# Blind Spectrum Sensing in Cognitive Radio Using Right Anderson Darling Test

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**Abstract.** Goodness of Fit tests have been used to find available spectrum with excellent detection performance in Cognitive Radio System. To extend those works, in this paper, we reformulate the spectrum sensing as a unilateral Goodness of Fit testing problem. With difference to previous available works, a random variable that obeys central F distribution with presence of primary user (PU) signal and a non-F distribution with absence of PU signal, which provides technical support for achieving blind spectrum sensing; furthermore, inspired by the thought of unilateral hypothesis test, we apply Right Anderson Darling (RAD) test to achieve bind spectrum sensing and derive a blind spectrum sensing called RAD sensing. Finally, the validness of proposed algorithm is proved by enormous Monte Carlo simulations.

**Keywords:** Cognitive radio · Blind spectrum sensing · F distribution · Right-Anderson Darling test

## 1 Introduction

Cognitive radio (CR) is a dynamic spectrum management technology by means of finding "spectrum holes" and making full use of idle spectrum, which is designed to solve spectrum shortage. Spectrum sensing, as a prerequisite and basis technology for CR, is to monitor spectrum state and detect "spectrum holes" in order to avoid interference to the primary user (PU) [1].

To this end, the common spectrum sensing algorithms consist of Cyclostationary Feature Detection (CFD) algorithm, Matched-Filter detection (MF), Energy Detection (ED), eigenvalue based spectrum sensing and Goodness of Fit (GoF) based spectrum sensing [2]. For examples, the PU signal (i.e., signal waveform, modulation, etc.) must be as a prior information, in addition, MF has relatively high requirement for synchronization [3]; ED algorithm is the most common method because of its low complexity, however it is sensitive to noise uncertainty, which results in low robustness and difficulty in setting threshold [4].


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To overcome this difficulty, algorithms based on eigenvalue are proposed such as Maximum Minimum Eigenvalue (MME) [5] and generalized likelihood ratio test (GLRT) [6] based spectrum sensing. The proposed algorithms are free of noise uncertainty but at the cost of high computational complexity.

Recently, Goodness of Fit (GoF) test, as a nonparametric hypothesis test, has been widely used in cognitive radio system via testing whether the received signal comes from the assumed distribution [7–13]. In this case, the spectrum sensing problem is transformed into a GoF testing problem and GoF test (i.e., AD criterion) is used to examine it. To be more explicit, we firstly assume the received signal obeys a particular distribution with the absence of PU signal and deviates from the distribution with the presence of PU signal, and then exploit the GoF test to solve the above problem. For instance, Wang assumes that the PU signals remain the same during the sampling period due to the fact that PU signals are often narrowband signal whose envelope changes very slowly after down-conversion, low pass filtering and sampling at CR system, in such hypothesis, the author first presents Anderson Darling (AD) sensing, where the spectrum sensing is converted into check whether the received signal obeys the normal distribution or not [7]. In addition, the performance of AD sensing is evaluated through enormous Monte Carlo simulations and prove that AD sensing outperforms ED. However, the noise power is needed for AD sensing [8]. To achieve blind spectrum sensing, a new random variable is constructed and the spectrum sensing problem is reformulated as a Students testing problem; then the AD test is used to achieve spectrum sensing [9]. Similarly, the characteristic function is also exploited into spectrum sensing; then a blind spectrum sensing based on characteristic function and AD test (CAD) is proposed [9]. Afterwards, Shen extended it into MISO [10] and MIMO CR system [11]. However, the spectrum sensing algorithms based on characteristic function have heavy complexity compared to AD sensing [7] and Students distribution based spectrum sensing [9].

In this paper, from another perspective to view the spectrum sensing based on GoF test, we extend the above works [7–11] and reformulate the spectrum sensing as a unilateral F distribution testing problem. In addition, we construct a random variable and prove that it obeys a central F distribution when the PU signal is not transmitted and a non-F distribution when the PU signal is transmitted. The constructed random variable provides technical support for achieving blind spectrum sensing since it is free of noise power. Inspired by [14], the AD criterion is suited to two-sided hypothesis test problem due to the fact that the AD criterion gives equal weight to differences between empirical and theoretical distribution functions corresponding to all the observation and the spectrum sensing problem is transformed as a unilateral F distribution testing problem in this paper. Therefore, we apply the Right AD (RAD) criterion [14], which addresses the difference between the empirical distribution and assumed distribution in right tail, to check whether the constructed random variable comes from central F distribution or not and present a blind spectrum dubbed RAD sensing. Finally, the validness of proposed algorithm is proved by the enormous Monte Carlo simulations.

