

# A Modified LFF Method for Direct P-Code Acquisition in Satellite Navigation

Xinpeng Guo<sup>(✉)</sup>, Hua Sun, Hongbo Zhao, and Wenquan Feng

Beihang University, Xueyuan Road No. 37, Haidian District, Beijing, China  
buaagxp@126.com, bhzhb@126.com,  
{sun, buaafwq}@buaa.edu.cn

**Abstract.** Due to the high dynamic in Satellite Navigation, the vital restriction for P-code Acquisition should be the Doppler frequency offset and large uncertainty of P code. To speed up the P-code acquisition, this paper proposed a novel acquisition method, based on the Local Frequency Folding method (LFF), by folding local frequency cells and generating code with little burden increase. Meanwhile, the coherent integration results storing structure of the method was modified to fit the parallel non-coherent integration, which accelerated the detection process of P-code acquisition. Preliminary result shows that the mean acquisition reduces significantly, with only  $-2$  dB degradation in detection performance. Furthermore, it can be eliminated when the SNR exceed  $-10$  dB.

**Keywords:** P-code acquisition · Satellite communication · Mean acquisition time

## 1 Introduction

The direct sequence spread spectrum (DSSS) communication is widely used in Global navigation satellite system. To receive the signal successfully, the Doppler frequency offset and uncertainty of the code phase, which caused by the relative motion between satellites and earth, should be determined by acquisition at a receiver. To achieve the quick acquisition, the downlink information is modulated by C/A code because of its short code period. However, the C/A code could be jammed and spoofed easily. Considering the security, the important link such as the uplink and the military link should take the advantages of precision code (P-code) to modulate the information. The code-searching range of P-code acquisition is much longer than C/A code. Therefore the traditional acquisition method is inapplicable to P-code. Meanwhile the high dynamic phenomenon is existent generally. It is necessary to propose practical direct P-Code acquisition method in high dynamic.

To accelerate the process of P-code searching, some direct P-Code acquisition method were proposed [1, 3]. The Extended Replica Folding Acquisition Search Technique (XFAST) was discussed widely [2], which extended and folded local generated code to achieve parallel code-searching. However because of the coherent combining loss caused by the large Doppler offset, the acquisition performance improvement of the XFAST was still subject to frequency-search. Recently Local Frequency Folding method (LFF) was introduced which folded local frequency cells

together [4]. The LFF method solved the problem of coherent combining loss without increasing the acquisition time. However, because the LFF method focused on the efficiency of frequency-searching, the acquisition process with linear code-searching became laborious when the P-code period was very long. As solution to this problem, the Two-dimension Folding (TDF) method was presented in this paper which combined the LFF method with XFAST to accelerate the process of code-searching of the LFF method. Meanwhile a novel storing and detection method was proposed for the complex coherent integration, which could accurate the detecting process significantly.

The remaining sections of this paper are organized as follows: Sect. 2 analyzes the improvement of LFF method in parallel Doppler frequency search. Section 3 presents the TDF method. The acquisition performance in terms of detection probability and mean acquisition time is described in Sects. 4 and 5 demonstrates numerical results about some important factors. Finally, the conclusion is given in Sect. 6.

## 2 Analysis of the LFF Method

The LFF method achieves the parallel Doppler frequency search in high dynamic environment by folding local blocks in frequency domain. However there is lack of research in the LFF method, only the detection probability curve and the mean acquisition time are given. This paper will analyze the improvement of LFF method regarding to the coherent combining loss which is the point of achieving the parallel search.

The radio frequency (RF) signal is processed by the RF module in receiver, and the intermediate frequency (IF) signal down-sampled to twice chip rate are multiplied by local carrier. Than the base-band signal with the Doppler frequency offset is expressed as:

$$S(k) = D_i(k)P(k)e^{[2\pi(f_d/f_s)k + \varphi_i]j} + n_k, k = 1, 2, \dots \quad (1)$$

where the  $D_i(k)$  represents the data in modulated signal.  $P(k)$  is the P-code.  $f_d$  and  $f_s$  represent the Doppler frequency offset and twice chip rate sampling frequency.  $n_k$  is the additive White Gaussian Noise (AWGN):

