Robust Power Allocation Scheme in Cognitive Radio Networks

Hongzhi Wang, Meng Zhu, and Mingyue Zhou(✉)

College of Computer Science and Technology,
Changchun University of Technology,
Yan'an Street 2055, Changchun 130012, China
{hzwang, zmyjlu}@ccut.edu.cn, zhumengvipzm@163.com

Abstract. Considering that the spectrum resources are becoming increasingly demand, maximum channel capacity is very crucial for future wireless communication systems, especially for cognitive radio networks (CRNs). However, most existing works usually assume that channel parameter estimation is perfect, which is often damped in practical systems. In this paper, we investigate the robust maximum channel capacity problem in the CRNs. Then assuming that channel parameter uncertainty is bounded, we consider that all channel parameter uncertainties are described by ellipsoid sets. From the perspective of worst-case optimization, we formulate it as a semi-infinite programming (SIP) problem. Furthermore, an optimal iterative algorithm based on the dual decomposition theory and Lagrange multiplier algorithm is applied. Simulation results validate that our robust scheme can achieve the channel capacity maximization considering the worst-case and strictly guarantee the power interference requirement of second users (SUs) under all parameters' uncertainties.

Keywords: Capacity maximization · CRNs · Ellipsoidal set · Distributed algorithm · Robust optimization

1 Introduction

In recent years, with the rapid development and the wide application of radio communication technology, the demand for wireless spectrum resources becomes exceedingly urgent. According to the report by Federal Communication Commission (FCC), the authorized spectrum utilization is obviously inefficient since the fixed spectrum allocation approach [1]. Dr. Mitola first proposed the concept of cognitive radio technology [2], which is to establish communication among unauthorized users without exceeding the interference that primary users (PUs) can tolerate.

The problem of assigning power to different SUs has recently been an area of active research. There are many papers [3, 4] addressing the problem of channel capacity maximization under the assumption that the parameters and constraints are perfect. However, this information is subject to errors due to measurement uncertainties in practical systems. We often call the corresponding problems for the "nominal" problem [5]. However, these parameters are time-varying, imperfect or uncertainty. Several researches on the problem of parameter uncertainties have been investigated in the

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CRNs. The authors investigate state estimation problems for nonlinear systems with parameter uncertainties. A new robust unscented Kalman filter is devised by analyzing the influence which parameter uncertainties give to covariance matrix [6]. Robust power control strategies for cognitive radios in the presence of sensing delay and model parameter uncertainty is investigated [7]. The authors use a discrete-time Markov chain (DTMC) to characterize the primary users' dynamics as well as the fading channel. Furthermore, most of existing algorithms for power control mechanism problem are centralized [8, 9], where the parameter control and transmission is completed by the base station. Nevertheless, the centralized scheme has obvious computation and transmission overhead that is a shortcoming of the centralized way. Orthogonal frequency division multiplexing (OFDM) has been considered a potential transmission technology for CR systems. We investigated robust power allocation by considering an OFDM framework with transmit power budget and interference threshold into account [10]. In this paper, we investigate the worst case robust formulation under distributed way [11] in cognitive radio wireless ad-hoc networks to maximize channel capacity while keeping the SINR amount of SU within relatively high range.

Considering the above problems, robust optimization techniques are more appropriate obviously. Firstly, we define an uncertainty set, which is an ellipsoid set that captures the parameter uncertainty. Secondly, the robust capacity maximization problem can be converted into a SIP problem, which is transformed into a second order cone programming (SOCP) problem [12] under the worst-case. Thirdly, a distributed algorithm is proposed based on dual decomposition theory and Lagrange multiplier algorithm [13] is proposed to achieve an optimal solution. Finally, the equivalent constraint and the iterative algorithm derived from parameter uncertainty are proposed to acquire optimal solution [14], and the theoretical discussions between robust algorithm and non-robust algorithm are demonstrated by simulation results.

2 System Model and Robust Distributed Formulation

2.1 System Model

We consider an ad-hoc cognitive radio network as Ref. [15], i.e., each link consists of a transmitter node and a receiver node. Assume that there are $\mathbf{K} = \{1, 2, 3, \ldots, K\}$ cognitive links and only one primary link in the region of interest.

In this paper, we pay attention to the underlay paradigm in CRNs. In this model, SUs can always access the channel that is assigned to PUs, in which the total interference introduced to PUs is strictly less than a predefined threshold which PUs can tolerate as follows, i.e.,

$$\sum_{i=1}^{k} g_i p_i \leq I_{th}. \tag{1}$$

where g_i denotes the channel gain between the cognitive transmitter (CR-Tx) of link i to the PU receiver (PU-Rx). p_i denotes the transmission power of the CR-Tx for link i. I_{th} represents the permissible interference threshold for PU-Rx.

