u'}||), \tag{3}$$

where p_v is the ranging error model. Only the single-hop neighbor nodes are considered in localization process. Therefore, when $o_{uu'} = 0$, corresponding $\psi_{uu'}(\mathbf{x}_u, \mathbf{x}_{u'}) = 0$. Then the joint posterior PDF for the locations of all the nodes is denoted as

$$p(\mathbf{x}_1, ..., \mathbf{x}_N | \{o_{uu'}, d_{uu'}\}) \propto \prod_{u=1}^{N} \psi_u(\mathbf{x}_u) \prod_{u' \in \Gamma_u, u} \psi_{uu'}(\mathbf{x}_u, \mathbf{x}_{u'}), \tag{4}$$

where Γ_u denotes the neighbor node set of node n_u. For each node n_u, by marginalizing this PDF, we can obtain the corresponding position which is characterized by the posterior marginal PDF $p(\mathbf{x}_u | \{d_{uu'}\})$, where $\{d_{uu'}\}$ is the set of the distances between node n_u and its neighbor nodes.

3 Proposed Scheme

3.1 Anchor Node Selection Phase

In the network, the distribution of anchor nodes influences the performance of localization process [1]. For the previously mentioned MANETs, the issue of selecting anchor nodes with reasonable distribution is quite challenging, especially without a central controller of the whole network.

Cluster Formation Process. To achieve a reasonable distribution of anchor nodes, firstly, all the nodes in the network are clustered distributively. Affinity propagation clustering algorithm [9] is a distributed clustering scheme and only relies on the similarity $s(u, u')$ between node $n_{u'}$ and node n_u. The negative value of the distance between n_u and $n_{u'}$ is used as $s(u, u')$. After the clustering process, all the nodes are aggregated into several clusters, and the cluster header node (CH) of each cluster is responsible for anchor node selection process.

Anchor Node Selection Process. In [8], an optimal anchor node selection algorithm is proposed to select three existing anchor nodes for each agent node. Inspired by this work, based on the clustered network, we consider selecting three nodes implementing an approximate regular triangle in each cluster to act as anchor nodes. The selection process is conducted by CH of each cluster and bases on the gathered positions of the nodes in the cluster.

In cluster C, the number of nodes is N_C, C_i $(i = 1, ..., N_C)$ represents the i-th node, the centroid is represented as o and d_{io} denotes the distance between C_i and o. In addition, the set of the distance between o and each node in C is denoted as S_d, the median value and the maximum value in S_d are d_m and d_{max}.

In each cluster C, if the distance between o and node C_i is between d_m and $(d_m + d_{max})/2$, node C_i is selected. Any three selected nodes C_{i_1}, C_{i_2}, C_{i_3} can form a triangle, whose area can be calculated with Heron's formula

$$Area_{tri} = \sqrt{p(p - d_{i_1 i_2})(p - d_{i_2 i_3})(p - d_{i_3 i_1})}, \tag{5}$$

where $p = (d_{i_1 i_2} + d_{i_2 i_3} + d_{i_3 i_1})/2$; $d_{i_j i_k}$ $(j, k = 1,\ 2,\ 3,\ j \neq k)$ denotes the distance between node C_{i_j} and C_{i_k}. Suppose a triangle, the distances between o and its vertexes are assigned as $d_{i_1 o}$, $d_{i_2 o}$, $d_{i_3 o}$, keeping o inside the triangle, the maximum area of the triangle can be achieved when it is a regular triangle

$$Area_{max} = \frac{3\sqrt{3}}{4} \times (\frac{d_{i_1 o} + d_{i_2 o} + d_{i_3 o}}{3})^2. \tag{6}$$

To evaluate how approximate the triangle composed of C_{i_1}, C_{i_2}, C_{i_3} is to the equilateral triangle, approximation ratio λ is introduced and denoted as

$$\lambda = Area_{tri}/Area_{max}, \tag{7}$$

the three nodes with the largest λ are selected as anchor nodes.

Anchor Node Re-selection Process. Since nodes in the network keep moving, the topology of the network is influenced dynamically and randomly. From the perspective of positioning accuracy, anchor nodes may not be suitable for the localization process all the time, the anchor node re-selection process should be introduced to mitigate the influence of node mobility. When the number of agent nodes, which are not well located in the localization process, reaches a predefined threshold, the re-selection process of anchor nodes begins. In the process, based on the estimated locations of agent nodes and the real positions of anchor nodes, clustering process is performed again and new anchor nodes are selected using the anchor node selection algorithm.

