Multiantenna Based Blind Spectrum Sensing via Nonparametric Test

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Abstract. Multiantenna based spectrum sensing algorithms are widely used in cognitive radio networks on account of improving the system reliability. Utilizing the difference between the received signal and the noise statistical covariances, two kinds of novel spectrum sensing algorithms, binomial distribution based detection (BD) and wilcoxon signed rank test based detection (WSD), are proposed based on the sample covariance matrix calculated from a limited number of received signal samples. BD and WSD algorithms do not need any priori information of the primary signal and the noise. In addition, their thresholds are found via the statistical theory. Compared with energy detection (ED), maximum-minimum eigenvalue (MME) and covariance absolute value (CAV), those two algorithms can obtain better performance. Finally, the performance of the proposal is verified by simulations.

Keywords: Cognitive radio · Spectrum sensing · Multiantenna · Covariance matrix · Binomial distribution · Wilcoxon signed rank test

1 Introduction

High-rate wide-band wireless communication will bring a huge demand of spectrum. Limited available spectrum resources and inefficient static spectrum allocation policy result that the lack of spectrum resources become increasingly serious. On the one hand, the rapid growth of radio communication service and the appearance of new system, protocol and network result in further competition of wireless spectrum. On the other hand, most of the available spectrum have been assigned to the licensed users. Therefore, unlicensed users can only use the a little spectrum which is nearly saturation.

The statically spectrum allocation way, which is widely used in the current spectrum management system, causes that a large amount of idle spectrum are not applied, and then leads to low spectrum efficiency. To solve the problem of the shortage of spectrum resources, regulators consider using dynamic allocation for spectrum management.

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Cognitive Radio (CR) [1], which achieves the dynamic allocation of spectrum resources, can improve the spectrum utilization. As the fundamental task for CR network, spectrum sensing [2] allows secondary users to use the licensed spectrum to primary users when they are not active.

It has been shown that multiantenna based blind spectrum sensing algorithms are not affected by noise uncertainty to which the classical energy detection (ED) [3,4] is known to be sensitive. Those algorithms are mainly divided into two categories, the eigenvalue based algorithms [5,6] and the covariance based algorithms [7,8]. The eigenvalue based algorithms, such as the maximum minimum eigenvalue (MME) [5] and the maximum eigenvalue trace (MET) [6] algorithms, exploit the difference between the eigenvalue of sample covariance matrices of the primary signals and noises. Nevertheless, it is difficult to obtain the accurate decision threshold. Meanwhile, high computational intensity is required for those detectors.

The covariance absolute value (CAV) [7] algorithm is proposed in order to overcome the above defects. The test statistic is set according to the ratio of the sum of all elements in the absolute value of covariance matrix and the sum of the absolute value of diagonal elements. The accurate decision threshold can be obtained and the computational complexity is low because it does not need eigen-decomposition. However, their performance is worse than ED.

In this paper, we present two kinds of novel nonparametric tests based spectrum sensing algorithms, which utilize the upper triangular elements of sample covariance matrix. The proposed BD and WSD algorithms require no prior information of noise and primary signal. In other words, they are robust to noise uncertainty. To compare with BD, WSD gets better performance but needs higher computational complexity. It is revealed in the simulation that the performance of proposed detectors is superior to traditional multiantenna based blind spectrum sensing algorithms and ED algorithm.

The rest of this paper is organized as follows. System model is given in Sects. 2 and 3 gives the proposed algorithm. Computational complexity of various algorithms are compared in Sect. 4. Simulation results are presented in Sect. 5 and concluding remarks are made in Sect. 6.

2 System Model

2.1 Signal Model

Consider a multiantenna cognitive radio system with one primary user with single antenna and one secondary user with M antennas. As shown in Fig. 1, the essential problem of spectrum sensing is to detect the primary user in the noise environment. Generally, the spectrum sensing can be expressed as a binary hypothesis test problem. H_0 denotes the null hypothesis (absence of the primary user) and H_1 stands for the alternative hypothesis (presence of the primary user). The signal model of spectrum sensing in array antenna receiver is formulated as follows:

$$x_m(n) = \begin{cases} w_m(n), & H_0 \\ hs(n) + w_m(n), & H_1 \end{cases}, n = 1, 2, \cdots, N \tag{1}$$

where $x_m(n)$ and $w_m(n)$ represent the received signal and additive white Gaussian noise (AWGN) from the mth$(1 \leq m \leq M)$ antenna, $w_m(n)$ $N(0, \sigma^2)$. $s(n)$ denotes the primary signal. h represents channel gain. Without loss of generality, we assume $h = 1$ in array antennas system. N is the number of samples. It is assumed that the primary signal is independent of the noise. Let us express the receive signals as a $M \times N$ matrix.