To guarantee the normal work of the system, the transmission power of each SU should not exceed its power budget. We have

$$p_i \leq p_i^{\max}, \ \forall i \in \mathbf{K}. \tag{2}$$

where $p_i^{\max}$ is the maximum transmission power cognitive receiver i.

The signal to interference ratio plus noise (SINR) at the cognitive receiver i is

$$SINR_i = \frac{h_{ii}p_i}{\sum\limits_{j \neq i} h_{ij}p_j + \sigma_i}, \ \forall i \in \mathbf{K}. \tag{3}$$

where h_{ij} denotes the channel gain from cognitive transmitter j to receiver i. σ_i is the background noise at cognitive receiver i which includes both the thermal noise and interference caused by the primary transmission.

The utility function chosen by each SU to be maximized is the data capacity since spectrum efficiency is the main target of cognitive radio. While guaranteeing constraints both (1) and (3), the problem is formulated as

$$\begin{aligned} &\max \sum_{i=1}^{k} \log(1 + \frac{h_{ii}p_i}{\sum\limits_{j \neq i} h_{ij}p_j + \sigma_i}) \\ &s.t. \quad C1 : \sum_{i=1}^{k} g_i p_i \leq I_{th}, \quad C2 : p_i \leq p_i^{\max}, \ \forall i \in \mathbf{K}. \end{aligned} \tag{4}$$

where the variable $p_i \geq 0$ for all i. Moreover, $\alpha = [\alpha_{ij}]$ can be denoted by the formula as follows

$$\alpha_{ij} = \begin{cases} 0 \ , & if \ \ i = j, \\ \dfrac{h_{ij}}{h_{ii}}, & if \ \ i \neq j. \end{cases} \tag{5}$$

Then, objective function in (4) can translate to as follows

$$\begin{aligned} &\max \sum_{i=1}^{k} \log(1 + \frac{p_i}{\sum\limits_{j \neq i} \alpha_{ij}p_j + \sigma_i / h_{ii}}) \\ &s.t. \quad C1 : \sum_{i=1}^{k} g_i p_i \leq I_{th}, \quad C2 : p_i \leq p_i^{\max}, \quad \forall i \in \mathbf{K}. \end{aligned} \tag{6}$$

In the rest of the section, we will formulate the robust optimization problems considering uncertainties which include both the channel coefficient α and channel gain g.

2.2 Robust Distributed Formulation Under Ellipsoid Uncertainty Set

We firstly consider the uncertainties of channel parameter matrix α and channel gain g_i, and use the ellipsoid set to depict the corresponding parameter uncertainty.

Let α_i denote the uncertainty set of the *ith* row for matrix α, which can capture the perturbation of interfering channel gains relative to the main channel gain of link i. Denote the actual standardized channel gain between user j's transmitter and user i's receiver as $\bar{\alpha}_{ij} + \Delta\alpha_{ij}$, where $\bar{\alpha}_{ij}$ is the nominal value, and $\Delta\alpha_{ij}$ is the corresponding uncertainty part. Hence, the uncertainty set of α_i for $\bar{\alpha}_{ij}$ under ellipsoid approximation can be expressed as:

$$\alpha_i = \left\{ \bar{\alpha}_i + \Delta\alpha_i : \left\| \Delta\alpha_{ij} \right\|_2^2 \leq \varepsilon_0^2,\ \forall j \neq i,\ i \in \mathbf{K} \right\}. \tag{7}$$

where $\|x\|_2$ refers to the Euclidean norm [16], and ε_0 is the positive upper bound on the uncertainty region.

Let g_i denote the uncertainty set of the *ith* row of matrix g, g_i describes the channel gain between cognitive transmitter i and PU receiver, and $g_i = \bar{g}_i + \Delta g_i$, where $\bar{g}_i$ is the nominal value, and the corresponding uncertainty part is Δg_i. Then the certainty set g_i under ellipsoid approximation is formulated as:

$$g_i = \left\{ \bar{g}_i + \Delta g_i : \|\Delta g_i\|_2^2 \leq \varepsilon_i^2,\ \forall i \in \mathbf{K} \right\}. \tag{8}$$

where ε_i is the upper bound on the uncertainty region.