## 2 System Model

Without loss of generality, spectrum sensing is transmitted as a binary hypothesis testing problem: $H_0$ denotes the null hypothesis (absence of the primary user) and $H_1$ stands for the alternative hypothesis (presence of the primary user). To be more explicit, the spectrum sensing mathematical model can be described as

$$\begin{cases} x_i = w_i, & H_0 \\ x_i = \sqrt{\rho}u + w_i, & H_1 \end{cases} \tag{1}$$

Where $x_i$ is the received signal at time $i$ $(i = 1, 2, ..., N)$; $w_i$ represents additive white Gaussian noise (AWGN) with mean zero and variance $\sigma^2$, that is, $w_i \sim \mathrm{N}(0, \sigma^2)$; $u$ is the signals transmitted by PU, $\rho$ meets $\mathrm{SNR} = 10\lg(\rho u^2/\sigma^2)$ and SNR is signal-to-noise ratio (SNR). We adopt the PU signal model in [7–12] and assume that $u = 1$. Therefore, when SNR remains unchanged, $x_i$ obeys Gaussian distribution, that is,

$$x_i \sim \begin{cases} N(0, \sigma^2)\,, & H_0 \\ N(k\sigma, \sigma^2)\,, & H_1 \end{cases} \tag{2}$$

where $k = \sqrt{\rho}/\sigma = \sqrt{10^{0.1SNR}}$.

## 3 Spectrum Sensing as Goodness of Fit Testing

Generically, achieving blind spectrum sensing based on GoF test has two steps. The first step is to construct a random variable, which is free of noise variance and has obvious difference between $H_1$ and $H_0$, so as to formulate the spectrum sensing as GoF testing problem. The second step is to find a powerful GoF criterion to verify the above problem. In the following, we will obtain a random variable with irrelevance of noise variance and formulate spectrum sensing as GoF testing problem.

First, we divide the received signals $x_1, x_2, ..., x_N$ into $L(L < N)$ parts, each part has $M = N/L$ data. Thus, the mean and variance of $l_{th}$ $(l = 1, 2, ..., L)$ sample can be expressed as, respectively

$$\bar{x}_l \triangleq \frac{1}{M} \sum_{i=1+M(l-1)}^{Ml} x_i \tag{3}$$

$$s_l^2 \triangleq \frac{1}{M-1} \sum_{i=1+M(l-1)}^{Ml} (x_i - \bar{x}_l)^2 \tag{4}$$

**Lemma 1.** *Let's denote a new random variable* $T_l \triangleq \frac{M\bar{x}_l^2}{s_l^2}$, *if* $x_i \sim N(0, \sigma^2)$, *the variable* $T_l$ *obeys the central F distribution with 1 and* $M-1$ *degrees of freedom respectively.*

*Proof.* If $\bar{x}_l \sim N(0, \sigma^2/M)$, it is obvious that $M\bar{x}_l/\sigma^2 \sim \chi_1^2$ after normalization and square. Note that $\chi_1^2$ is central chi-square distribution with 1 degree of freedom and noncentral parameter $k^2M$. In addition, the random variable $(M-1)s_l^2/\sigma^2$ obeys central chi-square distribution with $M-1$ degrees of freedom according to Cochran Theorem [15]. Based on the above analysis, it is easily obtained that $M\bar{x}_l^2/s_l^2$ has central F distribution with 1 and $M-1$ degrees of freedom respectively, that is, $T_l \sim F_{1,M-1}$.

**Lemma 2.** *If $x_i \sim N(k\sigma, \sigma^2)$, the variable $T_l$ obeys the noncentral F distribution with 1, $M-1$ degrees of freedom and noncentral parameter $k^2M$ respectively, that is, $T_l \sim F_{1,M-1,k^2M}$.*

*Proof.* If $\bar{x}_l \sim N(k\sigma, \sigma^2/M)$, it is easily to find that $M\bar{x}_l/\sigma^2 \sim \chi^2_{1,k^2M}$ after normalization and square. Note that $\chi^2_{1,k^2M}$ is non-central chi-square distribution with 1 degree of freedom and noncentral parameter $k^2M$. Similarly, according to Cochran Theorem [15], the random variable $(M-1)s_l^2/\sigma^2$ obeys chi-square distribution with $M-1$ degrees of freedom. Therefore, it is easily obtained that $M\bar{x}_l^2/s_l^2$ has noncentral F distribution with 1, $M-1$ degrees of freedom and noncentral parameter $k^2M$ respectively, that is, $T_l \sim F_{1,M-1,k^2M}$.

