A Joint Bandwidth and Power Allocation Scheme for Heterogeneous Networks

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Abstract. Heterogeneous networks (HetNets) composed of macrocells and small cells are expected to improve the transmission performance of users significantly. In this paper, a joint bandwidth and power allocation scheme is proposed for femto base stations (FBSs) in HetNets. By taking into account bandwidth requirements of femto user equipments and bandwidth resource characteristics of the network, a bankruptcy game based bandwidth resource scheme is proposed for the FBSs, based on which a multi-objective optimization based power allocation scheme is proposed in which the energy efficiency optimization problem of each FBS is formulated respectively and is solved via ideal point method and genetic algorithm. Simulation results demonstrate the efficiency of the proposed scheme.

Keywords: Heterogeneous networks · Bandwidth allocation · Power allocation · Bankruptcy game · Multi-objective optimization

1 Introduction

The rapidly growing demand for mobile Internet applications poses great challenges on traditional cellular networks. Cellular heterogeneous networks (HetNets) which consist of both macrocells and small cells such as femtocells, picocells and relay nodes, etc., are proposed and expected to improve user quality of service (QoS) and network performance [1].

Macrocells and small cells in HetNets may share the same bandwidth of telecom operators, therefore bandwidth allocation scheme should be carefully designed to achieve efficient resource utilization and transmission performance enhancement. Two types of bandwidth allocation schemes have been proposed for HetNets, i.e., orthogonal and co-channel bandwidth allocation scheme, where, co-channel bandwidth allocation scheme is capable of achieving higher bandwidth usage compared to orthogonal spectrum allocation [2]. However, bandwidth sharing among cells may cause severe inter-cell interference, which highly limits the transmission performance of both cells. To achieve efficient interference management and network performance enhancement, reasonable bandwidth allocation and power allocation schemes should be designed.

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Several bandwidth allocation schemes and power allocation schemes have been proposed for HetNets in recent years. In [3], a semi-static hybrid spectrum allocation scheme is proposed which achieves the maximum capacity of macro user equipments (MUEs) and femto UEs (FUEs). The authors in [4] study joint bandwidth allocation and call admission control problem for a cellular HetNet, which minimizes the handover rate of UEs. In [5], the transmit power of macro base station (MBS) is determined through maximizing the minimum data rate of MUEs. Reference [6] considers power allocation problem for uplink transmission in HetNets, a sum-rate optimization problem is formulated under the constraints of cross-tier interference between HetNets and user QoS requirements.

Some research works jointly consider the impacts of allocated bandwidth and transmit power in HetNets. In [7], the authors design a subchannel allocation and BS selection scheme to achieve the maximal network throughput. Reference [8] proposes a joint user transmit power and BS association scheme to maximize the utility function of user data rate. However, this may result in large power consumption and low energy efficiency. In [9], the authors aim to maximize the energy efficiency of UEs to obtain the optimal joint bandwidth and power allocation policy. However, the authors assume that the UEs may compete over the shared bandwidth, which may result in lower energy efficiency. In addition, it is assumed that the bandwidth resource is sufficient in most of the previous research works, this may not be the case for the bandwidth requirement from small cells is highly dynamic, hence, designing bandwidth and power allocation scheme according to bandwidth resource characteristics is of great importance.

In this paper, we consider the joint bandwidth allocation and power allocation for HetNets, proposing a two-step resource allocation algorithm, which first allocates bandwidth to various FBSs according to different bandwidth requirements and bandwidth resource characteristics, then an energy efficiency based power allocation algorithm is designed for FBSs.

The rest of this paper is organized as follows. Section 2 introduces the system model considered in this paper. Sections 3 and 4 propose bandwidth and power allocation scheme, respectively. The performance evaluation and simulation results are presented in Sect. 5. Section 6 concludes this paper.

2 System Model

In this paper, we consider a HetNet consisting of one MBS and multiple femto BSs (FBSs) as shown in Fig. 1. Assuming that the MBS covers whole area while each FBS covers a small region of the area. Downlink transmission from BSs to UEs is considered in this paper, particularly, we assume that at interested time duration, the MBS transmits to one MUE, and each FBS transmits to one FUE, for convenience, the ith FBS and the ith FUE are denoted as FBS_i and FUE_i, respectively, $1 \leq i \leq N$, N denotes the number of FBSs.

