A Computationally Efficient 2-D DOA Estimation Approach for Non-uniform Co-prime Arrays

Fenggang Sun[1,2(✉)], Lei Zhao[3], Xiaozhi Li[4], Peng Lan[2], and Yanbo Zi[2]

[1] College of Communications Engineering, PLA University of Science and Technology, Nanjing 210007, China
sunfg@sdau.edu.cn

[2] College of Information Science and Engineering, Shandong Agricultural University, Tai'an 271018, China
lanpeng@sdau.edu.cn

[3] Ji'nan Wenwang Center, Jinan 250100, China

[4] Shandong Medicine Technician College, Tai'an 271000, China
lfr1972@sina.com

Abstract. This paper investigates the problem of two dimensional (2-D) directions-of-arrival (DOA) estimation of multiple signals in co-prime planar arrays. The array consists of two uniform planar arrays with their respective inter-element spacing being both larger than half wavelength, which can enhance the resolution but at the cost of phase ambiguity. The phase ambiguity problem can be addressed by combining the results of two subarrays. Specifically, we apply the multiple signal classification (MUSIC) algorithm to each subarray to acquire their respective spectrum; then we obtain the joint spatial spectrum, which is defined as the product of the respective spatial spectrums; Finally, according to co-prime property, we search over the angular field for the spectral peaks to estimate the DOA uniquely. Finally, we verify the effectiveness of the proposed method via simulations.

Keywords: Directions-of-arrival estimation · Two dimensional (2-D) · Co-prime planar array · Phase ambiguity · Joint spatial spectrum

1 Introduction

Over several decades, direction of arrival (DOA) estimation has become a crucial problem in array signal processing and has been widely used in various fields such as radar, sonar, and wireless communications [1–4]. Among various DOA estimation methods, two dimensional (2-D) DOA estimation has drawn a remarkable amount of attention [5–8], because their array models are more practical in actual applications. Traditionally, the most commonly used array is mainly uniform array geometry; however, an appropriate non-uniform array geometry can provide higher estimation accuracy than the uniform geometry [9].

Q. Chen et al. (Eds.): ChinaCom 2016, Part II, LNICST 210, pp. 183–192, 2018.
DOI: 10.1007/978-3-319-66628-0_18

The history of research on non-uniform array geometry can date back to minimum-redundancy arrays [10]. The introduction of co-prime linear arrays in [11] has created renewed interest in such geometries. A co-prime linear array is composed of two uniform subarrays, which have co-prime integers M and N sensors, respectively and the corresponding inter-element spacings are N and M of half wavelength. It is proved that a co-prime array consisting of $M + N - 1$ sensors can provide $\mathcal{O}(MN)$ degrees of freedom and therefore enhance the estimation performance [11]. Thereby, the co-prime array structure can reduce the cost of array design and motivates the study of DOA estimation with non-uniform co-prime arrays [12–17]. In [12], Zhou *et al.* proposed to search over the total angular field for each subarray and uniquely determine the true DOAs by finding the common peaks. To reduce the computational complexity, Weng and Djuric in [13] proposed a projection-like approach to avoid spectral search. In [14], the authors generalized the co-prime arrays from the viewpoint of difference co-array equivalence and verify the performance. In [15], Tan *et al.* proposed a super resolution DOA estimation approach for co-prime arrays. Recently, a partial search based method is proposed in [16] to reduce the complexity.

To the best of our knowledge, most of the works on co-prime arrays focus on 1-D DOA estimation and cannot be directly extended to estimate 2-D DOAs. Among various 2-D DOA methods, the classical multiple signal classification (MUSIC) method for 2-D case as [6] can provide a reasonable resolution and is regarded as one of the most popular techniques. In this paper, we study the 2-D DOA estimation method for co-prime planar arrays to enhance estimation performance with reduced complexity.