3.2 NBP-Based Cooperative Localization Scheme

NBP Implementation. With the defined statistical framework in Section II and the selected anchor nodes, NBP is utilized to estimate the locations of the agent nodes with two updating rules, namely the belief updating rule and the message updating rule. The belief (or the estimated posterior distribution of the position) of agent node n_t in the l-th iteration is computed by taking a product of the local potential $\psi_t(\mathbf{x}_t)$ with the messages from the neighbor nodes participating the localization process of n_t

$$b_t^{(l)}(\mathbf{x}_t) \propto \psi_t(\mathbf{x}_t) \prod\nolimits_{n_u \in \Gamma_t} m_{ut}^{(l)}(\mathbf{x}_t), \tag{8}$$

where Γ_t denotes the neighbor nodes participating the localization process of n_t; $m_{ut}^{(l)}(\mathbf{x}_t)$ is the message from neighbor node n_u, which can be anchor node or agent node, to n_t. In the l-th iteration, the message sent from agent node $n_{t'}$ is

$$m_{t't}^{(l)}(\mathbf{x}_t) \propto \sum_{\mathbf{x}_{t'}} \psi_{tt'}(\mathbf{x}_t, \mathbf{x}_{t'}) \frac{b_{t'}^{(l-1)}(\mathbf{x}_{t'})}{m_{tt'}^{(l-1)}(\mathbf{x}_{t'})}, \tag{9}$$

and the message from anchor node n_a is given by $m_{at}^{(l)}(\mathbf{x}_t) \propto \psi_{at}(\mathbf{x}_a, \mathbf{x}_t)$.

In NBP, stochastic approximations are used when computing the belief and the message: for node n_u $(n_u \in \Gamma_t)$, firstly, samples are drawn from the belief $b_u^{(l-1)}(\mathbf{x}_u)$, and these samples are used to approximate the message $m_{ut}^{(l)}(\mathbf{x}_t)$ sent to agent node n_t. In the l-th iteration, weighted samples $\{\mathbf{x}_u^{(lj)}, \omega_u^{(lj)}\}_{j=1}^{M}$ are drawn from the belief $b_u^{(l-1)}(\mathbf{x}_u)$, each sample $\mathbf{x}_u^{(lj)}$ is moved in a random direction $\theta_{ut}^{(lj)}$ by a noisy measurement $d_{ut}^{(lj)}$ of the distance between n_u and n_t

$$\mathbf{x}_{ut}^{(lj)} = \mathbf{x}_u^{(lj)} + d_{ut}^{(lj)} \cdot [sin(\theta_{ut}^{(lj)}), cos(\theta_{ut}^{(lj)})]^T, \tag{10}$$

where $d_{ut}^{(lj)} = \|\mathbf{x}_u - \mathbf{x}_t\| + v_{ut}^{(lj)}$, $v_{ut}^{(lj)} \sim p_v$; $\theta_{ut}^{(lj)} \sim U[0, 2\pi]$. The weight of $\mathbf{x}_{ut}^{(lj)}$ is

$$\omega_{ut}^{(lj)} = \frac{\omega_u^{(lj)} P_o(\mathbf{x}_u^{(lj)}, \mathbf{x}_t)}{m_{tu}^{(l-1)}(\mathbf{x}_u^{(lj)})}. \tag{11}$$

Modifications Based on NBP. To decrease the complexity of NBP, modifications are considered. In the localization process of agent node n_t, only when there are no less than three reference nodes (anchor nodes or located agent nodes),