To achieve efficient spectrum utilization, we assume that full spectrum sharing between MBS and FBSs is allowed, i.e., all the FBSs are allowed to use the whole spectrum of the MBS. To avoid transmission interference among FBSs, we assume various FBSs are assigned different portion of the spectrum of the MBS.

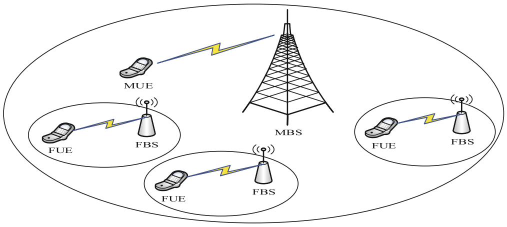

Fig. 1. System scenario

In this paper, we assume the total amount of the bandwidth and the transmit power of the MBS are given constants and study the bandwidth allocation and transmit power allocation problem of the FBSs under the constraints of the transmission interference between the MBS and the FBSs and the QoS requirement of both the MUE and the FUEs. We propose a two-step joint bandwidth allocation and transmit power allocation algorithm for the FBSs. In the following sections, the bandwidth allocation and power allocation algorithms are discussed respectively.

3 Bankruptcy Game Based Bandwidth Allocation Scheme for FBSs

In this paper, we assume that network bandwidth resource is limited compared to the bandwidth requirement of the FUEs, the maximal bandwidth requirement of the FUEs can not be reached. Under the condition that the FBSs may cooperate in sharing the bandwidth resource of the MBS, the bandwidth allocation problem can be modeled as a Bankruptcy cooperative game problem and solved via Shapley value method [10]. As the transmit power constraints of the FBS and the data rate requirement of the FUEs and the MUE may jointly affect the bandwidth requirement of the FBSs and the bandwidth allocation strategy in turn, we first examine the bandwidth allocation constraints.

3.1 Bandwidth Allocation Constraints of FBSs

Bandwidth allocation constraints of the FBSs consists of both lower bound and upper bound constraints of the allocated bandwidth of FBSs.

Lower Bound Constraint of Allocated Bandwidth: To guarantee signal receiving successfully, the signal-to-interference plus noise ratio (SINR) of the MUE should be greater than certain threshold, which may pose constraint on the transmit power of the FBSs in turn. Assume that the MBS and FBS_i share a portion of spectrum with the bandwidth being B_i, the SINR of the MUE on the spectrum can be calculated as

$$\mathrm{SINR}_{\mathrm{m},i} = \frac{P_{\mathrm{m}} h_{\mathrm{m}}}{P_i g_{i,\mathrm{m}} + \sigma^2} \tag{1}$$

where P_{m} and P_i denote the transmit power of the MBS and FBS_i, respectively, h_{m} denotes the link gain from the MBS to the MUE, $g_{i,\mathrm{m}}$ denotes the link gain from FBS_i to the MUE, and σ^2 denotes the noise power of the link, which is assumed to be a constant for all the links in this paper. The SINR should meet certain constraint, i.e.,

$$\mathrm{SINR}_{\mathrm{m},i} \geq \mathrm{SINR}_{\mathrm{m}}^{\mathrm{th}} \tag{2}$$

where $\mathrm{SINR}_{\mathrm{m}}^{\mathrm{th}}$ denotes the SINR threshold of the MUE. Combining (1) and (2), we can obtain that

$$P_i \leq \frac{P_{\mathrm{m}} h_{\mathrm{m}}}{\mathrm{SINR}_{\mathrm{m}}^{\mathrm{th}} g_{i,\mathrm{m}}} - \frac{\sigma^2}{g_{i,\mathrm{m}}}. \tag{3}$$

For convenience, we denote

$$P_i^{\max,1} = \frac{P_{\mathrm{m}} h_{\mathrm{m}}}{\mathrm{SINR}_{\mathrm{m}}^{\mathrm{th}} g_{i,\mathrm{m}}} - \frac{\sigma^2}{g_{i,\mathrm{m}}}. \tag{4}$$

