A Hybrid Automatic Repeat reQuest Scheme Based on Maximum Distance Separable Codes

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Abstract. For communication systems, a good error control technology is expected to get higher data transmission rate without reducing the quality of service. This paper presents a Hybrid Automatic Repeat reQuest (HARQ) scheme based on Maximum Distance Separable (MDS) codes to improve the ability of error correction. We divide the file into several segments, i.e., information packets and get a check packet using a kind of MDS codes before we transmit the file. Then we try our best to recover the file with the help of the check packet and information packets at the receiver. It is shown that our proposed HARQ scheme has better Block Error Rate (BLER) performance when compared to the traditional HARQ scheme, but the average slots cost per file does not increase significantly.

Keywords: Error control · HARQ · MDS

1 Introduction

In communication systems, higher data transmission rate and higher quality of service are the two important aspects of both research and application. Error control technologies are developed to get higher data transmission rate, without reducing the quality of service. Forward Error Correction (FEC) and Automatic Repeat reQuest (ARQ) are two of the most important error control technologies [1].

But in wireless communication systems, the channel condition is complicated and unpredictable, while the interference and the distortion is serious. These factors reduce the quality of communication systems significantly, making simple FEC and ARQ useless.

As the combination of FEC and ARQ, Hybrid Automatic Repeat reQuest (HARQ) is one of the important link adaptive technology which could improve the efficiency of data transmission. Nowadays, there are three kinds of HARQ technologies used in error control retransmission mechanism, namely Type-I HARQ, Type-II HARQ and Type-III HARQ [2–4].

Lots of people have done the research about HARQ and some achievements have been made. In [4], the authors obtained the close-form solutions of Type-I HARQ and Type-II HARQ, and the integral solution of Type-III HARQ

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by applying the latest theory and channel coding theorem. Furthermore, they proved that the performance of Type-III HARQ scheme is always better than that of Type-II HARQ scheme, and the performance of Type-II HARQ scheme is always better than that of Type-I HARQ scheme.

In another paper [5], someone confirmed that a variable-rate HARQ-IR scheme would provide gains when compared to a fix-rate transmission in terms of increased throughput and decreased average number of transmissions.

In [6], the paper investigates the throughput performance of HARQ systems under finite block length constraint and presents a framework to compute the maximum achievable rate with HARQ over the Rayleigh fading channel for a given probability of error.

Paper [7] studied the throughput of a power-limited communication system using incremental redundancy HARQ. The results show that, for a large range of HARQ feedback delays, the throughput is increased by finite-length coding incremental redundancy HARQ, if the sub-codeword lengths are properly designed.

This paper is organized as follows. Section 2 puts forward the system model of our work and the encoding process of the proposed scheme is also given in this part. Section 3 introduces the decoding process of the proposed scheme in detail and some analysis is done. Simulations are presented in Sect. 4. And Sect. 5 concludes the paper.

2 System Model and Encoding

As stated earlier in this paper, Hybrid Automatic Repeat reQuest (HARQ) is an important technology which can reduce the block error rate effectively and improve the system throughput performance as well. The system block diagram of HARQ is shown in Fig. 1.

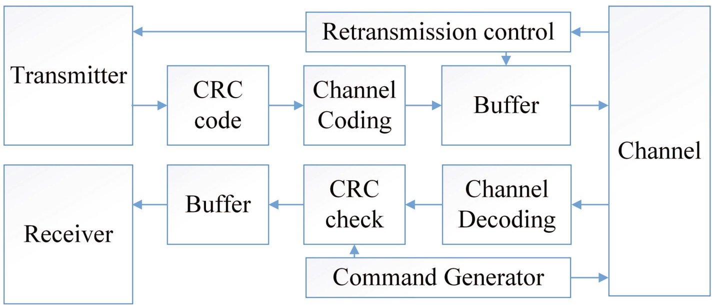

Fig. 1. System block diagram of HARQ

There are three kinds of HARQ. In the Type-I HARQ scheme, those packets which fail to match the Cyclic Redundancy Check (CRC) are discarded directly and the retransmission packets are exactly the same as the packet which is transmitted at the first time. So the Type-I HARQ scheme can't adapt to the channel condition well. In order to overcome the disadvantages of the Type-I HARQ scheme, the Type-II HARQ scheme changes the code rate adaptively according to the current channel condition. Specially, in the Type-II HARQ scheme, the packets which fail to match the CRC are deposited into the buffer at the receiver, rather than discarded directly. If the packet does not match the CRC, the receiver would try to complete the decoding process after receiving a retransmission packet, using the retransmission packet as well as the information stored in the buffer. This is the so-called soft-combined. But the Type-II HARQ scheme still has shortcomings that the retransmission packet can't be decoded alone. Then the Type-III HARQ scheme is proposed to solve the problem. The main advantage of the Type-III HARQ scheme is that any retransmission packets can be self-decoding.

