Capacity Analysis in the Cognitive Heterogeneous Cellular Networks with Stochastic Methods

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Abstract. Small cells are widely being deployed to enhance the performance of cellular networks, which results in a random distribution of base stations as well as a complex interference problem. Therefore, it becomes considerably challenging to derive a closed-form expression for the capacity of small cell enhanced heterogeneous cellular network especially when the cognitive radio (CR) technology is utilized to mitigate the possible interference. In this paper, we first use the discrete time Markov chain (DTMC) to achieve the spectrum mobility of macro base station (MBS) users, i.e. primary users (PUs) in the cognitive heterogeneous cellular networks (CHCNs). Meanwhile, by modeling MBSs and small base stations (SBSs) as two independent homogeneous Poisson point processes (HPPPs), we propose an integral way based on stochastic geometry (SG) to get the calculation of the interference. Simulation results show that our capacity analysis method of CHCNs serves well in approximating the network capacity by conquering the complex interference and the uncertainty of spectrum mobility, which turns out to be an efficient and promising approach.

Keywords: Cognitive heterogeneous cellular networks (CHCNs) · Markov chain · Stochastic geometry · Homogeneous Poisson point process (HPPP)

1 Introduction

The last few years have witnessed the proliferated deployment of small cells in the cellular network, e.g., pico cell, femto cell. The small cells can bring a high network capacity by providing heterogeneous access for indoor and outdoor hotspots. However, they also arouse a considerably complex problem, i.e., the cross-tier interference as well as co-tier interference. Meanwhile, the emerging of the small cells also aggravates the irregularity of network coverage. Therefore, the traditional regular hexagon network is not enough to provide the diverse rate requirement of different coverage.

Fortunately, by enabling cognitive radio (CR) technology on small cells to be aware of and adapt to communication environments, the interference issue can be tackled [1, 2]. The CR-enabled small base stations (SBSs) can actively acquire the information about the channel by spectrum sensing mechanism, which conduces to avoid the possible co-channel interferences and enhance the entire network performance. However, due to the irregular coverage of base stations and the troublesome interference problem,

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few works can give out a closed-form expression for the capacity of cognitive heterogeneous cellular networks (CHCNs). In addition, it becomes even more complicated when spectrum mobility is involved. Therefore, it's far from easy to weigh the contribution of CR technology to the improvement of network capacity mathematically.

Recently, stochastic geometry (SG) wins its popularity through capturing the topological randomness as well as acquiring tractable numerical results in the increasingly complex networks [3–6]. Motivated by the favorable conclusions in previous works, SG tends to be used in CHCNs. In [7], Hesham et al. utilize the SG to model and analyze heterogeneous cellular networks from two aspects. They first exploit SG to evaluate the load of each network tier, and then obtain the maximal frequency reuse efficiency with spectrum sensing design for channel access by the assumption of hard core point process (HCPP). But they only focus on the performance of cognitive small cell network and get the outage probability for a small cell user. In [8], the authors provide a performance analysis of two-tier HetNets with cognitive small cells under the SG model with respect to the outage probability. They obtain the opportunistic spectrum access probability for small cell access points conditioned by the spectrum sensing threshold. However, there are three weaknesses in the work. First, the contribution of [8] is elaborated in the underlay fashion where the outage event of primary users (PUs) is mainly caused by the aggregated interference from secondary users (SUs) and noise environment. Second, only one PU is assumed in the derivation. Third, the close-form derivation of interference from SUs to PU is obtained only in the large-scale environment. [9] summarizes the previous work related to SG in the literatures for single-tier, multi-tier, as well as cognitive cellular wireless networks. The author points out that only few results in the context of multi-tier cellular networks are available to CHCNs and indicates that there are opportunities for innovating techniques which facilitate the SG modeling.

After analyzing these, we can clarify some facts and difficulties in obtaining the capacity derivation of cognitive multi-tier cellular networks. (i) The objective of CHCNs is different from the conventional cognitive radio networks (CRNs) (i.e., cognitive networks with licensed and unlicensed users). That is, we need to focus on the capacity aggregation of both MBSs (similar to PUs in the conventional CRNs) and cognitive small cell base stations (similar to SUs in the conventional CRNs) rather than an opportunistically utilization of SUs on the unlicensed band subject to a tolerable performance degradation for the PUs. (ii) The spectrum access is sensitive to its mobility model which affects the capacity derivation significantly. (iii) The calculation of interference becomes even more difficult in the CHCNs. For one thing, the small cell infrastructure confuses traditional regular deployment, also the spectrum reuse policy, and the interference becomes complicated to be observed. For another, it is never too easy to get the channel state information (CSI) in the multi-cell system, especially in the multi-tier networks.