(3) can be rewritten as:

$$P_i \leq P_i^{\max,1}. \tag{5}$$

In addition, due to hardware limitation, the transmit power of FBSs should meet certain maximum power constraint, i.e.,

$$P_i \leq P_i^{\max,2} \tag{6}$$

where $P_i^{\max,2}$ denotes the maximum allowable transmit power of FBS_i. Denote

$$P_i^{\max} = \min\{P_i^{\max,1}, P_i^{\max,2}\}. \tag{7}$$

Combining (5) and (6), we can express the upper bound constraint of FBS_i as

$$P_i \leq P_i^{\max}. \tag{8}$$

Denote R_i as the data rate of FBS_i when transmitting to FUE_i, it can be expressed as

$$R_i = B_i \log_2 \left(1 + \frac{P_i h_i}{P_{\mathrm{m}} g_{\mathrm{m},i} + \sigma^2}\right) \tag{9}$$

where h_i denotes the link gain from FBS_i to FUE_i, $g_{\mathrm{m},i}$ denotes the link gain from the MBS to FUE_i. Denoting $R_i^{\min}$ as the minimum data rate requirement of FBS_i, the data rate of FBS_i should meet the data rate constraint

$$R_i \geq R_i^{\min}. \tag{10}$$

Combining (8)–(10), we obtain

$$B_i \geq \frac{R_i^{\min}}{\log_2(1 + \frac{P_i^{\max} h_i}{P_{\mathrm{m}} g_{\mathrm{m},i} + \sigma^2})}. \tag{11}$$

Denote

$$B_i^{\min} = \frac{R_i^{\min}}{\log_2(1 + \frac{P_i^{\max} h_i}{P_{\mathrm{m}} g_{\mathrm{m},i} + \sigma^2})}, \tag{12}$$

we obtain the lower bound constraint of the allocated bandwidth of FBS_i, i.e.,

$$B_i \geq B_i^{\min}. \tag{13}$$

Upper Bound Constraint of Allocated Bandwidth: To guarantee successful information transmission, the SINR of FUE_i should meet certain constraint, i.e.,

$$\mathrm{SINR}_i = \frac{P_i h_i}{P_{\mathrm{m}} g_{\mathrm{m,i}} + \sigma^2} \geq \mathrm{SINR}_i^{\mathrm{th}} \tag{14}$$

where $\mathrm{SINR}_i^{\mathrm{th}}$ denotes the SINR threshold of FUE_i. From (14), we can obtain that

$$P_i \geq \frac{\mathrm{SINR}_i^{\mathrm{th}}(P_{\mathrm{m}} g_{\mathrm{m,i}} + \sigma^2)}{h_i}. \tag{15}$$

Set

$$P_i^{\min} = \frac{\mathrm{SINR}_i^{\mathrm{th}}(P_{\mathrm{m}} g_{\mathrm{m},i} + \sigma^2)}{h_i}, \tag{16}$$

we obtain the lower bound constraint of P_i

$$P_i \geq P_i^{\min}. \tag{17}$$

We further assume that the service of FUEs has a maximum data rate requirement, which is resulted from the network architecture or resource management schemes. Denoting the maximum data rate of FUE_i as $R_i^{\max}$, the maximum data rate requirement of FUE_i can be expressed as

$$R_i \leq R_i^{\max}. \tag{18}$$

Combining (16)-(18), we obtain the upper bound constraint of the allocated bandwidth of FBS_i

$$B_i \leq \frac{R_i^{\max}}{\log_2(1 + \frac{P_i^{\min} h_i}{P_{\mathrm{m}} g_{\mathrm{m},i} + \sigma^2})}. \tag{19}$$

Denote

$$B_i^{\max} = \frac{R_i^{\max}}{\log_2(1 + \frac{P_i^{\min} h_i}{P_{\mathrm{m}} g_{\mathrm{m},i} + \sigma^2})}, \tag{20}$$

the upper bound constraint of B_i can be rewritten as:

$$B_i \leq B_i^{\max}. \tag{21}$$

The bandwidth requirement of FUE_i can be considered in (13) and (21). Assuming the network bandwidth resource is limited compared to the bandwidth requirement of the FUEs. A brief introduction of bankruptcy game theory is presented, then the game model for FBSs bandwidth allocation is established and the optimal solution is presented.

3.2 Bankruptcy Game Formulation

The theory of bankruptcy game can be dated back to an estate allocation problem of a bankruptcy company. Assume that a company with estate E becomes bankrupt, it owes money to creditors, and the amount of the money claimed for all the creditors is D. Hence, the money E is needed to be divided among N creditors, as the estate of the bankrupt company is less than the sum of the claims from the creditors, i.e., $E < D$. This conflicting situation leads an N-person cooperative game, where the optimal solution for dividing the money can be obtained through solving the game model.