From the Lemmas 1 and 2, when there is no PU signal within the desired frequency band, the random variable $T_l$ comes from the central F distribution with 1 and $M-1$ degrees of freedom; when the PU signal is transmitted within the desired frequency, the random variable $T_l$ obeys the noncentral F distribution with 1, $M-1$ degrees of freedom and noncentral parameter $k^2M$. Note that the probability density function (PDF) of the noncentral F distribution deviates rightward from the central F distribution.

To sum up, the spectrum sensing can be described as the following GoF testing problem,

$$\begin{cases} T_l \, obeys \, F_{1,M-1}, & H_0 \\ T_l \, deviates \, rightward \, from \, F_{1,M-1}, & H_1 \end{cases} \tag{5}$$

## 4 Right-Anderson Darling Sensing

In this section, in order to find "spectrum holes", GoF test is used to examine the above problem that is described in (5) via measuring the distance between empirical Cumulative Distribution Function (CDF) and assumed CDF. Most previous works utilize AD criterion to achieve spectrum sensing due to effectiveness for two-sided hypothesis testing problem. The AD criterion can be written as

$$A_L^2 = L \int_{-\infty}^{+\infty} [G_L(T) - G_0(T)]^2 \frac{dG_0(T)}{G_0(T)(1-G_0(T))} \tag{6}$$

where $L$ is the number of constructed random variable $T_l$; $(G_0(T)(1-G_0(T))^{-1}$ is nonnegative weight function. $G_0(T)$ is the assumed CDF; $G_L(T)$ is the empirical CDF and can be calculated by

$$G_L(T) = |\{i : T_i \leq T, 1 \leq i \leq L\}| \tag{7}$$

where |•| is indicates cardinality. From (6) and [14], AD criterion is obviously not the best choice for unilateral hypothesis test since it gives equal weight to both tails of distributions and not utilizes all of unilateral hypothesiss feature(i.e., $G_L(T) - G_0(T)$ is always less than zero in theory when there has transmitted PU signal, in this case, the absolute of $G_L(T) - G_0(T)$ is bound to lead to performance loss). To surmount this problem, based on the AD criterion, Jin et al. [13] proposes a unilateral AD (UAD) criterion using $G_L(T) - G_0(T)$ rather than the absolute of $G_L(T) - G_0(T)$ in (6); furthermore, the author verify that the UAD criterion is more powerful for unilateral hypothesis via Monte Carlo simulation compared to AD criterion. However, the theoretical detection threshold only is gotten via Monte Carlo simulation, which hinder the field of application.

Subsequently, sinclair proposed a more power GoF test dubbed as RAD criterion for unilateral hypothesis via modifying the nonnegative weight function and giving large weight to the right tail [14]. Moreover and fortunately, the author gives the way to calculate accurate theoretical detection threshold and prove that RAD test is more powerful for unilateral hypothesis compared to AD criterion.

In this paper, we select RAD criterion to test unilateral hypothesis due to its effectiveness for testing (5), and apply it to spectrum sensing, yielding a blind spectrum sensing dubbed RAD sensing. The RAD test statistic is given by

$$R_L^2 = L \int_{-\infty}^{+\infty} \left[G_L(T) - G_0(T)\right]^2 \frac{dG_0(T)}{(1 - G_0(T))} \tag{8}$$

By breaking the whole integral in (8) into $L$ parts, it is easy to show that it can be rewritten as

$$R_L^2 = \frac{L}{2} - 2\sum_{n=1}^{L} Z_n - \frac{1}{L}\sum_{n=1}^{L} (2n-1)\ln(1 - Z_{L+1-n}) \tag{9}$$

where $Z_n = G_0(T_l)$. Once the $R_L^2$ is acquired, it will be compared with a threshold $\lambda_{RAD}$ using the following detection criterion

$$\begin{cases} R_L^2 \geq \lambda_{RAD}, H_1 \\ R_L^2 < \lambda_{RAD}, H_0 \end{cases} \tag{10}$$

According to [14], we can find a function to describe the relationship between threshold and false alarm probability ($P_f$) for RAD criterion. The functions is described as,

$$P_f = 0.889(1.835\lambda_{RAD})^{-1/2} \exp(-1.835\lambda_{RAD}) \tag{11}$$

where the $\lambda_{RAD}$ is the detection threshold. For a given $P_f$, we can approximately calculate the $\lambda_{UAD}$ using formula (11). For examples, when $P_f = 0.05$, $\lambda_{UAD} = 1.303$; when $P_f = 0.1$, $\lambda_{UAD} = 2.060$.