Traditional acquisition method do the coherent integration between the receiving signal and local generated code. Assume that the  $D_i(k)$  will not change in the coherent integration process and the code delay is zero. The result is:

$$\begin{aligned} I_{fd} &= D_i(k_0) \sum_{k=k_0}^{k_0+N-1} \cos(2\pi(f_d/f_s)k + \varphi_i) + \sum_{k=k_0}^{k_0+N-1} n_k P_0(k) \\ &= \frac{D_i(k_0) \sin(\pi f_d N / f_s)}{\pi f_d N / f_s} \cos(2\pi f_d \frac{k_0+N/2}{f_s} + \varphi_i) + \sum_{k=k_0}^{k_0+N-1} n_k P_0(k) \end{aligned} \quad (2)$$

In the Eq. 3,  $P_0(k)$  represent the local generated code.  $N$  is the integration point. If we designate  $\sum_{k=k_0}^{k_0+N-1} n_k P_0(k) = N_k$ , then:

$$I_{f_d} = D_i(k_0) \text{sinc}(f_d N / f_s) \cos(2\pi f_d \frac{k_0 + N/2}{f_s} + \varphi_i) + N_k \quad (3)$$

The amplitude of  $I_{f_d}$  submits to  $\text{sinc}(f_d N / f_s)$ . When  $f_d N / f_s = K$ ,  $K = 1, 2, 3, \dots$ ,  $\text{sinc}(f_d N / f_s) = 0$ , the gain of coherent integration disappeared. And the peak of the correlation is lower than the noise floor [5]. The situation will happen when the Doppler frequency offset is about 10 kHz. So the Doppler frequency space is divided to cells, which is tested sequentially to avoid the situation happening, but acquisition speed is decelerated.

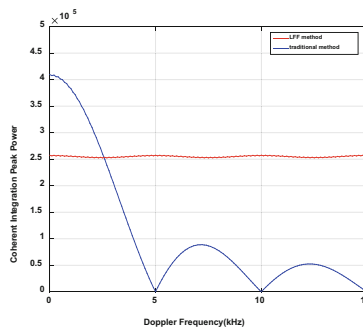
The LFF method folds the frequency cells to solve the problem. The Doppler frequency lies within  $[-f_{d\max}, f_{d\max}]$ , then the frequency cell can be expressed as  $f_{\delta} = -f_{d\max} + \delta f_s / 2N$ ,  $\delta = 1, 2, 3, \dots, F$  and the folding times  $F = 4f_{d\max} N / f_s$ . After adding all frequency cells and sampled, the result modulated by local generated P-code is given, where we designate  $\theta_k = \frac{2\pi f_{d\max} k}{f_s}$ :

$$L(k) = P_0(k) \sum_{\delta=1}^F e^{-j\frac{2\pi f_{\delta} k}{f_s}} = P_0(k) (\cos \theta_k + \sin \theta_k \cot \frac{\theta_k}{F}) = P_0(k) m(k), \quad k = 1, 2, 3, \dots, N \quad (4)$$

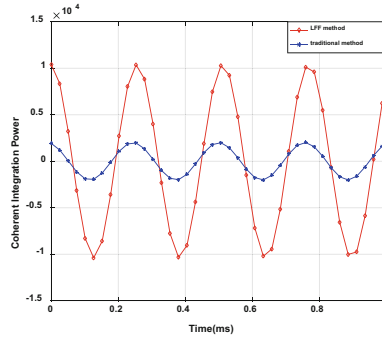
The traditional local generated code is replaced by the new sequence  $L(k)$ , where the  $m(k)$  can be generated through two numerically controlled oscillators (NCO). So the coherent integration result between the incoming signal and the new sequence is given, we designate  $N_{m,k} = \sum_{k=k_0}^{k_0+N-1} n_k P_0(k) m(k)$ :

$$I_{m,f_d} = D_i(k_0) \sum_{k=k_0}^{k_0+N-1} e^{[2\pi(f_d/f_s)k + \varphi_i]j} m(k) + N_{m,k} = D_i(k_0) R_{I,k_0} + N_{m,k} \quad (5)$$

The numerical simulation results of  $R_{I,k_0}$  is given in Figs. 1 and 2. The results show that the correlation peak of LFF method is more consistent than traditional method in