$$\mathrm{X} = \begin{bmatrix} x_1(1) & x_1(2) & \cdots & x_1(N) \\ x_2(1) & x_2(2) & \cdots & x_2(N) \\ \vdots & \vdots & \ddots & \vdots \\ x_M(1) & x_M(2) & \cdots & x_M(N) \end{bmatrix} \tag{2}$$

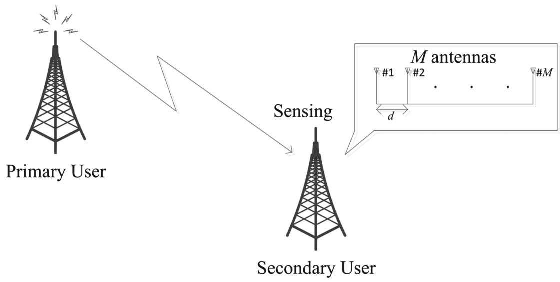

Fig. 1. System model for cognitive radio network with multiple antennas

The sample covariance matrix is defined as

$$\mathrm{R} = \frac{1}{\mathrm{N}} \mathrm{X}\mathrm{X}^{\mathrm{H}} \tag{3}$$

$r_{i,j}$ is the $(i, j)_{th}$ element of $\mathbf{R}$.

$$r_{i,j} = \frac{1}{N} \sum_{n=1}^{N} x_i(n) x_j(n) \tag{4}$$

There are two probabilities can measure the performance of different algorithms in spectrum sensing:

- probability of detection P_d, which defines the probability of the algorithm deciding the presence of primary signal exist under H_1.
- probability of false alarm P_{fa}, which defines the probability of the algorithm claiming the presence of the primary signal under H_0.

2.2 Previous Works

Energy Detection: multi-antenna assisted ED algorithm is a classical sensing method, which does not need any information of the primary signal. The total energy of received signal is regarded as the test statistic, namely

$$T_{ED} = \sum_{n=1}^{N} \sum_{m=1}^{M} |x_m(n)|^2 \underset{H_0}{\overset{H_1}{\gtrless}} \gamma_{ED} \tag{5}$$

Nevertheless, to calculate the decision threshold, the noise power is required. In practice, it is not easy to obtain accurate noise power so that the performance will decrease. The SNR wall, which means a minimum SNR below that a signal cannot be reliably detected, is also cased by noise uncertainty. This indicates that ED is influenced by the noise uncertainty.

Eigenvalue Based Detection: MME algorithm, which compares the ratio of the maximum eigenvalue and the minimum eigenvalue with a threshold, is proposed to overcome the impact of noise uncertainty in ED algorithm. The test statistic is given by

$$T_{MME} = \frac{\lambda_{\max}}{\lambda_{\min}} \tag{6}$$

where λ_{max} and min are the maximum and minimum eigenvalue of sample covariance matrix $\mathbf{R}$. MME algorithm is popular due to the fact that it does not need of any prior knowledge and is free of noise uncertainty. The drawback is its complexity caused by eigen-decomposition.

Covariance Based Detection: In order to reduce the complexity of MME algorithm, the difference of statistics covariance matrix between H_0 and H_1 is employed to detect whether PU exits or not. The test statistic is

$$T_{CAV} = \frac{\frac{1}{M} \sum_{i=1}^{M} \sum_{j=1}^{M} |r_{i,j}|}{\frac{1}{M} \sum_{i=1}^{M} |r_{i,i}|} \tag{7}$$

CAV algorithm avoids calculating the eigenvalue, therefore, the computational complexity decreased.

3 Nonparametric Test Based Detection

Although MME and CAV is robust to noise uncertainty, the performance is worse than ED. Two kinds of blind spectrum-sensing algorithms, which do not need any prior information of the primary signal and the noise, are proposed in this paper.

We consider the upper triangular elements of $\mathbf{R}(r_{i,j}, ij)$ to design test statistics. The total number of $r_{i,j}(i < j)$ in $\mathbf{R}$ is $M(M-1)/2$, which is been signed as L. Try to seek the difference of $r_{i,j}$s symcenter under H_0 and under H_1.