In this paper, we first formulate the co-prime planar array for the 2-D case, which includes two uniform planar subarrays of sizes $M \times M$ and $N \times N$, and the inter-element spacings for each subarray are N and M times of half wavelength, respectively. Similar to [12], there exist multiple ambiguous peaks for each real DOA in MUSIC spectrum. However, due to the co-prime property, the common peaks correspond to the real DOAs and there are no other common peaks except for the real DOAs. Therefore, we introduce the notation of joint spatial spectrum, which is defined as the product of spatial spectrum of the two subarrays. Accordingly, the joint spatial spectrum generates peaks only at the positions of real DOAs. Consequently, by searching for the peaks of the joint spectrum, the real DOAs can be uniquely estimated with phase ambiguity being removed successfully. We list the main novelty of this paper as follows:

- We formulate a special array geometry of 2-D co-prime planar array, which consists of two uniform subarrays. The non-uniform structure provides higher resolution than the uniform structure.
- We introduce the notation of the joint spectrum, by searching the peaks of which, the DOAs can be uniquely estimated. The proposed method can achieve a better performance-complexity tradeoff.
- We conduct extensive simulations to verify the effectiveness of the proposed method.

The rest of this paper is organized as follows. Section 2 gives the model of 2-D co-prime planar array. In Sect. 3, the proposed 2-D DOA estimation method is presented. Simulation results are provided in Sects. 4 and 5 concludes this paper.

2 System Model

Consider a co-prime planar array geometry that includes two uniform subarrays with size of $M_{s,i} = M_i \times M_i \ (i = 1, 2)$ sensors, where M_i denotes the sensor number along the x and y-axis of the ith subarray, the integers M_1 and M_2 are mutually co-prime. The spacing of the ith subarray is denoted as $d_i = M_{\widetilde{i}}\lambda/2$, where λ is wavelength and $\widetilde{i} = 1, 2, \widetilde{i} + i = 3$. The sensor elements of the ith subarray are located in the set $L_i = \{(md_i, nd_i), 0 \leq m, n \leq M_i - 1\}$. Therefore, the sensor elements of the entire array are located in $L = L_1 \cup L_2$. Figure 1 shows a specific co-prime planar array geometry with $M_1 = 4$ and $M_2 = 3$. The elements of two subarrays only overlap at the location (0, 0), therefore, the total number of sensor elements is $M_c = \sum_{i=1}^{2} M_i^2 - 1$.

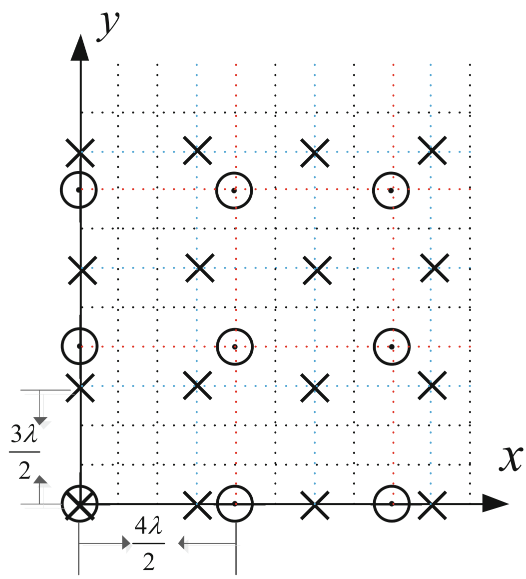

Fig. 1. An example of the co-prime planar array model with $M_1 = 3$ and $M_2 = 4$.

Assume K far-field narrowband sources imping on the array from different directions simultaneously. Specifically, the kth source is from angle $(\theta_k \phi_k)$, where the azimuth angle $\theta_k \in [0, \pi]$ is measured counterclockwise from the x-axis, and the elevation angle $\phi_k \in \left[0, \frac{\pi}{2}\right]$ is the angle between the incident direction and the z-axis.

The signal received by the entire array at time t $(t = 1, 2, \ldots, T)$ is denoted as

$$\mathbf{x}(t) = \mathbf{A}(\theta, \varphi)\mathbf{s}(t) + \mathbf{n}(t) = \sum_{k=1}^{K} \mathbf{a}(\theta_k, \phi_k) s_k(t) + \mathbf{n}(t) \in \mathcal{C}^{M_c \times 1} \tag{1}$$

where the $M_c \times K$ steering matrix $\mathbf{A}(\theta, \phi)$ is given as

$$\mathbf{A}(\theta, \phi) = [\mathbf{a}(\theta_1, \phi_1), \ldots, \mathbf{a}(\theta_K, \phi_K)] \tag{2}$$