In order to design a fair and efficient bandwidth allocation scheme for FBSs, the problem of bandwidth allocation of can be modeled as a bankruptcy game of N persons. Assuming that $\sum_{i=1}^{N} B_i = B$, B denotes the total amount of bandwidth of the MBS. From discussion in previous subsection, we can obtain

$$B_i^{\min} \le B_i \le B_i^{\max}. \tag{22}$$

3.3 Shapley Value Based Solution

Shapley value method is commonly applied for solving bankruptcy game problem. According to the Shapley value method, assuming that the alliance formed by the FBSs constitutes a finite set N, with S denoting a subset of N, i.e., $S \subset N$, the characteristic function $v(S)$ of the union S can be calculated as

$$v(S) = \max(0, B - \sum_{i \notin S} B_i^{\max}), \tag{23}$$

$v(S)$ holds the largest number of the allocated bandwidth for the union S, then the Shapley value of the bankruptcy game model can be defined as

$$B_i = \sum_{S \subset N} \frac{(|S|-1)!(N-|S|)!}{N!} [v(S) - v(S - \{i\})] \tag{24}$$

where $|S|$ denotes the number of elements in the set S. Assuming that FBS$_i$ is in the coalition S, $v(S) - v(S - \{i\})$ represents the contribution that FBS$_i$ makes to the coalition and $\frac{(|S|-1)!(N-|S|)!}{N!}$ represents the weight of the contribution that FBS$_i$ makes to the coalition, which is dependent on the size of the S and the total number of the game players. From above formula, the Shapley value B_i, which corresponds to the bandwidth allocation scheme for FBS$_i$ can be obtained.

4 Power Allocation Scheme for FBSs

Given the allocated bandwidth of the FBSs, we further design the power allocation scheme for the FBSs. The power allocation problem is formulated as an multi-objective optimization problem which is then solved via ideal point method and genetic algorithm (GA).

4.1 Proposed Optimization Scheme

In this subsection, the multi-objective power allocation optimization problem is formulated. From (9), it is apparent that the transmit power of FBS_i, the characteristics of the channel, including the bandwidth, the channel gain and the noise power jointly determine the transmission performance of the FBS. Particularly, given the channel characteristics, to maximize the data rate of FBS_i, the maximum transmit power should be applied. However, this may result in large power consumption and low energy efficiency, which are highly undesired. To jointly consider the transmission performance and the power consumption, we formulate the energy efficiency of the FBSs and design transmit power allocation strategy to achieve the maximum energy efficiency of each FBS in this paper.

The energy efficiency of FBS_i is defined as the ratio of the data rate and the power consumption of FBS_i, i.e.,

$$\eta_i = \frac{R_i}{P_i + P_{\mathrm{cir}}},\ 1 \le i \le N \tag{25}$$

where P_{cir} denotes the circuit power of FBS_i, which is assumed to be a constant for all FBSs in this paper.

In order to maximize the energy efficiency of all the FBSs, the multi-objective optimization problem can be formulated as

$$\begin{aligned} \max_{P_i} \quad & \eta_i, \quad 1 \le i \le N \\ \text{s.t.} \quad & \mathrm{C1}: R_i \ge R_i^{\min}, \\ & \mathrm{C2}: R_i \le R_i^{\max}, \\ & \mathrm{C3}: P_i \ge P_i^{\min}, \\ & \mathrm{C4}: P_i \le P_i^{\max}, \\ & \mathrm{C5}: R_{\mathrm{m}} \ge R_{\mathrm{m}}^{\min}. \end{aligned} \tag{26}$$

4.2 Solution to the Optimization Problem

The problem formulated in (26) is a multi-object optimization problem, the optimal solution of which is in general difficult to obtain. In this section, the ideal point method [11] is applied to solve the optimization problem.

The basic idea of the ideal point method is that for each objective function, the locally optimal solution, referred to as ideal result can be obtained independently without considering the joint constraints and the feasibility of the solutions, then a single objective optimization problem which minimizes the distance between the feasible solutions and the ideal solutions is formulated and solved based on GA.