**Fig. 1.** The correlation peaks of LFF method and traditional method in different Doppler frequency.



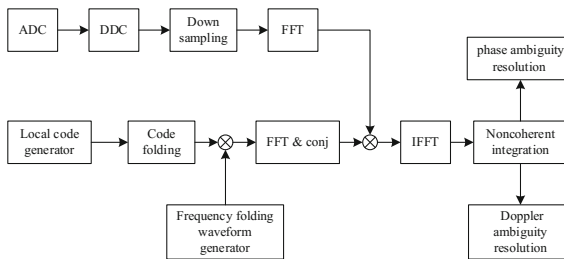
**Fig. 2.** The waves of coherent integration power of LFF method and traditional method when the Doppler frequency is 4 kHz.

different Doppler frequencies and the amplitude of the correlation peak is sufficient. Meanwhile the Doppler frequency of incoming signal can be detected through the coherent integration result by Fig. 2. The simulation result proves that the LFF method can be used to search the Doppler frequency offset parallelly in high dynamic environments.

### 3 Two-Dimension Folding Method

#### 3.1 General Method Description

The LFF method can solve the problem of coherent combining loss. However, to achieve the fast P-code acquisition in high dynamic, the code-search process of LFF method should be accelerated. To solve the problem the novel TDF method is presented next. Figure 3 shows the architecture of the novel method and the process of acquisition is put as follows:



**Fig. 3.** The architecture of the TDF method

Step 1. Intermediate frequency signal is produced by Analog-digital converter (ADC).  
 Step 2. Incoming Intermediate frequency signal is shifted to base band by digital down converter (DDC).

- Step 3. FFT is performed on the base band signal.
- Step 4. The local generated P-code is folded at the same time.
- Step 5. Fold Local frequency cells together to produce a waveform, which is then modulated by local folded code.
- Step 6. The code-modulated waveform from Step 5 goes through FFT and complex conjunction.
- Step 7. Results of step 4 and step 6 are complex-multiplied and IFFT is conducted on the multiplication results.
- Step 8. The IFFT result is stored in a deinterleaver and then integrated coherently.
- Step 9. Test the power of coherent integration result, and the ambiguity of the code phase is resolved by simple correlation check.
- Step 10. The Doppler offset is determined by taking FFT on the stored different temporal IFFT results at the same code phase.

### 3.2 Storing and Detection Process of Coherent Integration Results

The coherent integration result by FFT will be not sufficient when the SNR of the incoming signal is low. Therefore the non-coherent integration is necessary to amplify the correlation result [6, 7]. Early LFF method proposed to achieve non-coherent integration by FFT. The process of the method is showed in Fig. 4(a).  $P$  blocks of coherent integration results are stored in the deinterleaver. Then the results in the same code-phase are processed by  $P$  points FFT. The power of the correlation results can be focus on a certain point among the FFT results. To complete the test of  $N$  code-phases, there are  $N$  times FFT of  $P$  points and  $NP$  times detection to find the max FFT result. It increased the detecting time substantially.

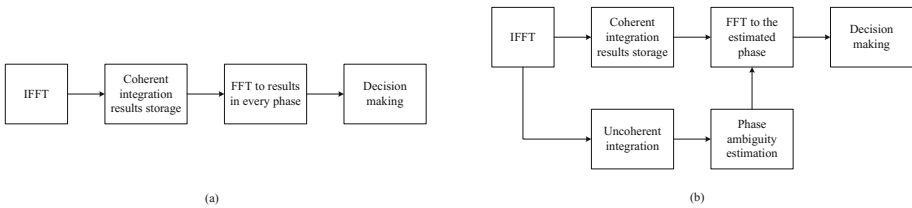


Fig. 4. Coherent integration storing and detection process

A novel coherent integration storing and detection process is given in Fig. 4(b), which will accelerate the detecting process and the matched storing architecture is showed in Fig. 5. The process is:

Step1: Write the  $N$  points of coherent integration results given by the IFFT module in the rows of deinterleaver. Meanwhile the absolute value of the results are added into the non-coherent integration result memory in the order of code-phase. In the same time, the next block of coherent integration results are being computed.

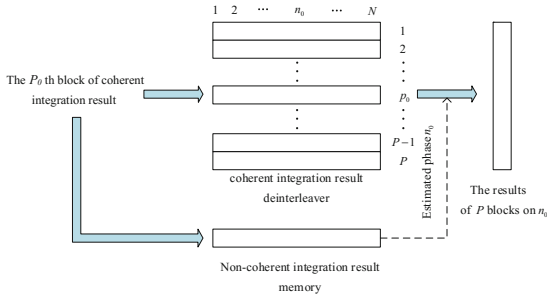


Fig. 5. Coherent integration storing architecture

Step2: After  $P$  blocks results added together, test the non-coherent integration results. The index of the max result is considered as the estimated code-phase.