Under H_0, the mean value is given by

$$\begin{aligned} E\left[r_{i,j}\right] &= E\left[\frac{1}{N}\sum_{n=1}^{N} x_i(n)x_j(n)\right] \\ &= \frac{1}{N}\sum_{n=1}^{N} E\left[w_i(n)\right] E\left[w_j(n)\right] = 0 \end{aligned} \tag{8}$$

and under H_1, we have

$$\begin{aligned} E\left[r_{i,j}\right] &= E\left[\frac{1}{N}\sum_{n=1}^{N} x_i(n)x_j(n)\right] \\ &= \frac{1}{N}\sum_{n=1}^{N} E\left[s(n)^2\right] \end{aligned} \tag{9}$$

Comparing (5) and (6), we obtain that $r_{i,j}$s symcenter is zero under H0 but not equals to zero under H_1. We can realize spectrum sensing via evaluating the distributional difference of the data greater than zero and less than zero.

As the symcenter of the whole data, there are two points to consider.

- The data volume on both sides of the symcenter is equal.
- The distribution of data on both sides of the symcenter is identical.

3.1 Binomial Distribution Based Detection

BD algorithm only takes advantage of the first point to realize spectrum sensing. When H_0, the symcenter of $r_{i,j}$ is zero, so there is approximate one half data volume in both sides of zero. The number of the data greater than zero is half of the total data in theory, which equals to $L/2$.

When H_1, the symcenter of $r_{i,j}$ is larger than zero, so the number of the data greater than zero exceeds $L/2$ in theory. Based on the above analysis, we can determine the state of primary user by the means of whether the number of $r_{i,j}$ greater than zero equals to $L/2$. The test statistic for BD method is given as

$$T_{BD} = \sum_{i=1}^{M} \sum_{j=1, j>i}^{M} u(r_{i,j}) \tag{10}$$

where $u(\bullet)$ represents step function. It is obvious that spectrum sensing can be transformed to a hypothesis test problem.

$$\begin{cases} T_{BD} = L/2, H_0 \\ T_{BD} > L/2, H_1 \end{cases} \tag{11}$$

Once the T_B is computed, it will be compared to a predefined threshold λ_B. The statistical test problem transforms into

$$\begin{aligned} H_0 &: T_{BD} \leq \lambda_B \\ H_1 &: T_{BD} > \lambda_B \end{aligned} \tag{12}$$

To derive the false alarm probabilities for the BD detector, the cumulative distribution function (CDF) of the test statistic T_{BD} should be derived under hypotheses H_0. $u(r_{i,j})$ can be seen as a Bernoulli experiment. Therefore, T_B can be seen as $M(M-1)/2$ Bernoulli experiment.

$$T_B \sim B(L, p) \tag{13}$$

where B denotes binomial distribution. p represents the probability of $r_{i,j}$ greater than zero, and then according to the above analysis, $p = 0.5$ under H_0. Hence, the cumulative distribution function (CDF) [9] of T_{BD} is defined as

$$F_B(t) = \sum_{l=0}^{t} C_L^l 0.5^L \tag{14}$$

For the given P_{fa}, the threshold can be obtained as

$$\gamma_B = F_B^{-1}(1 - P_{fa}) \tag{15}$$

- From the received signal matrix $\mathbf{X}$, calculate the sample covariance matrix $\mathbf{R}$ according to (3).
- Count the elements larger than zero from the upper triangle of $\mathbf{R}$ as T_{BD}.
- Find the threshold λ_B for a given probability of false alarm according to (12).
- Accept the null hypothesis H_0 if $T_{BD} > \lambda_B$. Otherwise, reject H_0 in favor of the presence of the primary user signal.

It is worth mentioning that the BD only need count the element larger than zero from $\mathbf{R}$ and does not make full use of $\mathbf{R}$. In other words, BD can get low computational complexity and suboptimum performance.

3.2 Wilcoxon Signed Rank Test Based Detection

In the following, we describe the wilcoxon signed rand test, which is a nonparametric statistical test, based spectrum sensing detector. The WSD algorithm compares the distribution of the measures on the different sides of symcenter. When H_0, the distribution of the measures on the different sides of zero are the same. Nevertheless, when H_1, the two distributions are different.