In this paper, we focus on a closed-form expression for the capacity of CHCNs, which including two parts, i.e., deriving the distribution of time slots by the discrete time Markov chain (DTMC) and approximating the co-tier and cross-tier interference by an integral method. In order to get the interference of the whole coverage, the integral method takes the integral of the probability of the interference according to the distribution of the distance from the user to its serving base station. Further, under the adopted homogeneous Poisson point process (HPPP) model, a series of simulations illustrate that our proposed approach shows its superiority of weighing the network capacity accurately and efficiently, which also takes the spectrum mobility of macro base stations (MBSs) users into account.

The remaining sections of this paper are organized as follows. In Sect. 2, our model with SG modeling techniques is presented. Section 3 describes the DTMC for the purpose of analyzing the distribution of time slots and captures the capacity of CHCN by giving a closed-form expression. In Sect. 4, we provide numerical results and finally, Sect. 5 concludes the paper.

2 System Model

In this paper, we focus on a downlink two-tier CHCN where the users associate with the BS (MBS or SBS) which provides the highest reference signal receiving power (RSRP). Meanwhile, we assume that MBSs and their users use the licensed spectrum band, while SBSs and their users act as the unlicensed users that access the channels if sensing the vacant spectrum. Thereby, they work in an overlay fashion composing a typical cognitive radio system apparently, i.e., MBS and their users work as PUs while SBS and their users are SUs. To note that, they both serve as the cellular network, thus, both the capacity of MBSs and SBSs need to be taken into consideration in the CHCNs. Each time slot T_{slot} is divided into two periods, i.e., sensing period T_s, where SBSs and their users scan throughout the spectrum band and data transmission period, where SBSs access the channel if finding vacancy for MBSs. The structure of the frame is illustrated in Fig. 1. The time for merging the sensing results and their feedback to the serving SBSs is ignored. Here, we assume that only one user is allowed to access the channel during the time slot within each macro cell or small cell and there is no free time slot.

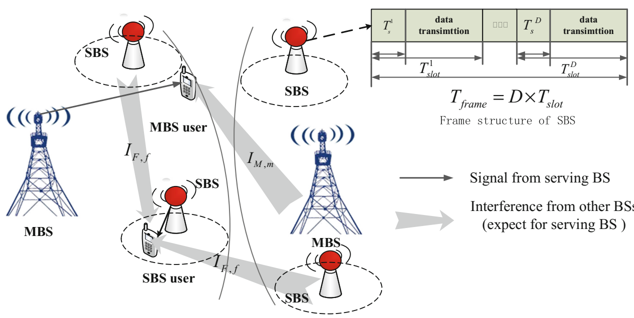

Fig. 1. Illustration of a two-tier heterogeneous network.

Instead of assuming the MBSs and SBSs are placed deterministically in a grid model, we adopt a HPPP model in this paper, where MBSs and SBSs accord to two independent HPPPs with density λ_M and λ_F respectively. Also, users located in the CHCNs accords to another HPPP with density λ_U, which includes X MBS users and Y SBS users. The idea of HPPP derives from SG which aims at weighing the network topologies from an average perspective rather than one single base station or user. It has been shown by [9] as an equally accurate model to capture the performance of the network compared with conventional grid model, whereas, the former is more preferable for its tractability to describe the increasing opportunistic placed base stations in the future.

As is shown in Fig. 1, since the overlay fashion is applied in this paper, there is no interference between two tiers, thus only co-tier interference remains. We assume that all the MBSs and SBSs simultaneously transmit to their associated users with the same power P_M and P_F. g_M denotes the transmission gain of MBS user from its serving MBS. Similarly, g_F is the transmission gain of SBS user from its serving SBS. Note that we only consider the large-scale fading for simplicity. The noise is assumed to be zero-mean complex additive white Gaussian random variables with power P_{Noise}. Then the signal to interference plus noise ratio (SINR) of MBS and SBS users are given respectively by (1) and (2).

$$SINR_m = \frac{P_M g_M}{I_{M,m} + P_{Noise}} \tag{1}$$

$$SINR_f = \frac{P_F g_F}{I_{F,f} + P_{Noise}} \tag{2}$$

where $I_{M,m}$ is the interference for MBS user m from other MBSs. Likewise, $I_{F,f}$ is the interference caused by other SBSs to SBS user f.