Here the vector $\mathbf{s}(t) = [s_1(t), s_2(t), \ldots, s_K(t)]^T$ is the corresponding transmit signal vector, and $s_k(t)$ denotes the complex-valued transmit signal for source k; $\mathbf{n}(t) \in \mathcal{C}^{M_c \times 1}$ denotes the additive white Gaussian noise (AWGN) vector with zero means and variance σ_n^2. The steering vector $\mathbf{a}(\theta_k, \phi_k) \in \mathcal{C}^{M_c \times 1}$ is the response with respect to the angles θ_k and ϕ_k. For the sensor at the location (x, y), the corresponding response can be denoted as $a_{x,y}(\theta_k, \phi_k) = \exp(i \sin\phi_k [x \cos\theta_k + y \sin\theta_k])$. Therefore $\mathbf{a}(\theta_k, \phi_k)$ is given as

$$\mathbf{a}(\theta_k, \phi_k) = \left[a_{x_1, y_1}(\theta_k, \phi_k), \cdots, a_{x_{M_c}, y_{M_c}}(\theta_k, \phi_k)\right]^T \tag{3}$$

As in Fig. 1, we consider the two subarrays. The steering vectors corresponding to the kth source are

$$\begin{cases} \mathbf{a}_{s,1}(\theta_k, \phi_k) = \left[a_{x_{1,1}, y_{1,1}}(\theta_k, \phi_k), \cdots, a_{x_{1,M_{s,1}}, y_{1,M_{s,1}}}(\theta_k, \phi_k)\right]^T, \\ \mathbf{a}_{s,2}(\theta_k, \phi_k) = \left[a_{x_{2,1}, y_{2,1}}(\theta_k, \phi_k), \cdots, a_{x_{2,M_{s,2}}, y_{2,M_{s,2}}}(\theta_k, \phi_k)\right]^T \end{cases} \tag{4}$$

where $(x_{i,j}, y_{i,j})$ is the location of sensors, and the subscript i and j denote the ith subarray and jth sensor, where $i = 1, 2$ and $1 \leq j \leq M_{s,i}$. Then the received signal vectors are respectively defined as

$$\begin{cases} \mathbf{x}_{s,1}(t) = \mathbf{A}_{s,1}(\theta, \varphi)\mathbf{s}(t) + \mathbf{n}(t) \in \mathcal{C}^{M_{s,1} \times 1} \\ \mathbf{x}_{s,2}(t) = \mathbf{A}_{s,2}(\theta, \varphi)\mathbf{s}(t) + \mathbf{n}(t) \in \mathcal{C}^{M_{s,2} \times 1} \end{cases} \tag{5}$$

where $\mathbf{A}_{s,1}(\theta, \phi) = [\mathbf{a}_{s,1}(\theta_1, \phi_1), \ldots, \mathbf{a}_{s,1}(\theta_K, \phi_K)] \in \mathcal{C}^{M_{s,1} \times K}$ and $\mathbf{A}_{s,2}(\theta, \phi) = [\mathbf{a}_{s,2}(\theta_1, \phi_1), \ldots, \mathbf{a}_{s,2}(\theta_K, \phi_K)] \in \mathcal{C}^{M_{s,2} \times K}$ are the steering matrices for the first and second subarrays, respectively.

3 MUSIC Spectrum and Proposed Method

To estimate 2-D DOAs, in this section, we first use the classic 2-D MUSIC approach for the two subarrays, where phase ambiguity problem arises due to the larger inter-element spacing. According to the co-prime property, the two spatial spectrums have common peaks only at the positions of real DOAs. Therefore, we can uniquely estimate the DOAs by finding the peaks of joint spatial spectrum, which is defined as the product of the spectrums of the two subarrays.

3.1 MUSIC Spectrum

The covariance matrices of data vector (5) are obtained as

$$\begin{cases} \mathbf{R}_1 = E\left[\mathbf{x}_{s,1}(t)\mathbf{x}_{s,1}^H(t)\right] = \mathbf{A}_{s,1}(\theta,\phi)\mathbf{R_{ss}}\mathbf{A}_{s,1}^H(\theta,\phi) + \sigma_n^2\mathbf{I}_{M_{s,1}} \\ \mathbf{R}_2 = E\left[\mathbf{x}_{s,2}(t)\mathbf{x}_{s,2}^H(t)\right] = \mathbf{A}_{s,2}(\theta,\phi)\mathbf{R_{ss}}\mathbf{A}_{s,2}^H(\theta,\phi) + \sigma_n^2\mathbf{I}_{M_{s,2}} \end{cases} \tag{6}$$