In this paper we mainly consider Type-II HARQ using incremental redundancy combining. That is to say, each retransmission packet contains the same information bits and different parity bits. Actually we use rate matching to obtain different redundant version which satisfies the retransmission request.

Although retransmission could reduce the block error rate effectively, it doesn't solve everything. A packet may be incorrect in a very poor channel condition, even if the retransmission number reaches the maximum retransmission number. On the other hand, a file is usually divided into several packets and the file is invalid as long as any of the packets is incorrect.

In order to further improve the performance of HARQ, we develop a HARQ scheme based on maximum distance separable (MDS) codes [8–10].

In the situation that the code length n and the code dimension k is fixed, MDS codes have the best error correction ability among all (n,k) codes. The minimum distance d of MDS codes is the code length n minus the code dimension k plus 1, i.e., MDS codes meet the Singleton-type bound. An (n,k) MDS code has an important property: if a file is divided into k segments and then coded into n segments using an (n,k) MDS code, any k out of n segments could recover the whole file.

Single Parity Check (SPC) code [11,12] is a class of MDS codes. In this case, the number of information bits is k, and the check bit is the exclusive-or of all the information bits.

In traditional HARQ scheme, a file is divided into several information packets. Similarly to SPC code, we can get the exclusive-or of all the information packets and the exclusive-or result is the so-called check packet. If the file is still unrecovered after the maximum retransmission number of all the information packets is reached, the check packet is transmitted to the receiver. According to the property of SPC code, receiver could try to recover the entire file using the information packets and the check packet as well.

An example is given to show how to calculate the check packet. Assuming that a file is divided into 5 information packets, and then the 6th packet, i.e. check packet is obtained. The 5 information packets are marked as I_1, I_2, I_3, I_4 and I_5, respectively, while the check packet is marked as C. The relationship between the information packets and the check packet can be represented as follows.

$$C = I_1 + I_2 + I_3 + I_4 + I_5 \tag{1}$$

3 Decoding Process

The flow diagram of our proposed HARQ scheme is shown in Fig. 2. In our proposed scheme, the transmitter sends the information packets to the receiver at first. If an information packet doesn't match the CRC, this information packet is retransmitted until the maximum retransmission number is reached. The file is recovered successfully if all the information packets match the CRC. In this case, the check packet is not necessary for the file recovery and there is no need to send it. In fact, the transmitter sends the information packets of the next file to the receiver immediately.

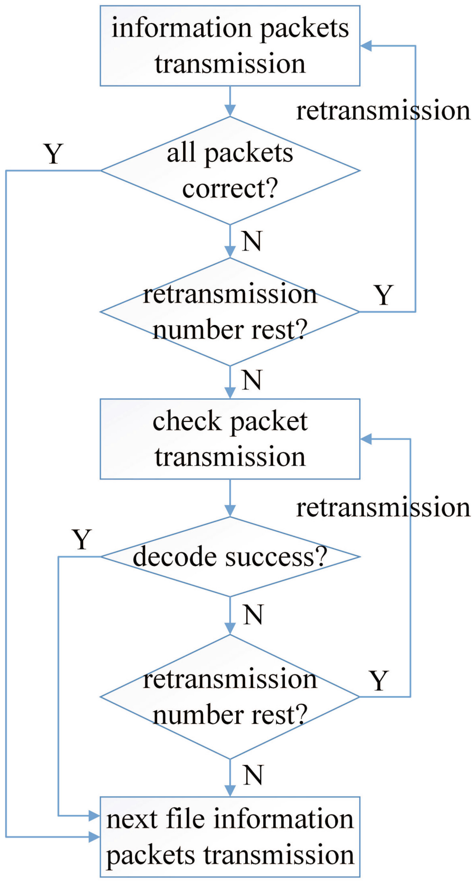

Fig. 2. Flow chart of our proposed HARQ scheme

Sometimes, some of the information packets couldn't match the CRC even if the retransmission number is used up, due to the poor channel condition. Compared with retransmitting the whole file, transmitting one more packet (check packet) would be quickly and effectively and the relationship between the information packets and the check packet can be used to recover the file.