Step3: Read the coherent integration results at the estimated code-phase in every block and do FFT for  $P$  point. If the max point of these FFT results exceeds the threshold, the detection is declare and the Doppler frequency can be detected by the index of the max point. Then resolve the ambiguity of code-phase causing by folding local generated code. If the max point does not exceed the threshold, clear all the memories and test next block of signal.

## 4 Theoretical Performance Analysis

### 4.1 Coherent Integration Performance

Assuming local generated code are folded  $X$  times, the local sequence is:

$$L_x(k) = m(k) \sum_{i=0}^{X-1} P_0(k + iN) \tag{6}$$

According to the theory of pseudo-random code [8] and the analysis in Sect. 2, the coherent integration result of TDF method when the code-phase difference  $\tau = 0$  can be given:

$$\begin{aligned} I_{m,k_0} &= \sum_{k=k_0}^{k_0+N} (P(k) e^{[2\pi(f_d/f_s)k + \varphi_i]j} + n_k) L_x(k) \\ &= \frac{N+X-1}{N} \sum_{k=k_0}^{k_0+N-1} e^{[2\pi(f_d/f_s)k + \varphi_i]j} m(k) + \sum_{k=k_0}^{k_0+N-1} \sum_{i=0}^{X-1} P_0(k + iN) n_k m(k) \end{aligned} \tag{7}$$

Compare with Eq. 7, we can get:

$$I_{m,k_0} = \frac{N+X-1}{N} R_{I,k_0} + \sum_{k=k_0}^{k_0+N-1} n_k m(k) \sum_{i=0}^{X-1} P_0(k+iN) \quad (8)$$

When the  $\tau \neq 0$ , the coherent integration result is:

$$I_{m,k_0} = \frac{X}{N} R_{I,k_0} + \sum_{k=k_0}^{k_0+N-1} n_{k+\tau} m(k) \sum_{i=0}^{X-1} P_0(k+iN) \quad (9)$$

Because the local P-code is not correlative with noise, the  $I_{m,k_0}$  is decided by  $R_{I,k_0}$ . Therefore the characteristic of the coherent integration result is similar as which is described in Sect. 2.

## 4.2 Detection Probability

To calculate the detection probability, the non-coherent integration result of the signal and noise are necessary. The non-coherent integration result can be expressed:

$$I_u = \sum_{i=1}^P |I_{m,k_0+(i-1)P}| \quad (10)$$

Assume that  $N$  is large enough to ensure that  $P_0(k)$  is orthogonal with  $P_0(k+\tau)$ . Then the mean and variance of  $I_u$  are:

$$E(I_{u,\tau}) = \begin{cases} \sum_{i=0}^{P-1} |R_{m,iN+1}|, \tau = 0 \\ 0, \tau \neq 0 \end{cases} = \begin{cases} S_u, \tau = 0 \\ 0, \tau \neq 0 \end{cases} \quad (11)$$

$$D(I_{u,\tau}) = \begin{cases} D\left(\sum_{i=0}^{P-1} \sum_{k=iN+1}^{(i+1)N} n_k m(k) \sum_{j=0}^{X-1} P_0(k+jN)\right), \tau = 0 \\ D\left(\sum_{i=0}^{P-1} \sum_{k=iN+1}^{(i+1)N} n_{k+\tau} m(k) \sum_{j=0}^{X-1} P_0(k+jN)\right), \tau \neq 0 \end{cases} = \sum_{i=0}^{P-1} \left( \sum_{k=1}^N m^2(k) \right) \sigma^2 \\ = PM\sigma^2 \quad (12)$$

The result of the FFT of  $P$  point coherent integration result when  $\tau = 0$  is considered next. According to former search the, the max result will appear at  $No.Pf_e/2f_c$  point of the FFT result. The mean and variance of the result are:

$$\begin{cases} E(I_{F,\tau,i}) = \begin{cases} P \sum_{k=1}^N e^{[2\pi(f_a/f_s)k + \varphi_i]j} m_k, i = Pf_e/2f_c \\ 0, i \neq Pf_e/2f_c \end{cases} = \begin{cases} PS_0, i = Pf_e/2f_c \\ 0, i \neq Pf_e/2f_c \end{cases} \\ D(I_{F,\tau,i}) = PM\sigma^2 \end{cases} \quad (13)$$