where $\mathbf{R_{ss}} \overset{\Delta}{=} E\left\{\mathbf{s}(t)\,\mathbf{s}^H(t)\right\}$ is the source covariance matrix. In practice, the theoretical array covariance matrix $\mathbf{R}_i$ given in Eq. (6) is unavailable and it is usually estimated by

$$\begin{cases} \widehat{\mathbf{R}}_1 = \frac{1}{T}\sum\limits_{t=1}^{T}\mathbf{x}_{s,1}(t)\mathbf{x}_{s,1}^H(t) \\ \widehat{\mathbf{R}}_2 = \frac{1}{T}\sum\limits_{t=1}^{T}\mathbf{x}_{s,2}(t)\mathbf{x}_{s,2}^H(t) \end{cases} \tag{7}$$

where T is the number of snapshots. The eigenvalue decomposition (EVD) of (7) can be expressed as

$$\begin{cases} \widehat{\mathbf{R}}_1 = \widehat{\mathbf{S}}_1\widehat{\mathbf{\Lambda}}_{1,s}\widehat{\mathbf{S}}_1^H + \widehat{\mathbf{G}}_1\widehat{\mathbf{\Lambda}}_{1,n}\widehat{\mathbf{G}}_1^H \\ \widehat{\mathbf{R}}_2 = \widehat{\mathbf{S}}_2\widehat{\mathbf{\Lambda}}_{2,s}\widehat{\mathbf{S}}_2^H + \widehat{\mathbf{G}}_2\widehat{\mathbf{\Lambda}}_{2,n}\widehat{\mathbf{G}}_2^H \end{cases} \tag{8}$$

where $\widehat{\mathbf{S}}_i$ and $\widehat{\mathbf{G}}_i$ are the estimated signal- and noise-subspace matrices, respectively. The MUSIC [6] spatial spectrum are obtained as

$$\begin{cases} P_{MUSIC,1}(\theta,\phi) = \frac{1}{\mathbf{a}_{s,1}^H(\theta,\phi)\widehat{\mathbf{G}}_1\widehat{\mathbf{G}}_1^H\mathbf{a}_{s,1}(\theta,\phi)} \\ P_{MUSIC,2}(\theta,\phi) = \frac{1}{\mathbf{a}_{s,2}^H(\theta,\phi)\widehat{\mathbf{G}}_2\widehat{\mathbf{G}}_2^H\mathbf{a}_{s,2}(\theta,\phi)} \end{cases} \tag{9}$$

We plot the MUSIC spectrum of each subarray in Fig. 2. In the next section, we analyze the problem of phase ambiguity, which is caused by the larger distance between adjacent elements.

3.2 Joint Spectrum and Proposed Method

In the two decomposed subarrays, due to the large distance between adjacent sensors, there exists the problem of phase ambiguity [12]. Figure 2(a) and (b) shows the MUSIC spectrum for each decomposed subarray. As can be seen, there exist multiple peaks for each source. The DOA cannot be uniquely estimated by a single subarray. Then we consider combining the results of the two subarrays. For each source, it must generate a peak at the position of real DOA, i.e., there exists at least one common peak for the two subarrays. However, due to the special property of the co-prime structure, there are no common peaks except for the real DOAs. Therefore, we define the joint spectrum as

$$P_{MUSIC}(\theta,\phi) = P_{MUSIC,1}(\theta,\phi) \times P_{MUSIC,2}(\theta,\phi) \tag{10}$$

In the joint spectrum $P_{MUSIC}(\theta,\phi)$, the peaks are only generated at the positions of real DOAs. We plot the joint spectrum $P_{MUSIC}(\theta,\phi)$ as shown