The check packet can also be retransmitted. The retransmission check packet is soft-combined with the former ones and then the combined result is used to calculate the revised sequence. Here the method is described in detail.

Assuming that the information packets I_1, I_2, I_3 and I_4 is correctly received, as well as the check packet C. That is to say, the information packet I_5 is the only one which could not match the CRC after all the retransmission times is used. According to Eq. (1), we can see that

$$I_5 = I_1 + I_2 + I_3 + I_4 + C \tag{2}$$

Then we obtain the information packet I_5 by simple exclusive-or operation and we recover the whole file.

The poorer the channel condition is, the more information packets are likely to be incorrect. If two information packets is incorrect, the check packet is still be sent to the receiver.

As is known to all, in the decoding part, soft information or the so-called Logarithmic Likelihood Ratio (LLR) is widely used. So the receiver could choose the packet which is more likely to be correct from the two incorrect packets and update its LLRs.

To find the packet which is more likely to be correct, we calculate the sum of squares of all the LLRs in each incorrect packet. Assuming that two information packets I_1 and I_4 are incorrect, and we calculate the sum of squares of all the LLRs in packet I_1 and I_4. The length of each LLR sequence is represented as L.

$$LLR_{sum}(I_j) = \sum_{k=1}^{L} LLR(k)^2, j = 1, 4 \tag{3}$$

The packet whose result of the summation is larger is the one we want to find. If packet I_1 is more likely to be correct, we then calculate a revised sequence of the LLRs of information packet I_1 and update it. Finally, the information packet I_1 can be decoded again with the help of other information packets and check packet.

$$\begin{aligned} LLR(I_1)_{revised} = 2 * arctanh \Bigg\{ tanh \left[\frac{LLR(C)}{2} \right] \\ * \prod_{j=2}^{5} tanh \left[\frac{LLR(I_j)}{2} \right] \Bigg\} \end{aligned} \tag{4}$$

$$LLR(I_1)_{new} = LLR(I_1)_{old} + LLR(I_1)_{revised} \tag{5}$$

If packet I_4 is more likely to be correct, we do the similar operations to packet I_4. In a situation that the packet we choose from the two incorrect packets is decoded successfully, we could recover the whole file using Eq. (2).

If the number of incorrect packets is three or more, we would find the packet which is most likely to be correct using Eq. (3). Then we try to decode the packet we choose using Eqs. (4) and (5). If the packet we choose is decoded successfully, we would try to find the next packet among all the remaining incorrect packets in the same way. The file is recovered as soon as the receiver recovers k packets with the help of the check packet. The maximum retransmission number of the check packet is set to be the same as the information packets.

According to Eq. (5), $LLR(I_1)_{old}$ and $LLR(I_1)_{revised}$ can be seen as two different transmission results of the same packet I_1 in different channel conditions. Although the results of $LLR(I_1)_{old}$ or $LLR(I_1)_{revised}$ can't be decoded correctly, their soft-combined result may be decoded successfully. Now we explain the reason why the updated LLRs may be decoded successfully.

According to the definition of the LLR, we can know that

$$LLR = \frac{2y}{\sigma^2} = \frac{2(x + \sigma N)}{\sigma^2} \tag{6}$$

where σ^2 is the channel noise power and N represents a Gaussian variable whose mean value is zero and variance is 1. The value of x is -1 or 1 and y is the received result at the receiving end. It is easy to prove that LLR is also a Gaussian variable. The mean value and variance of LLR can be calculated.

$$\mu_{LLR} = \frac{2x}{\sigma^2} \qquad \sigma^2_{LLR} = \frac{4}{\sigma^2} \tag{7}$$

where σ^2 is the channel noise power and the value of x is -1 or 1.