Algorithm 1. NBP-based cooperative localization scheme for agent nodes

Input: Γ_r: the reference node set of n_t;
$\{d'_{r_i t}\}_{i=1}^{|\Gamma_r|}$: range measurements between n_t and reference nodes.
Output: $\widehat{\mathbf{x}}_t$: the final estimated position of n_t.
Initialization: Set $[\theta_{min}^{r_i,1}, \theta_{max}^{r_i,1}]$ for reference node n_{r_i} $(i = 1, ..., |\Gamma_r|)$ as $[0, 2\pi]$;
for *iteration* $l = 1$ *to* L **do**
  Message computing in each reference node n_{r_i}:
  Draw random values $\{v_{r_i t}^{(lj)}, \theta_{r_i t}^{(lj)}\}_{j=1}^{M}$: $v_{r_i t}^{(lj)} \sim p_v$, $\theta_{r_i t}^{(lj)} \sim U[\theta_{min}^{r_i,l}, \theta_{max}^{r_i,l}]$;
  Calculate $\mathbf{x}_{r_i t}^{(lj)}$ with (12), and set corresponding weight $\omega_{r_i t}^{(lj)}$ as $1/M$;
  Broadcast message $\{\mathbf{x}_{r_i t}^{(lj)}, \theta_{r_i t}^{(lj)}, \omega_{r_i t}^{(lj)}\}_{j=1}^{M}$ to n_t;
  Belief computing in n_t:
  for *each sample* $\{\mathbf{x}_{r_i t}^{(lj)}, \theta_{r_i t}^{(lj)}, \omega_{r_i t}^{(lj)}\}$ *received by* n_t **do**
    Update the weight $\omega_{r_i t}^{(lj)}$ with (13);
  for $i =$ *1 to* $|\Gamma_r|$ **do**
    Filter $M/|\Gamma_r|$ samples with maximum weights from samples from n_{r_i};
    Get the range $[\theta_{min}, \theta_{max}]$ of the directions in reserved samples;
    Update $[\theta_{min}^{r_i,l+1}, \theta_{max}^{r_i,l+1}]$ of n_{r_i} as $[\theta_{min}, \theta_{max}]$;
  Normalize the weights of remaining samples with $\omega_t^{(lk)} = \omega_t^{(lk)} / \sum_{k=1}^{M} \omega_t^{(lk)}$;
  Update the belief $b_t^{(l)}$ of n_t using (8);
  Calculate the estimated position $\widehat{\mathbf{x}_t^{(l)}}$ of n_t with (15);
  if $l > 1$ **then**
    Check the convergence condition using (16);
    if *converged or* $l == L$ **then**
      Set $\widehat{\mathbf{x}_t^{(l)}}$ as the final estimated position $\widehat{\mathbf{x}}_t$;
      Terminate the iteration process.

in the neighboring node set Γ_t of n_t, can n_t locate itself by ranging with the reference nodes, and it will become a reference node for other agent nodes when well located. In this way, all the agent nodes are located incrementally.

Specifically, the generation principle (10) of samples is changed as

$$\mathbf{x}_{rt}^{(lj)} = \mathbf{x}_r + d_{rt}^{(lj)} \cdot [sin(\theta_{rt}^{(lj)}), cos(\theta_{rt}^{(lj)})]^T, \tag{12}$$

where if reference node n_r is an anchor node, $\mathbf{x}_r$ is the real position of n_r, and otherwise $\mathbf{x}_r$ is the estimated position. Through storing the random direction $\theta_{rt}^{(lj)}$ with $\mathbf{x}_{rt}^{(lj)}$ and initialising the weight $\omega_{rt}^{(lj)}$ of $\mathbf{x}_{rt}^{(lj)}$ as $1/M$, samples $\{\mathbf{x}_{rt}^{(lj)}, \theta_{rt}^{(lj)}, \omega_{rt}^{(lj)}\}_{j=1}^{M}$ can be achieved.

For each weighted sample $\{\mathbf{x}_{rt}^{(lj)}, \theta_{rt}^{(lj)}, \omega_{rt}^{(lj)}\}$ generated by n_r, the weight $\omega_{rt}^{(lj)}$ is updated by the deviation degrees between $\mathbf{x}_{rt}^{(lj)}$ and the real position of n_t evaluated by other reference nodes of n_t. The update principle is

$$\omega_{rt}^{(lj)} = \omega_{rt}^{(lj)} \prod_{n_{r'} \in \Gamma_r \setminus n_r} p_v(r't), \tag{13}$$

where Γ_r is the reference node set of n_t; $p_v(r't)$ is given by

$$p_v(r't) = p_v(d'_{r't}, d'_{r't} - d''_{r't}), \tag{14}$$

where $d'_{r't}=||\mathbf{x}_{rt}^{(lj)}-\mathbf{x}_{r'}||$ and $d''_{r't}$ is a noisy measurement of the distance between n_t and reference node $n_{r'}$, and regarded as the real distance between them for further evaluation. To evaluate the deviation degree between $\mathbf{x}_{rt}^{(lj)}$ and the real position $\mathbf{x}_t$ of n_t, firstly, $\mathbf{x}_{rt}^{(lj)}$ is assumed as $\mathbf{x}_t$, then $d'_{r't}$ can be considered as a distance measurement between $n_{r'}$ and n_t, thus $d'_{r't}-d''_{r't}$ is ranging error of $d'_{r't}$ and $p_v(r't)$ denotes the deviation degree between $\mathbf{x}_{rt}^{(lj)}$ and $\mathbf{x}_t$ evaluated by $n_{r'}$.