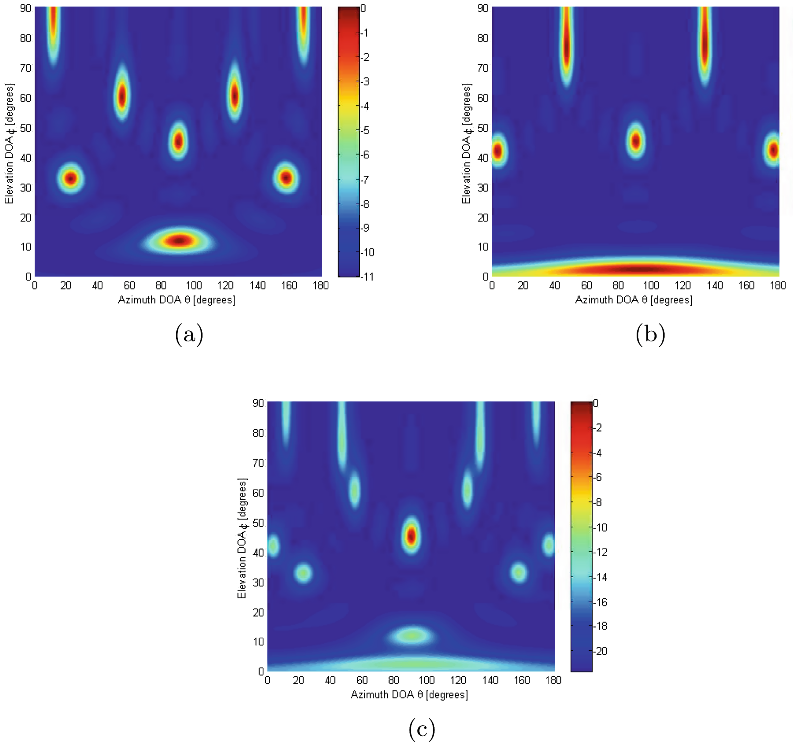

Fig. 2. The normalized spatial MUSIC spectrum with $M_1 = 3$ and $M_2 = 4$: (a) the spectrum of the first subarray, (b) the spectrum of the second subarray, and (c) the joint spectrum. The true DOA of the source is $(\theta, \phi) = (90^\circ, 45^\circ)$.

in Fig. 2(c). It is quite clearly that although there exists the problem of phase ambiguity in each subarray, we can eliminate it by combing the results of each subarray. Consequently, by searching for the peaks of the joint spectrum, the real DOAs can be uniquely estimated.

In 2-D MUSIC based DOA estimation methods, since they all involve a tremendous 2-D spectral search step, the complexity of which is much heavier than that of EVD [18]. Therefore, we approximately compute the complexity in terms of spectral search. The complexity of 2-D MUSIC is roughly denoted as $\mathcal{O}\left(J\left(M_1^2 + M_2^2\right)^2\right)$, where J denotes the number of spectral points over the total angular field. The complexity of the proposed method is about $\mathcal{O}\left(J\left(M_1^4 + M_2^4\right)\right)$. Hence, the computational complexity of proposed method is reduced, as compared to the 2-D MUSIC method.

4 Numerical Results

We validate the estimation performance of the proposed method via simulations in this section. We exhibits numerical results to compare the proposed method for co-prime planar arrays with that of 2-D MUSIC method for uniform planar arrays. The two decomposed uniform arrays of the co-prime planar arrays are of the size $M_1 \times M_1$ and $M_2 \times M_2$, with $M_1 = 3$ and $M_2 = 4$. For fair comparison, the traditional uniform planar array is set with the size of 4×6. The searching interval is set as $0.1°$ for all methods. The root mean square error (RMSE), defined as

$$RMSE = \sqrt{\frac{1}{QK}\sum_{q=1}^{Q}\sum_{k=1}^{K}\left(\left(\theta_k - \widehat{\theta}_k^{(q)}\right)^2 + \left(\phi_k - \widehat{\phi}_k^{(q)}\right)^2\right)} \tag{11}$$

is used as the performance metric. Here Q is the total number of Monte-Carlo trials. $\left(\widehat{\theta}_k^{(q)}, \widehat{\phi}_k^{(q)}\right)$ is the estimate result of the kth true DOA at the qth trial, $q = 1, 2, \ldots, Q$. In the section, $Q = 400$ rounds of Monte-Carlo runs are conducted.

First, we consider the case that there exists only one source at the DOA $(20°, 30°)$. Figure 3 shows the RMSE performance of different methods versus SNR when the snapshot number $T = 100$ and $T = 400$, respectively. Figure 4 plots he RMSE performance different methods versus the snapshot number when $SNR = 0$ dB and $SNR = 5$ dB, respectively. As is shown, with the increase of SNR and the snapshot number, the RMSE performance of the two methods improves gradually. As compared with 2-D MUSIC, the proposed method

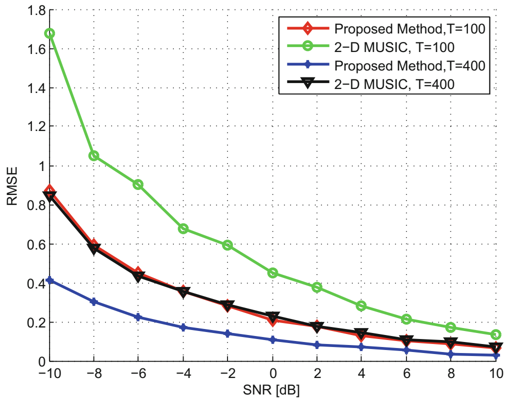

Fig. 3. RMSEs of different methods versus SNR with a single source at the DOA $(20°, 30°)$.