3 Capacity Derivation of Cognitive Heterogeneous Networks

In the CHCN model, it is hard to derive the capacity due to the uncertainty of spectrum mobility and the complex interference between users and the heterogeneous base stations. As the DTMC advantages in analyzing the reliability and performance of service portfolio, we consult to Markov chain to acquire the distribution of time slots. Meanwhile, the interference is captured with HPPP assumption in a SG way.

3.1 Markov Chain Model for the Spectrum Mobility

The method of DTMC to capture the spectrum mobility is defined by its state, transfer probability and steady probability. In what follows, we assume that MBS users arrive in the channel with a probability of λ_a and depart with a probability of λ_d.

(1) State
Let M time slots be occupied by MBS users on the channel, and use $\boldsymbol{\psi} = \{\psi(u) = 0\, or\, 1, u = 1, 2, \ldots, M\}$ to represent the occupancy state of all MBS users, in which $\psi(u) = 0$ means that the MBS user is on the channel while $\psi(u) = 1$ means it is absent. Then, $\phi(i) = \sum_{u=1}^{M} \psi(u) = i, 0 \leq i \leq D$ denotes that there are i MBS users in the frame and D is the number of time slots.

(2) Transfer probability

Here, the MBS users arrive and depart in a Poisson way, and once one MBS user arrives, a single time slot of consecutive frames will be occupied for an exponentially distributed time period until it leaves. The probability of k $(0 \le k \le i)$ MBS users' arrival during the frame is given by

$$P_A(k) = \mathbb{P}\{N_A = k\} = \frac{(\lambda_a T_{frame})^k}{k!} e^{-\lambda_a T_{frame}}. \tag{3}$$

where N_A is the number of MBS users who arrive at the channel during the frame and $T_{frame} = D \times T_{slot}$ means the total time of the frame.

Similarly, the probability of l $(0 \le l \le i+k)$ MBS users' departure during the frame is given by

$$P_D(l) = \mathbb{P}\{N_D = l\} = \frac{(\lambda_d T_{frame})^l}{l!} e^{-\lambda_d T_{frame}}. \tag{4}$$

where N_D is the number of MBS users leaving the channel during the frame.

Therefore, in the state $\phi(i)$, the probability of k MBS users' arrival is expressed by

$$A(i,k,l) = \begin{cases} P_A(k) & i - l + k < D \\ 1 - \sum_{d=0}^{k-1} P_A(d) & i - l + k = D \end{cases}. \tag{5}$$

Correspondingly, the probability of l departures in state $\phi(i)$ is

$$D(i,k,l) = \begin{cases} P_D(l) & i - l + k > 0 \\ 1 - \sum_{q=0}^{l} P_D(q) & i - l + k = 0 \end{cases}. \tag{6}$$

The transition probability matrix $\mathbf{P}$ can be derived by calculating all the state transforms probability P_{ij} from $\phi(i)$ to $\phi(i+H)$. The elements of the matrix P_{ij} is

$$P_{ij} = P_{i+H,i} = P((i+H)|i) = \sum_{k=\max(-H,0)}^{i} A(i, H+k, k) D(i, H+k, k), 0 \le i \le D, -i \le H \le D - i \tag{7}$$

(3) Steady probability $\pi(i)$

Constructing the steady state probability $\pi(i)$, $i = 1, \ldots, D$ as the elements of matrix $\mathbf{\Pi} = [\pi(0), \pi(1), \ldots, \pi(D)]$, we are able to obtain $\pi(i)$ by finding the solution with respect to the following condition equation.

$$\mathbf{\Pi} = \mathbf{\Pi} \bullet \mathbf{P} \tag{8}$$

(4) The number of MBS and SBS users

The average number of MBS users during the frame can be calculated by accumulating all the possible state.

$$N_p = \sum_{i=0}^{D} i \times \pi(i) \tag{9}$$

Therefore, the average number of SBS users is given by (10).

$$N_s = D - N_p \tag{10}$$

3.2 The Approximating Capacity of CHCN with HPPP Assumption

In this paper, assuming that interference is captured by the density of base stations, we consult to an integral way to derive the interference. The idea derives from "fluid model" in [10] where the integral of the density of BSs in the whole network is used to calculate the capacity. In the HPPP model, the probability density function (PDF) of the distance between the serving BS and the user (denoted as r) can be given by

$$f(r) = \lambda 2\pi r e^{-\lambda \pi r^2} \tag{11}$$

where $\lambda = \lambda_M \, or \, \lambda_F$.