With all the samples received by n_t, sample filtering process is conducted. For the M samples generated by n_r in this iteration, $M/|\Gamma_r|$ samples with the maximum weights are reserved, where $|\Gamma_r|$ is the number of nodes in Γ_r. Through recording the random directions of the reserved samples generated by n_r in this iteration, a direction range S_θ can be achieved, which will be the random direction range for n_r to generate samples of n_t in the next iteration. For n_t, through normalizing the weights of the remaining samples $\{\mathbf{x}_t^{(lk)}, \theta_t^{(lk)}, \omega_t^{(lk)}\}_{k=1}^M$, the estimated position in this iteration is calculated with

$$\widehat{\mathbf{x}_t^{(l)}} = \sum\nolimits_{k=1}^{M} \omega_t^{(lk)} \mathbf{x}_t^{(lk)}. \tag{15}$$

The iteration process terminates when convergence condition is met or maximum number L of the iterations is reached, and the convergence condition is

$$\|\widehat{\mathbf{x}_t^{(l)}} - \widehat{\mathbf{x}_t^{(l-1)}}\| \leq \varepsilon, \tag{16}$$

where ε is a predefined threshold. In the iteration process of agent node n_t, if the convergence condition is met, which indicates that n_t is well located and it will become a reference node for other unlocated agent nodes. And the final estimated position of n_t is assigned as the estimation of current iteration. Otherwise, n_t is unlocated in the localization process of current time slot and the estimation of the last iteration will be the final estimated position of n_t. The detailed NBP-based cooperative localization process is summarized in Algorithm 1.

4 Simulation Results

4.1 Simulation Setup

In the simulations, we consider a $100 \times 100\,\text{m}^2$ area with 150 nodes, including 18 anchor nodes (i.e., 6 clusters are established). And we assume that the movement of each node follows the Gaussian-Markov mobility model [10]. Table 1 lists the key parameters used in the simulations.

Results of the simulations are obtained from the localization process of the agent nodes in 500 continuous time slots.

Table 1. The key parameters in simulations

Parameter	Value
R: transmission radius	30 m
L: maximum number of iteration	10
ε: convergence threshold	0.1 m
Condition of anchor node re-selection	5 unlocated agent nodes

4.2 Performance Evaluation

Influence of Anchor Node Distribution. Three kinds of anchor node distribution are considered. The uniform distribution means that we choose the nodes that distributing in approximately uniform way in the network as anchor nodes, and the random distribution denotes that anchor nodes are randomly selected.

The performance on the positioning error of all the agent nodes in the network is valued by the root mean square error (RMSE)

$$\mathrm{RMSE} = \sqrt{\frac{1}{N_t} \sum\nolimits_{n_t \in S_t} \|\mathbf{x}_t - \widehat{\mathbf{x}}_t\|^2}, \tag{17}$$

where N_t is the number of agent nodes in the network; S_t represents the agent node set; $\widehat{\mathbf{x}}_t$ denotes the final estimated position of agent node n_t. Figure 1 shows the cumulative distribution functions (CDFs) of the RMSE performance of NBP with 200 samples based on the three kinds of anchor node distribution, we can see that compared with the random distribution case, the performance of NBP based on the proposed distribution is very close to the performance of the uniform distribution case, which can be regarded as the optimal distribution. The result indicates that the proposed anchor node selection algorithm can achieve a reasonable distribution of anchor nodes for the NBP-based localization schemes.

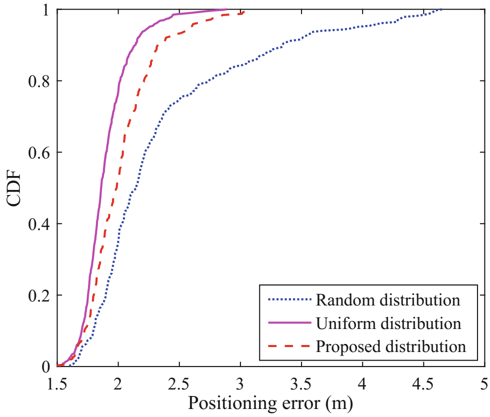

Fig. 1. Performance comparison on positioning error (RMSE) of NBP with different kinds of anchor node distribution.

Positioning Performance. Figure 2 shows the CDFs of the positioning error in the proposed scheme and NBP with different numbers of samples. The result reveals that more samples can improve the RMSE performance of both the proposed scheme and NBP. With same number of samples, the performance on positioning accuracy of the proposed scheme outperforms that of NBP. Compared with NBP, the change of sample number has less impact on the performance on positioning accuracy of the proposed scheme, and the proposed scheme with less samples can achieve a better performance on positioning accuracy.