First of all, obtain the absolute value $|r_{i,j}|\,(i < j)$, whose total number is L. Renumber those value: $|r_l|\,(l = 1, 2, , L)$. Then, rank $|r_l|$ and get order statistic $|r|_{(l)}$. Specifically, α_l represents the rank of $|r_l|$, in other words, $|r_l| = |r|_{(\alpha l)}$. If the symcenter of data is zero, the density should be approximately the same on both sides of the data. It further means that the original positive data and negative data should be staggered after taking absolute value. Therefore, the sum of rank of positive data is nearly equal to the sum of rank of negative data in the absolute value.

The test statistic of the WSD algorithm is given by

$$T_W = \sum_{l=1}^{L} \alpha_l u(r_l) \tag{16}$$

Under null hypothesis, the mean and variance of T_W can be expressed as

$$\begin{aligned} E(T_W) &= E(\sum_{l=1}^{L} \alpha_l u(r_l)) \\ &= \tfrac{1}{2}\sum_{l=1}^{L} E(\alpha_l) = \tfrac{1}{4}L(L+1) \end{aligned} \tag{17}$$

$$\begin{aligned} \mathrm{var}(T_W) &= \mathrm{var}(\sum_{l=1}^{L} \alpha_l u(r_l)) \\ &= \tfrac{1}{4}\sum_{l=1}^{L} \mathrm{var}(\alpha_l) = \tfrac{1}{24}L(L+1)(2L+1) \end{aligned} \tag{18}$$

A large sample modificatory test statistic of the WSD algorithm [10] is given by

$$T_{WSD} = \frac{T_W - \frac{1}{4}L(L+1)}{\sqrt{\frac{1}{24}L(L+1)(2L+1)}} \tag{19}$$

which under H_0 has a standard normal distribution. Given the required P_{fa}, the decision threshold is determined by

$$\gamma_W = F_W^{-1}(1 - P_{fa}) \tag{20}$$

where $F_W^{-1}(\bullet)$ is the inverse function of $F(\bullet)$ with $F(\bullet)$ being the CDF of T_W.

4 Implementation and Complexity Comparison

Figure 2 shows the procedures of WSD algorithm, BD algorithm, ED algorithm, MME algorithm and CAV algorithm. The computational complexity of calculateing sample covariance matrix is $\mathrm{O}(NM^2)$. WSD algorithm need to rank the upper triangle elements of $\mathbf{R}$, whose total number is $L = M(M-1)/2$. The rank complexity is $\mathrm{O}(Llog_2(L))$. The complexity of BD algorithms after calculate $\mathbf{R}$ is $\mathrm{O}(L)$. In the MME algorithm, a complexity of $\mathrm{O}(M^3)$ is required in the calculation of the eigen-decomposition. The complexity of MME algorithm is higher than the proposed algorithms. Although the procedure of ED algorithm is simple, it needs noise power as prior information and is influenced by noise uncertainty. The comparison of computational complexity is given in Table 1.

5 Simulation Results

To evaluate the performance of the proposed BD and WSD sensing algorithm, we have make simulations in MATLAB along with the ED, MME and CAV sensing methods. In the following simulations, $M = 4$, $N = 50$ and false alarm probability $P_{fa} = 0.1$ are assumed. When there is noise uncertainty α, the range of noise variance can be set as $[B^{-1}\sigma^2, B\sigma^2]$, where $B = 10^{0.1\alpha}$.

Figure 3 presents the performance comparison of those algorithms. The performance of ED descends dramatically with the increase of noise uncertainty,

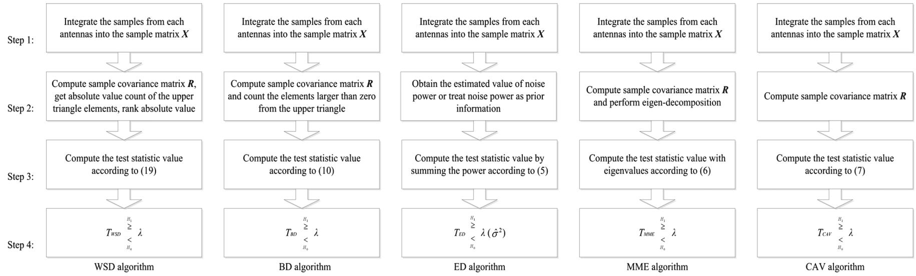

Fig. 2. Procedures of WSD algorithm, BD algorithm, ED algorithm, MME algorithm and CAV algorithm. Note that the knowledge of noise power is required in the ED algorithm.