As to the channel model, we only consider the power loss propagation for short. Hence, the transmission gain in (1) and (2) can be express as $g_M, g_F = r^{-\alpha}$ where α denotes the path loss exponent. Taking the integral in the coverage of interfering base stations, we can get the interference for MBS users and SBS users.

According to our assumption, only co-tier interference is considered. Therefore, there are two kinds of interference categorized by different tiers.

(1) When the channel is occupied by MBS users, for a specific MBS user, the interference from other MBSs is given by

$$I_{M,m} = \int_0^{2\pi}\int_r^{\infty} p_M \lambda_M 2\pi r e^{-\lambda_M \pi r^2} \times r^{-\alpha} dr d\theta = 2\pi p_M (\pi\lambda_M)^{\frac{\alpha}{2}}\left[\Gamma\left(1-\frac{\alpha}{2}\right) - \Gamma\left(1-\frac{\alpha}{2}, r\right)\right] \tag{12}$$

where $\Gamma(x) = \int_0^{\infty} t^{x-1}e^{-t}dt$ and $\Gamma(x, r) = \int_0^{r} t^{x-1}e^{-t}dt$ are standard gamma function and incomplete gamma function respectively.

(2) Similarly, when the channel is occupied by SBS users, thus, the interference from other SBSs is as follows

$$I_{F,f} = \int_0^{2\pi}\int_r^{\infty} p_F \lambda_F 2\pi r e^{-\lambda_F \pi r^2} \times r^{-\alpha} dr d\theta = 2\pi p_F (\pi\lambda_F)^{\frac{\alpha}{2}}\left[\Gamma\left(1-\frac{\alpha}{2}\right) - \Gamma\left(1-\frac{\alpha}{2}, r\right)\right] \tag{13}$$

Therefore, the whole network capacity can be expressed as

$$C = C_M + C_F = \frac{W\eta}{D}\left[N_{\mathrm{p}} \sum_{m=1}^{X} \log_2(1 + SINR_m) + N_s \sum_{f=1}^{Y} \log_2(1 + SINR_f)\right] \tag{14}$$

where W is the bandwidth of the channel, $\eta = 1 - T_s/T_{slot}$ is the sensing efficient. C_M and C_F stand for the capacity of macro cells and small cells respectively.

4 Simulation Results and Analysis

To testify the proposed capacity expressions, we present several numerical metrics and give relative analysis in this section. In the two-tier CHCN, MBSs, SBSs and users are distributed in a HPPP way with density $4 \times 10^{-5}/\text{m}^2$, $8 \times 10^{-4}/\text{m}^2$ and $1.6 \times 10^{-4}/\text{m}^2$ respectively in a circular coverage with radius 500 m. The rest of the simulation parameters are listed in Table 1.

Table 1. Simulation parameters.

Symbol	Definition	Default value
P_M	The transmit power of MBS	20 W
P_F	The transmit power of SBS	0.1 W
P_{Noise}	The power of noise	−174 dBm/Hz
W	The physical bandwidth	1 Hz
α	The path loss exponent	3
D	The number of time slots in a frame	20
T_{slot}	The lasting time of each time slot	0.577 ms
T_s	The lasting time of sensing time	25 μs
λ_a	The arrival probability of the MBS users	0.8
λ_d	The departure probability of the MBS users	0.5

4.1 Comparison Between Theoretical Analysis and Simulation Result

We simulate the capacity of the network derived by integral method (theoretical analysis) and sum method (simulation) in HPPP model in Fig. 2. It can be figured out that the integral method has a lower capacity than that of the sum method. This is because the BSs and users modeled by HPPPs are located in a more random way, which brings a more conservative result by theoretical analysis compared with the simulation. Thereby, integral method aggregates a higher interference resulting in the lower capacity. Also, with the increase of SBSs density, the gap between the two

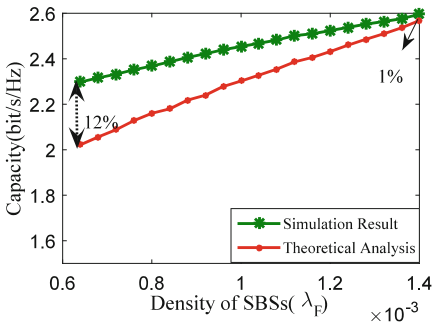

Fig. 2. Comparison between simulation result and theoretical analysis of network capacity.

methods gets smaller (when the density of SBSs increases from 0.00063 to 0.0014/m^2, the gap decreases from 12% to 1%). This reveals that the theoretical results approach the simulation at high density of SBSs.