In summary, RAD sensing algorithm can be concluded as follows

Step 1: For a given false alarm probability, calculate decision threshold $\lambda_{RAD}$. via formula (11);
Step 2: Compute via formula (9);
Step 3: Make a conclusion according to formula (10).

## 5 Simulation Results

In this section, simulation is implemented using Matlab and detailed analysis is given in order to compare the performance of five algorithms (i.e., AD sensing, ED method, CAD sensing and RAD sensing, AD-F sensing). Note that AD-F sensing presents that AD criterion is used to test (5). The performance is assessed via the maximum of detection probability in accord with a certain false alarm probability. Note that noise variance is not needed for RAD, AD-F, CAD sensing.

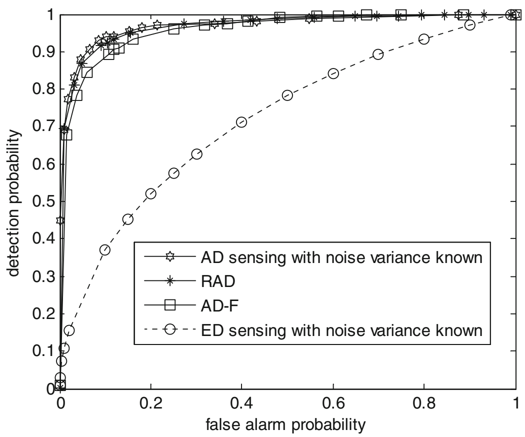


**Fig. 1.** ROC curves for four methods over AWGN channel with SNR = $-8$ dB, $L = 64$

For assessing the performance of RAD sensing, Fig. 1 gives receiver operating curves (ROC) of four algorithms with SNR $= -8$ dB and $N = 64$ over Additive White Gaussian Noise (AWGN) Channels. As is shown in Fig. 1, AD sensing has the best detection performance among four algorithms. For example, when $P_f = 0.1$, the detection probabilities of AD sensing, RAD sensing, AD-F sensing and AD are about 0.92, 0.91, 0.84 and 0.39 respectively. It is worth noting that RAD sensing outperforms AD-F sensing since the RAD criterion is more powerful than AD criterion for unilateral alternative hypothesis, which is corresponding to a practical case. Note that the noise variance is not needed for RAD sensing and AD-F sensing.

Figure 2 presents the detection probabilities of AD sensing, CAD sensing, RAD sensing with respect to different SNRs for $N = 64$ and $P_f = 0.05$ in the case of quasi static fading channel. In the quasi static fading channel, the channel gain is assumed to obey the standard normal distribution in this simulation. From Fig. 2, on one hand, the performance of AD sensing is also great than RAD and CAD because the noise variance is needed for AD sensing; on the other hand, it is not hard to find that RAD sensing has best detection probability when $L = 1$. Note that with the absence of noise variance, in this case, RAD algorithm has a marginal performance loss compared to AD sensing with the noise known and slightly outperforms the CAD sensing. Note that CAD sensing dose not need noise variance.

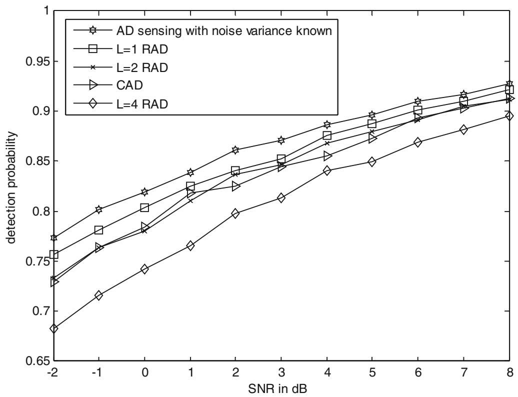


**Fig. 2.** Detection probability against SNRs for three methods over a quasi static fading channel with $P_f = 0.05$, $L = 64$

## 6 Conclusion

In this paper, we construct a variable random and formulate the spectrum sensing as a unilateral GoF testing problem. Then a powerful GoF test called RAD criterion is applied to examine it and a blind spectrum sensing dubbed RAD sensing is proposed. Both simulation and analysis demonstrate that the RAD sensing has excellent performance without the need of noise uncertainty. For instance, RAD sensing is better than CAD sensing and has a ignorable performance loss compared to AD sensing. Note that the noise variance is needed for AD sensing but is not needed for CAD sensing. In further, we are interested to extend our work into MISO CR system and MIMO CR system.

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