Ideal Solution to Individual Optimization Objective: For the ith FBS, $1 \le i \le N$, the energy efficiency optimization problem can be expressed as

$$\max_{P_i} \quad \eta_i \qquad (27)$$
$$\text{s.t.} \quad \text{C1} - \text{C4 in (26)}.$$

For convenience, we denote the maximum energy efficiency of FBS_i as

$$\eta_i^* = \frac{R_i(P_i^*)}{P_i^* + P_{\text{cir}}} = \max_{P_i} \ \{\frac{R_i(P_i)}{P_i + P_{\text{cir}}}\} \qquad (28)$$

where P_i^* denotes the optimal transmit power of the ith FBS. According to [12], the maximum energy efficiency η_i^* is achieved if and only if

$$\max_{P_i} \ \{R_i(P_i) - \eta_i^{\max}(P_i + P_{\text{cir}})\} = R_i(P_i^*) - \eta_i(P_i^* + P_{\text{cir}}) = 0. \qquad (29)$$

Hence, the optimization problem formulated in (27) can be equivalently transformed into following problem, i.e.,

$$\max_{\eta_i, P_i} \quad R_i - \eta_i(P_i + P_{\text{cir}}) \qquad (30)$$
$$\text{s.t.} \quad \text{C1} - \text{C4 in (26)}.$$

To obtain the optimal energy efficiency η_i^*, we apply an iterative algorithm and the convergence to optimal energy efficiency can be guaranteed.

Single Objective Optimization Problem Formulation: The locally optimal $\eta_i^*, 1 \leq i \leq N$ may not be feasible for the formulated multi-objective optimization problem. According to the ideal point method, we examine the distance between the feasible solutions and the locally optimal solution, denoted by Q,

$$Q = \sum_{i=1}^{N} (\eta_i - \eta_i^*)^2, \qquad (31)$$

then the original multi-objective optimization problem can be converted into a single object optimization problem as follows,

$$\min_{P_i} \quad Q \qquad (32)$$
$$\text{s.t.} \quad \text{C1-C5 in (26)}.$$

The formulated single objective optimization problem can be solved based on GA.

5 Simulation Results

In the simulation, we consider a HetNet consisting of one MBS and five FBSs. Assuming all UEs are randomly located in a rectangular region with the size being 500×500, the MBS is located in the position with the coordinate being (255,200), the positions of the FBSs are listed in Table 1. In the simulation, the

Table 1. Simulation parameters

FBS_1	FBS_2	FBS_3	FBS_4	FBS_5
(97,350)	(40,60)	(417,100)	(250,405)	(460,460)

Table 2. Simulation parameters

Parameters	Value
The minimum rate requirement of FBS_i	400 kbps
The minimum rate requirement of MBS	100 kbps
The maximum power of MBS	0.5 W
Noise power	−200 dBm

transmission gain of the link between BS and UE is modeled as $h = (c/4\pi f d)^2$, where d denotes the distance between the source node and the destination node of the link, c denotes the speed of light, f denotes the carrier frequency of the transmit signal. Other simulation parameters are summarized in Table 2.

Assuming that the bandwidth offered by MBS is 1.2MHz, and the maximum bandwidth requirement of each FBS are 244, 297, 288, 298, and 299KHz, respectively. Comparing the bandwidth requirement and the total amount of the bandwidth of the MBS, the bankruptcy game can be modeled and the allocated bandwidth for each FBS can be calculated according to the Shapley value method. In Fig. 2, the allocated bandwidth for FBSs is plotted and compared with the maximum bandwidth requirement of the FBSs. It can be seen from Fig. 2 that the allocated bandwidth of all the FBSs meets the minimum bandwidth requirement and a relatively fair bandwidth allocation with respect to bandwidth requirement can be achieved. Fig. 3 shows the allocated transmit power of FBS_1 to FBS_5 versus the number of generations of GA. It can be seen from the figure that the transmit power curves of FBSs converge to constants, demonstrating the effectiveness of the applied GA.