As mentioned earlier, LLR_{old} and $LLR_{revised}$ can be seen as two different transmission results of the same packet in different channel conditions. Obviously, the channel noise power σ^2 is different in different channel conditions. Then we can get the relationship between the variance of channel noise and the variance of LLRs.

$$\sigma^2_{LLR_{old}} = \frac{4}{\sigma^2_{old}} \qquad \sigma^2_{LLR_{revised}} = \frac{4}{\sigma^2_{revised}} \tag{8}$$

When we update the LLRs using Eq. (5), we add the variance of $LLR_{revised}$ to the variance of LLR_{old} and get the variance of LLR_{new}.

$$\sigma^2_{LLR_{new}} = \sigma^2_{LLR_{old}} + \sigma^2_{LLR_{revised}} \tag{9}$$

Then we can calculate the equivalent channel noise power after soft-combining.

$$\sigma^2_{new} = \frac{\sigma^2_{old}\sigma^2_{revised}}{\sigma^2_{old} + \sigma^2_{revised}} \tag{10}$$

We can easily prove that the variance after soft-combining is smaller than any of the variance before soft-combining.

$$\sigma^2_{new} < \sigma^2_{old} \qquad \sigma^2_{new} < \sigma^2_{revised} \tag{11}$$

As is shown in Fig. 3, the smaller the channel noise power is, the better the channel condition is. That is to say, soft-combining makes us complete the decoding process under a better channel condition. That is the reason why our proposed scheme has better block error rate performance than traditional HARQ scheme.

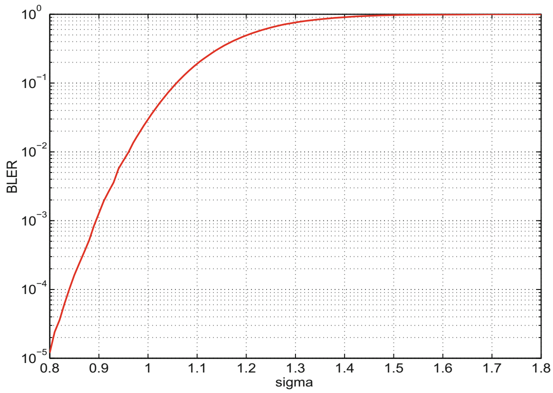

Fig. 3. Sigma vs. BLER in AWGN channel

4 Simulations

In this section, the advantages of our proposed Hybrid Automatic Repeat reQuest (HARQ) scheme based on maximum distance separable (MDS) codes is verified by simulating a simple communication system. A simplest MDS code, i.e., Single Parity Check (SPC) code is employed for simplicity.

The file is divided into 5 segments and each segment is seen as an information packet. In traditional HARQ scheme, we just transmit the information packets. But in our HARQ scheme, we transmit the information packets as well as the check packet. The maximum retransmission number of both the information packets and the check packet is 4. The channel used in the communication system is assumed to be Additive White Gaussian Noise (AWGN) channel and the channel coding method we choose is Turbo code.

The simulation is completed in two situations and the code rate of Turbo code is 1/3 and 1/2, respectively. The performances we focus on are block error rate which indicates the probability of a correct file transmission and the average slots cost per file in a file transmission.

4.1 Case 1

In this case, the code rate of Turbo code is 1/3. The block error rate (BLER) and average slots vs. the signal to noise ratio (SNR) are shown in Figs. 4 and 5, respectively. According to the results in Fig. 5, it can be seen that the poorer the channel condition is, the more average slots is cost to transmit a file in both traditional HARQ scheme and proposed HARQ scheme. For example, when SNR is −8 dB, the average slots cost per file in traditional HARQ scheme is 24.5010, while the average slots cost per file in proposed HARQ scheme is 28.5710. That is to say, almost every packet in both schemes has been retransmitted 4 times. As the channel condition gets better, the average slots cost per file of both schemes decrease.