Table 1. Computational complexity comparison

Algorithm	Computational complexity
WSD	$\mathrm{O}(NM^2 + Llog_2(L))$
BD	$\mathrm{O}(NM^2)$
ED	$\mathrm{O}(NM)$
MME	$\mathrm{O}(NM^2 + M^3)$
CAV	$\mathrm{O}(NM^2)$

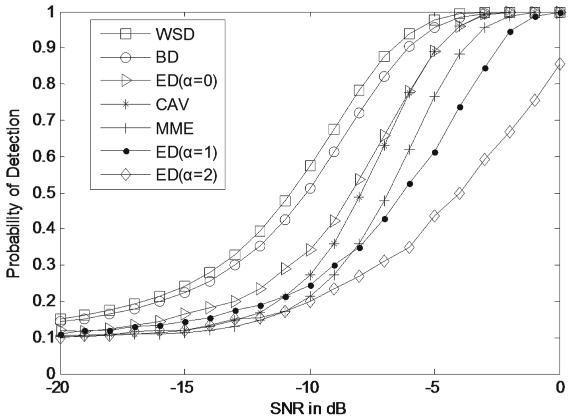

Fig. 3. P_d vs SNR with $P_{fa} = 0.1$ and $M = 4$, $N = 50$

however, other algorithms remains high performance no matter what the noise uncertainty is. It is clear that WSD and BD algorithms perform better than the ED, MME and CAV sensing methods. The WSD algorithm gets optimal performance compared with BD algorithm, which is consistent with theoretical analysis. Figure 4 shows the ROC performance comparison with the WSD, BD, ED, CAV, MME algorithms. There are four antennas and 40 sample points in

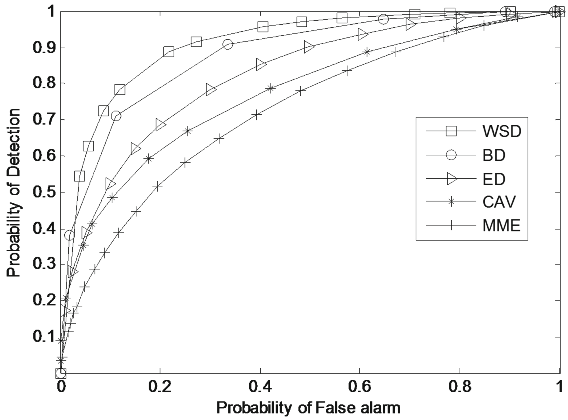

Fig. 4. ROC curves with SNR = −8 dB

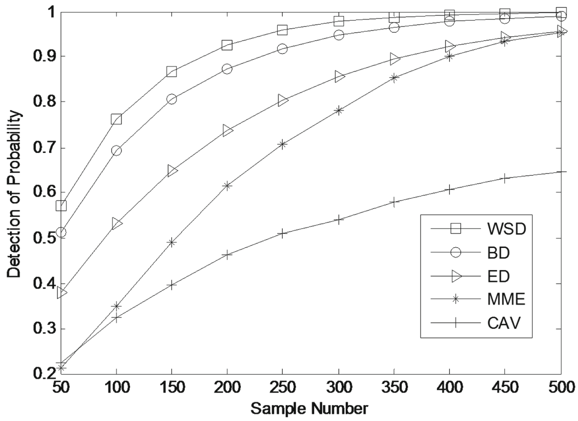

Fig. 5. Detection performances versus sample number with SNR = −10 dB

each antenna. The signal noise ratio(SNR) is −8 dB. It is clear that the WSD and BD algorithms perform better. In order to further compare several kinds algorithms, Fig. 5 gives the simulation performance for P_d at different sample size. The SNR is set as −10 dB. Compared with other algorithm, the performance of CAV algorithm improves slowly with the increase of sample number. In summary, the simulation result shows that the proposed algorithms can reach better performance than traditional algorithms.

6 Conclusion

In this paper, two sensing algorithms based on nonparametric test theory have been proposed for multi-antenna CR system. The upper triangular elements of sample covariance matrix are utilized to calculate test statistics and the threshold is set according to given probability of false alarm. It is worth noting that the proposed algorithms do not need any knowledge of signal and noise. In addition, the computational complexity of BD algorithm is just higher than ED algorithm. Simulation analysis have shown that the performance of the proposed algorithms is much better than ED, MME and CAV.

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