Figures 4 and 5 show the total energy efficiency versus the maximum transmit power of the FBSs ($P_i^{\max}$), and the results are obtained from the proposed scheme and the scheme proposed in [8]. In Fig. 4, we examine the total energy efficiency for different circuit power consumption of the FBSs. Comparing the results obtained from the proposed scheme and the scheme proposed in [8], we can see that for small $P_i^{\max}$, the energy efficiency increases with the increase of $P_i^{\max}$ for both schemes, indicating a larger power threshold is desired for achieving the maximum energy efficiency. However, as $P_i^{\max}$ reaches to a certain value, the energy efficiency obtained from our proposed algorithm becomes a fixed value for the transmit power being less than $P_i^{\max}$ has resulted in the optimal energy efficiency, which will no longer vary with $P_i^{\max}$, whereas the energy efficiency obtained from the other scheme decreases with the increase of $P_i^{\max}$. This is because the scheme proposed in [8] aims to achieve the maximum data rate, thus may require higher power consumption, resulting in undesired

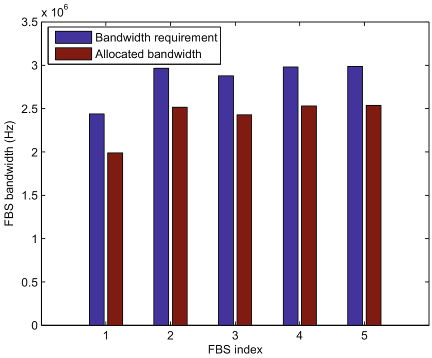

Fig. 2. Comparison of the maximum required bandwidth and allocated bandwidth of FBSs.

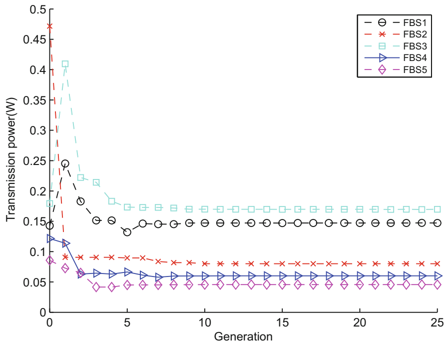

Fig. 3. Transmission power versus generations for FBSs

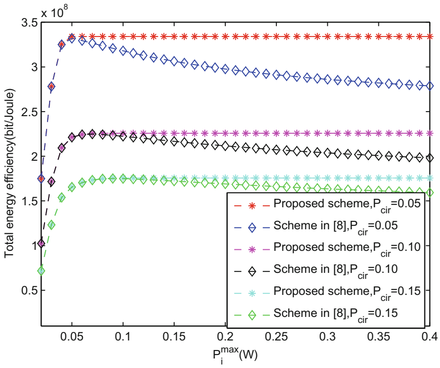

Fig. 4. Energy efficiency versus maximum transmit power (different circuit power)

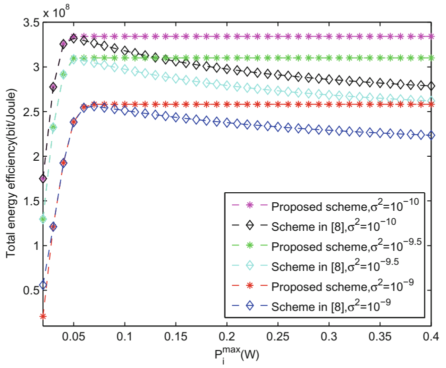

Fig. 5. Energy efficiency versus maximum transmit power (different noise power)

energy efficiency. Comparing the two curves in the figure, we can see that the proposed scheme outperforms the scheme proposed in [8].

In Fig. 5, we examine the total energy efficiency for different noise power of the links between BSs and UEs, i.e., σ^2. We can see from the figure as the value of σ^2 increases, the total energy efficiency decreases. This is because larger noise power results in worse transmission performance of the FBSs. Comparing the results obtained from the proposed scheme and the scheme proposed in [8], we can see that our proposed scheme outperforms previously proposed scheme.

6 Conclusion

In this paper, we study joint bandwidth and power allocation problem for HetNets. A bankruptcy game model is formulated and the amount of bandwidth is obtained for FBSs based on Shapley value method. To design the optimal power allocation strategy for FBSs, we then formulate a multi-objective optimization problem which maximizes the energy efficiency of the FBSs individually. The optimization problem is solved through applying ideal point method and GA. Simulation results demonstrate the proposed scheme is capable of guaranteeing QoS requirement of FUEs and achieving higher energy efficiency compared to previously proposed scheme.

$$v(S) = \max(0, E - \sum_{i \notin S} d_i^{\max}) \tag{33}$$

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