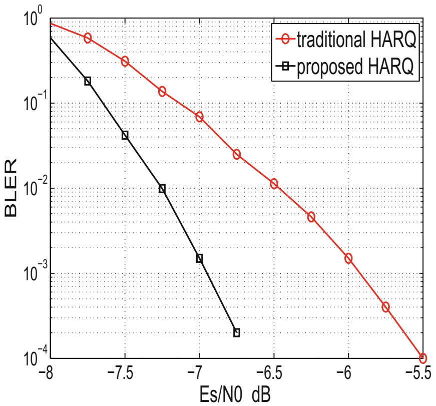

Fig. 4. BLER vs. SNR for 1/3 code rate Turbo code

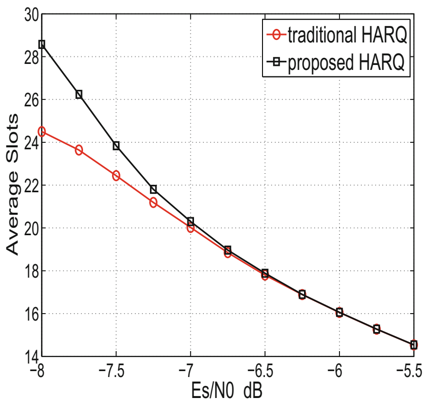

Fig. 5. Average Slots vs. SNR for 1/3 code rate Turbo code

In the region where SNR is greater than −7 dB, the gap of the average slots cost per file between traditional HARQ scheme and proposed HARQ scheme is very small, but the BLER performance in proposed HARQ scheme is much better than that in traditional HARQ scheme. We take the situation when SNR equals −7 dB as an example. The average slots cost per file in traditional HARQ scheme is 20.0220, while the average slots cost per file in proposed HARQ scheme is 20.2995. The BLER of the proposed HARQ scheme is 0.0015, and the BLER of the traditional HARQ scheme is 0.069. When BLER is 10^{-3}, our proposed HARQ scheme has about 1 dB gain compared to traditional HARQ scheme.

4.2 Case 2

In this case, the code rate of Turbo code is 1/2. The block error rate (BLER) and average slots vs. the signal to noise ratio (SNR) are shown in Figs. 6 and 7, respectively. As the same as case 1, almost every packet in both schemes has been retransmitted 4 times when the channel condition is poor and the average slots cost per file of both schemes decrease when the channel condition gets better.

In the region where SNR is greater than -4.5 dB, the gap of the average slots cost per file between traditional HARQ scheme and proposed HARQ scheme is negligible, but the BLER performance in proposed HARQ scheme is much better than that in traditional HARQ scheme. We take the situation when SNR equals -4.5 dB as an example. The average slots cost per file in traditional HARQ scheme is 18.6930, while the average slots cost per file in proposed HARQ scheme is 18.8777. The BLER of the proposed HARQ scheme is 0.0018, and the BLER of the traditional HARQ scheme is 0.067. When BLER is 10^{-3}, our proposed HARQ scheme has about 1.1 dB gain compared to traditional HARQ scheme.

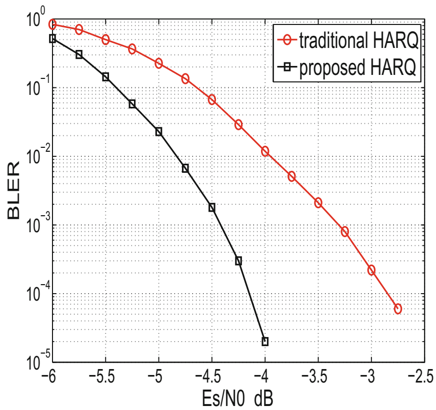

Fig. 6. BLER vs. SNR for 1/2 code rate Turbo code

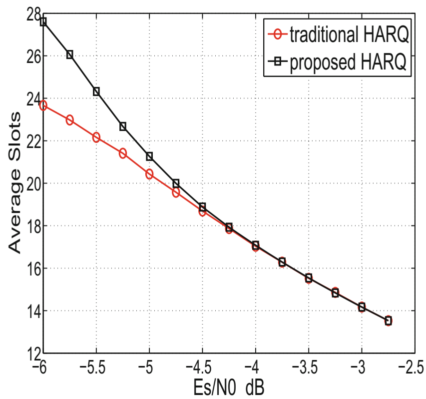

Fig. 7. Average slots vs. SNR for 1/2 code rate Turbo code

The results show that our proposed HARQ scheme has much better BLER performance than traditional HARQ scheme, and the average slots cost per file in proposed HARQ scheme does not increase.

5 Conclusion

In this paper, we have proposed a Hybrid Automatic Repeat reQuest (HARQ) scheme based on Maximum Distance Separable (MDS) codes to improve the ability of error correction. The system model and the detailed simulation results have been presented to verify the advantages of our proposed HARQ scheme. Compared to traditional HARQ scheme, our proposed HARQ scheme has better BLER performance, but the average slots cost per file does not increase significantly.

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