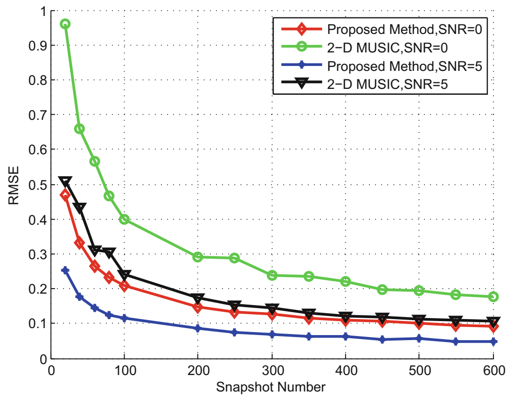

Fig. 4. RMSEs of different methods versus the snapshot number with a single source at the DOA $(20°, 30°)$.

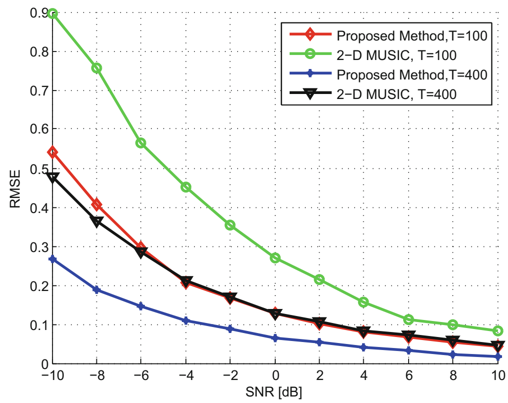

Fig. 5. RMSEs of different methods versus SNR with a single source at the DOAs $(20°, 30°)$, $(60°, 40°)$, and $(80°, 60°)$, respectively.

provides superior estimation performance. Since the complexity of the proposed method is smaller than 2-D MUSIC, therefore, the proposed method can achieve a better performance-complexity tradeoff.

For the case of three signal sources, Figs. 5 and 6 show the RMSE performance versus SNR and the snapshot number, respectively. The three desired sources are at the DOAs $(20°, 30°)$, $(60°, 40°)$, and $(80°, 60°)$, respectively. All the other simulation parameters are the same as the case of one signal source. As can be seen, the proposed method is always better than the 2-D MUSIC method across the whole SNR range and snapshot parameter range.

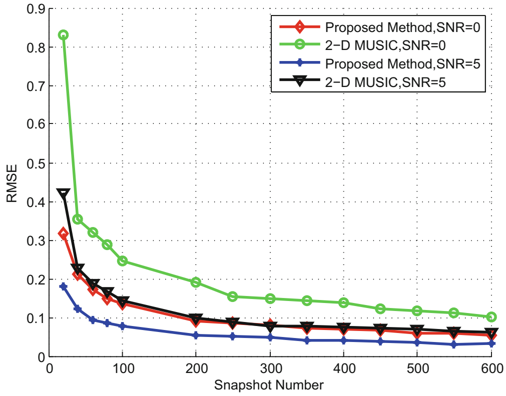

Fig. 6. RMSEs of different methods versus the snapshot number with three sources at the DOAs $(20°, 30°)$, $(60°, 40°)$, and $(80°, 60°)$, respectively.

5 Conclusion

In this paper, we have constructed a special array geometry of 2-D co-prime planar arrays, which can be decomposed into two uniform planar subarrays with the distance between adjacent sensors larger than the half-wavelength. For a single source, there exist multiple ambiguous DOAs in each subarray. To eliminate ambiguity, we introduced the notion of joint spectrum which is defined as the product of the 2-D MUSIC spectrum of the two subarrays. By utilizing the co-prime property, the real DOA can be uniquely estimated by searching for the peak of the joint spectrum. Extensive simulations have been conducted to verify the effectiveness of the proposed method and results are compared with that from the classic 2-D MUSIC method.