So the peak of the non-coherent integration result  $\gamma_u = I_{u,\tau=0}$  and the FFT result  $\gamma_F = I_{F,\tau=0,i=P_f/2f_c}$  follow the Rician distribution. The noise of non-coherent integration result and the FFT result  $\gamma_n = I_{u,\tau \neq 0} \cup I_{F,\tau \neq 0 \cup i \neq P_f/2f_c}$  follows Rayleigh distribution.

The threshold  $V_{th} = \sqrt{-2PM\sigma^2 \ln(P_{fa}^{(s)})}$  is set by Constant False Alarm Rate (CFAR) with the designed false alarm probability  $P_{fa}^{(s)}$  [9]. Then the Detection Probability can be obtained by referencing the process in Sect. 3.2.

$$P_d^{(c)} = \int_{V_{th}}^{\infty} [F_{\gamma_n}(r_F)]^{P-1} P_{\gamma_F}(r_F) dr_F \int_0^{\infty} [F_{\gamma_n}(r_u)]^{N-1} P_{\gamma_u}(r_u) dr_u \quad (14)$$

where  $P_{\gamma_F}(r_F)$  and  $P_{\gamma_u}(r_u)$  are the Probability Density Function the peak of the non-coherent integration result and the FFT result of coherent integration. And the  $F_{\gamma_n}(r_F)$  is the Cumulative Distribution Function of the noise of non-coherent integration result.

### 4.3 Mean Acquisition Time

The incoming signal is processed block by block in the TDF method. So the process unit time is  $T_d$  which in Eq. 7. Then the mean acquisition time considering single dwell time is [10, 11]:

$$\bar{T}_a = \frac{(\eta/X - 1)(1 + \kappa P_{fa}^{(s)})(2 - P_d^{(c)})}{2P_d^{(c)}} + \frac{1}{P_d^{(c)}} \quad (15)$$

where  $\kappa$  is the punishment factor of false alarm and  $\eta$  represents the number of blocks in each code period.

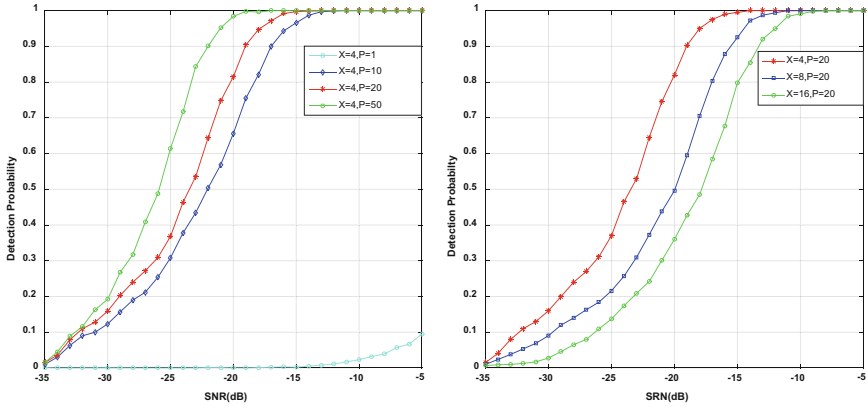
## 5 Simulation Results

Monte Carlo method and numerical simulation method are used to present the performance of TDF method. The detection probability and mean acquisition time are considered. The number of the coherent integration is set as 1024. The chip rate is 10.23 Mcps. According to practical situation in navigation satellite, the max Doppler offset is 250 kHz and the false alarm probability is set below  $10e(-6)$ .