4.2 Effect of the Spectrum Mobility of MBS Users

Figure 3 illustrates the network capacity under different departure probabilities with the increase of arrival probability of MBS users. It can be observed that the network capacity keeps growing with the increase of arrival probability of MBS users when the departure probability is 0.2, 0.5 and 0.9. Whereas, when the departure probability is 0.1, the network capacity exhibits a slight decline. Actually, at low departure probability values, high arrival probability of MBS users impacts the capacity by aggregating the interference, which results in a lower capacity (when the arrival probability goes from 0 to 1, the capacity decreases 5%). Conversely, in the case of high departure probability, there are more time slots occupied by MBS users with the increase of arrival probability of MBS users, which brings a higher capacity.

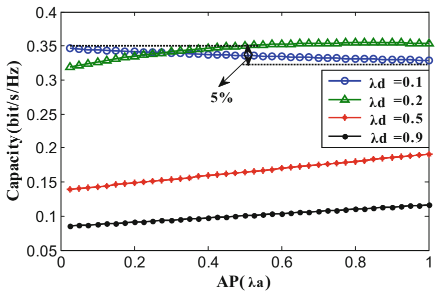

Fig. 3. Capacity comparison with variance of arrival probability (AP)

The capacity under different arrival probabilities with variance of departure probability of MBS users is shown in Fig. 4, in which we can see there is an optimal departure probability for each curve and it can be inferred from (4). Moreover, as is illustrated in Fig. 4, the curves flare up at low departure probability, but show a decline when the departure probability over certain values. The reason is that the whole capacity includes macro cell capacity and small cell capacity and both of the two parts have similar tendency with the whole capacity, as is depicted in Figs. 5 and 6. It can be observed that the macro cell capacity curve has the same tendency with the whole capacity curve while the small cell capacity curve acts conversely. In fact, in the case of low departure probability, more users can connect to MBSs rather than SBSs with the increasing departure probability, which leads to a higher macro cell capacity. Whereas, when the departure probability goes higher, the total number of the MBS users in the network becomes less, which results in a lower macro cell capacity. Opposite results can be derived for the SBS network since that they share the same frequency during each frame in the overlay fashion.

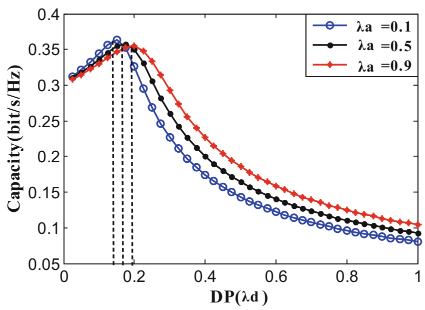

Fig. 4. Capacity comparison with variance of departure probability (DP).

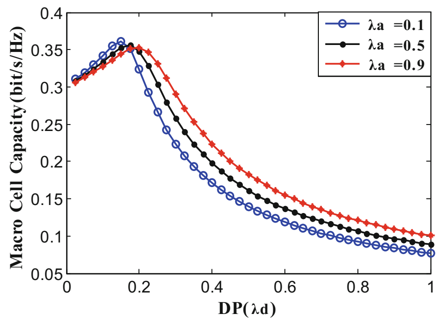

Fig. 5. Macro cell capacity with variance of departure probability.

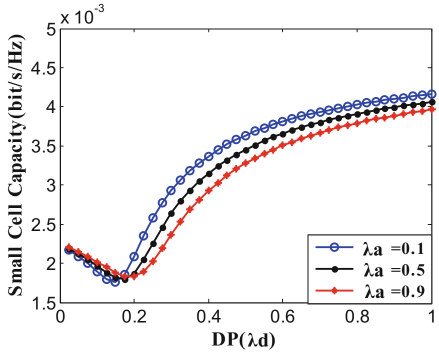

Fig. 6. Small cell capacity with variance of departure probability.

5 Conclusions

In this paper, we derive the network capacity by a stochastic method in a two-tier CHCN. A DTMC is employed to capture the spectrum mobility while an integral method is proposed to approximate the interference. Simulation is made in a HPPP network model, and results show that the proposed method turns out to be an efficient way to calculate the network capacity in CHCNs. Moreover, we also analyze the effect of the spectrum mobility of MBS users on the network capacity. It has shown that the arrival probability conduces to the network capacity at high departure probability and there is an optimal departure probability for each arrival probability.

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