Acknowledgment. This work is supported by Key Projects in the National Science and Technology Pillar Program during the Twelfth Fiveyear Plan Period (2011BAD32B02). Shandong Provincial Natural Foundation, China (ZR2017PF007 and ZR2016FB19).

References

1. Krim, H., Viberg, M.: Two decades of array signal processing research: the parametric approach. IEEE Sig. Process. Mag. **13**(4), 67–94 (1996)
2. Wu, Q., Ding, G., Wang, J., Yao, Y.-D.: Spatial-temporal opportunity detection for spectrum-heterogeneous cognitive radio networks: two-dimensional sensing. IEEE Trans. Wirel. Commun. **12**(2), 516–526 (2013)
3. Ding, G., Wu, Q., Yao, Y.-D., Wang, J., Chen, Y.: Kernel-based learning for statistical signal processing in cognitive radio networks: theoretical foundations, example applications, and future directions. IEEE Sig. Process. Mag. **30**(4), 126–136 (2013)

4. Wu, Q., Ding, G., Xu, Y., Feng, S., Du, Z., Wang, J., Long, K.: Cognitive internet of things: a new paradigm beyond connection. IEEE Internet Things J. **1**(12), 129–143 (2014)
5. Liang, J., Liu, D.: Joint elevation and azimuth direction finding using L-shaped array. IEEE Trans. Antennas Propag. **58**(6), 2136–2141 (2010)
6. Chen, Y.M., Lee, J.H., Yeh, C.C.: Two-dimensional angle-of-arrival estimation for uniform planar arrays with sensor position errors. IEE Proc. F Radar Sig. Process. **140**(1), 37–42 (1993)
7. Zhang, W., Liu, W., Wang, J., Wu, S.: Computationally efficient 2-D DOA estimation for uniform rectangular arrays. Multidimension. Syst. Sig. Process. **25**(4), 847–857 (2013)
8. Heidenreich, P., Zoubir, A.M., Rubsamen, M.: Joint 2-D DOA estimation and phase calibration for uniform rectangular arrays. IEEE Trans. Sig. Process. **60**(9), 4683–4693 (2012)
9. Chen, H., Wang, Y., Wan, S.: Performance improvement of estimation direction-of-arrival via array geometry arrangement. In: IEEE International Symposium Antennas and Propagation Society, vol. 3, pp. 1600–1603. IEEE (1999)
10. Moffet, A.: Minimum-redundancy linear arrays. IEEE Trans. Antennas and Propag. **16**(2), 172–175 (1968)
11. Vaidyanathan, P.P., Pal, P.: Sparse sensing with co-prime samplers and arrays. IEEE Trans. Sig. Process. **59**(2), 573–586 (2011)
12. Zhou, C., Shi, Z., Gu, Y., Shen, X.: DECOM: DOA estimation with combined MUSIC for coprime array. In: Proceedings of IEEE International Conference on Wireless Communications and Signal Process (WCSP), Hangzhou, pp. 1–5 (2013)
13. Weng, Z., Djuric, P.: A search-free DOA estimation algorithm for coprime arrays. Digit. Sig. Process. **24**, 27–33 (2014)
14. Qin, S., Zhang, Y., Amin, M.G.: Generalized coprime array configurations for direction-of-arrival estimation. IEEE Trans. Sig. Process. **63**(6), 1377–1390 (2015)
15. Tan, Z., Eldar, Y.C., Nehorai, A.: Direction of arrival estimation using co-prime arrays: a super resolution viewpoint. IEEE Trans. Sig. Process. **62**(21), 5565–5576 (2014)
16. Sun, F., Lan, P., Gao, B.: Partial spectral search-based DOA estimation method for co-prime linear arrays. Electron. Lett. **51**(24), 2053–2055 (2015)
17. Sun, F., Wu, Q., Sun, Y., Ding, G., Lan, P.: An iterative approach for sparse direction-of-arrival estimation in co-prime arrays with off-grid targets. Digit. Sig. Process. **61**, 35–42 (2016). http://dx.doi.org/10.1016/j.dsp.2016.06.007
18. Rubsamen, M., Gershman, A.B.: Direction-of-arrival estimation for nonuniform sensor arrays: from manifold separation to Fourier domain MUSIC methods. IEEE Trans. Sig. Process. **57**, 588–599 (2009)