### 5.1 Detection Probability

Figure 6 shows the detection probability of TDF method on incoming signals with different SNRs. The times of folding local code and frequency cells are  $X = 4$  and  $F = 256$ . The non-coherent integration times  $P$  are 1, 10, 20, 50 from right to left. The detection probability is improved evidently by non-coherent integration as the curve of  $P = 1$  is far backward as the others. As the non-coherent integration times increase, the





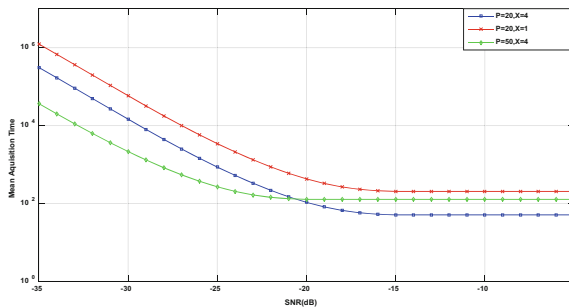
**Fig. 6.** Detection probability of TDF method as  $X = 4$  and  $P = 20$

detection performance of the proposed method is improved accordingly. However, when the integration times is larger than 50, the improvement brought in by non-coherent integration is retarded. On the contrary, the cost of storing resource increases rapidly and the mean acquisition time will enlarge, as showed in Fig. 7.

The detection probability of the method with different code folding times is showed in Fig. 6. The non-coherent integration times is set at 20 which is a rational choice by weighing the detection probability and resource cost. It can be seen that the detection probability reduces with the increase of code folding number. However the negative effects on of the cross-correlation diminish in high SNR which exceeds  $-10$  dB. In this situation, the larger folding times will accelerate the acquisition process.

### 5.2 Mean Acquisition Time

The mean acquisition time of the proposed method with different non-coherent integration is shown in Fig. 7, where the code folding times  $X = 4$  and  $X = 1$ . When  $X = 1$ , the TDF method degenerates to LFF method. There is a 6 dB difference of mean acquisition time between TDF method and LFF method because of the decreased times



**Fig. 7.** Mean acquisition time of TDF method

of code-searching brought in by Folding local code and novel detection process. And the mean acquisition time performance will be improved with the increase of non-coherent integration times when the SNR is low. However the method with fewer integration times will detect the signal in a shorter time by its simple calculation process as the SNR exceeds  $-20$  dB, according to the simulation results.

## 6 Conclusion

This paper proposes the TDF method. By folding local generated code and frequency cells, this method shows a way to achieve fast direct P-code acquisition in high dynamic. The process is accelerated by the novel algorithm of generating local sequence and detection process. Meanwhile the detection performance degradation due to code and frequency folding is made up for by non-coherent integration.

## References

1. Pang, J.: Direct global positioning system P-code acquisition field programmable gate array prototyping. Ohio University, Diss (2003)
2. Yang, C., Vasquez, M.J., Chaffee, J.: Fast direct P(Y)-code acquisition using XFAST. In: ION GPS, Nashville, TN, pp. 317–324 (1999)
3. Li, H., Cui, X., Lu, M., et al.: Dual-folding based rapid search method for long PN-code acquisition. *J. IEEE Trans. Wirel. Commun.* **7**(12), 5286–5296 (2008)
4. Zhao, H., Feng, W., Xing, X.: A novel PN-Code acquisition method based on local frequency folding for BeiDou system. In: The 2016 International Technical Meeting of ION, Monterey, California, pp. 940–947 (2015)
5. Xie, G.: Principles of GPS and receiver design. *Electron. Ind. Beijing* **7**, 61–63 (2009)
6. Ping, J., Wu, X., Yan, J.: Modified zero-padding method for fast long PN-code acquisition. In: 2014 IEEE 80th Vehicular Technology Conference (VTC Fall). IEEE (2014)
7. Spangenberg, S.M., Scott, I.: An FFT-based approach for fast acquisition in spread spectrum communication systems. *Wireless Pers. Commun.* **13**(1-2), 27–55 (2000)
8. Peterson, R.L., Ziemer, R.E.: Introduction to Spread Spectrum Communications. Prentice-Hall, Englewood Cliffs (1995)
9. Wu, X.C., Gong, P., Song, H.J.: An FFT-based approach for carrier frequency domain acquisition in spread spectrum TT&C system. *J. Appl. Mech. Mater.* **135–136**, 211–216 (2012)
10. Kong, S.H.: A deterministic compressed GNSS acquisition technique. *J. IEEE Trans. Veh. Technol.* **62**(2), 511–521 (2013)
11. Kim, B., Kong, S.H.: Design of FFT-based TDCC for GNSS acquisition. *J. IEEE Trans. Wirel. Commun.* **13**(5), 2798–2808 (2014)