Complexity Comparison. The performance on time complexity of the proposed scheme and NBP is evaluated by the normalized CPU running time. Figure 3 shows the comparison of the computational cost of the proposed scheme

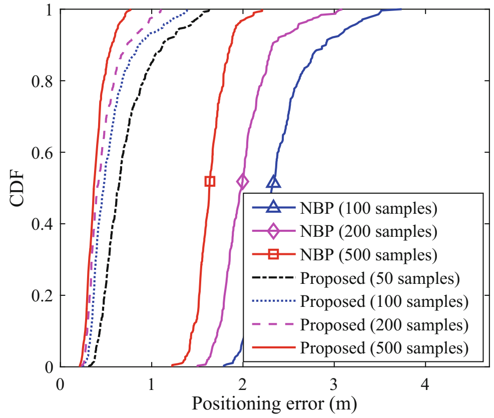

Fig. 2. Performance comparison on positioning error (RMSE) of the proposed scheme and NBP with different numbers of samples.

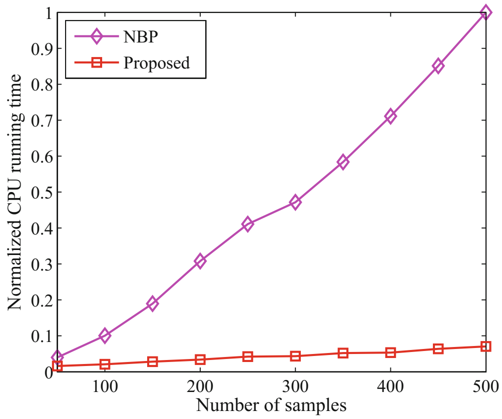

Fig. 3. Performance comparison on computational cost of the proposed scheme and NBP with different numbers of samples.

and NBP with different numbers of samples. Compared with NBP, improved performance on the computational cost of the proposed scheme can be clearly observed, which demonstrates the low computational cost of the proposed scheme.

5 Conclusions

This paper focuses on solving the problems of applying NBP in MANETs, where nodes are randomly deployed and keep moving, and proposes an efficient and practical NBP-based scheme. The proposed scheme considers the issues of anchor node selection, node mobility and non-Gaussian uncertainty to obtain a better performance of the localization process. Specially, anchor nodes are firstly selected based on the clustered network, which is established in a distributed way, then the cooperative localization process is conducted. And a practical ranging error model is adopted in the scheme. Furthermore, to tackle the issue of node mobility, a re-selection process of anchor nodes is conducted when necessary. The simulation results reveal that the proposed scheme has a significant effect on improving the positioning accuracy and reducing the computational cost of localization process compared with the traditional NBP algorithm.

References

1. Zekavat, R., Buehrer, R.M.: Handbook of Position Location: Theory, Practice and Advances. Wiley, Hoboken (2012)
2. Wymeersch, H., Jaime, L., Moe, Z.W.: Cooperative localization in wireless networks. Proc. IEEE **97**(2), 427–450 (2009)
3. Ihler, A.T., Fisher, J.W., Moses, R.L., Willsky, A.S.: Nonparametric belief propagation for self-localization of sensor networks. IEEE J. Sel. Areas Commun. **23**(4), 809–819 (2005)
4. Li, X., Gao, H., Cai, H., Lv, T.: Nonparametric belief propagation based cooperative localization: a minimum spanning tree approach. In: IEEE Wireless Communications and Networking Conference (WCNC), pp. 1775–1780. IEEE Press (2015)
5. Li, S., Hedley, M., Collings, I.B.: New efficient indoor cooperative localization algorithm with empirical ranging error model. IEEE J. Sel. Areas Commun. **33**(7), 1407–1417 (2015)
6. Lv, T., Gao, H., Li, X., Yang, S., Hanzo, L.: Space-time hierarchical-graph based cooperative localization in wireless sensor networks. IEEE Trans. Signal Process. **64**(2), 322–334 (2016)
7. Van de Velde, S., Arora, G., Vallozzi, L., Rogier, H., Steendam, H.: Cooperative hybrid localization using Gaussian processes and belief propagation. In: IEEE International Conference on Communication Workshop (ICCW), pp. 785–790. IEEE Press (2015)
8. Wang, X., Qiu, J., Fan, J., Dai, G.: MDS-based localization scheme for large-scale WSNs within sparse anchor nodes. In: IEEE International Conference on Communications (ICC), pp. 6609–6614. IEEE Press (2015)
9. Frey, B.J., Dueck, D.: Clustering by passing messages between data points. science **315**(5814), 972–976 (2007)
10. Liang, B., Haas, Z.J.: Predictive distance-based mobility management for multidimensional PCS networks. IEEE/ACM Trans. Netw. **11**(5), 718–732 (2003)