The robust power allocation algorithm with capacity maximization under ellipsoid set can be represented by

$$\begin{aligned} &\max \sum_{i=1}^{k} \log\Big(1 + \frac{p_i}{\sum\limits_{j \neq i} (\bar{\alpha}_{ij} + \Delta\alpha_{ij}) p_j + \sigma_i / h_{ii}}\Big) \\ &s.t. \quad C1 : \sum_{i=1}^{k} (\bar{g}_i + \Delta g_i) p_i \leq Ith,\ C3 : \left\| \Delta\alpha_{ij} \right\|_2^2 \leq \varepsilon_0^2, \quad \forall j \neq i, i \in \mathbf{K}, \\ &\qquad C2 : p_i \leq p_i^{\max},\ \ \forall i \in K,\ \ C4 : \|\Delta g_i\|_2^2 \leq \varepsilon_i^2, \qquad \forall i \in \mathbf{K}. \end{aligned} \tag{9}$$

The robust capacity maximization problem (9) is an infinite number of constraints relative to the sets α_i and g_i, i.e., it is a SIP problem. We can transform the SIP problem into an equivalent problem with finite constraints under the worst-case. Considering Cauchy–Schwartz inequality, the equivalent problem as follows

$$\max_{\alpha_i \in \alpha} \left\{ \sum_{j \neq i} \Delta\alpha_{ij} p_j \right\} = \varepsilon_o \sqrt{\sum_{j \neq i} p_j^2},\ \ \max_{g_i \in g} \left\{ \sum_{i} \Delta g_i p_i \right\} = \varepsilon_i \|p_i\|_2,\ \forall i \in \mathbf{K}. \tag{10}$$

Then, the problem (9) is transformed into an equivalent problem under the worst-case.

3 The Distributed Capacity Maximization Algorithm

In this section, we develop a worst-case distributed capacity maximization problem by dual decomposition theory. Hence, in this case, we transform constraint C1 in (9) into an equivalent problem as follows

$$\varepsilon_i \|p\|_2 \leq -\bar{g}^T p + I_{th}, \ \forall i \in \mathbf{K}. \tag{11}$$

However, it is difficult to decompose the coupled part $\varepsilon_i \|p\|_2$. Therefore, we propose a worst-case distributed capacity maximization algorithm along with the convergence, i.e., this constraint can be represented by

$$\sum_i (\bar{g}_i + \varepsilon_i) p_i \leq I_{th}, \ \forall i \in \mathbf{K}. \tag{12}$$

Taking into account the all parameters uncertainties, where the uncertainties is the worst-case level that is the upper bound of ellipsoid sets, the worst-case distributed capacity maximization problem can be expressed as

$$\begin{aligned} &\max \sum_{i=1}^{k} \log(1 + \frac{p_i}{\sum_{j \neq i} \bar{\alpha}_{ij} p_j + \varepsilon_o \sqrt{\sum_{j \neq i} p_j^2} + \sigma_i / h_{ii}}) \\ &s.t \quad C1 : \sum_{i=1}^{k} (\bar{g}_i + \varepsilon_i) p_i \leq I_{th}, \ C2 : p_i \leq p_i^{\max}, \ \ \forall i \in \mathbf{K}. \end{aligned} \tag{13}$$

Therefore, robust distributed algorithm take more conservative protection into account of the cognitive radio system. The problem (13) is not convex optimization problem. We may rewrite (13) as follows

$$\begin{aligned} &-\min \sum_{i=1}^{k} \log(1 + \frac{p_i}{\sum_{j \neq i} \bar{\alpha}_{ij} p_j + \varepsilon_o \sqrt{\sum_{j \neq i} p_j^2} + \sigma_i / h_{ii}}) \\ &s.t. \quad C1 : \sum_{i=1}^{k} (\bar{g}_i + \varepsilon_i) p_i \leq I_{th}, C2 : p_i \leq p_i^{\max}, \ \ \forall i \in \mathbf{K}. \end{aligned} \tag{14}$$

By using the Lagrange multiplier algorithm, a new objective Lagrange function is defined as

$$\begin{aligned} &L(\{p_i\}, u_i, v_i) = \\ &-\sum_{i=1}^{k} \log(1 + \frac{p_i}{\sum_{j \neq i} \bar{\alpha}_{ij} p_j + \varepsilon_o \sqrt{\sum_{j \neq i} p_j^2} + \sigma_i / h_{ii}}) \ + u_i(\sum_i (\bar{g}_i + \varepsilon_i) p_i / I_{th} - 1) + v_i(p_i / p_i^{\max} - 1). \end{aligned} \tag{15}$$

where $u_i \geq 0$ and $v_i \geq 0$ are Lagrange multipliers for the two constraints in (14), respectively. Furthermore, the updating function is defined as

$$u_i(t+1) = \max(u_i(t) + \alpha L\_u_i(t), 0), \ \forall i \in \mathbf{K}. \tag{16}$$

$$v_i(t+1) = \max(v_i(t) + \beta L\_v_i(t), 0), \ \forall i \in \mathbf{K}. \tag{17}$$

where α and β are the step size which are positive, and t is the iteration times. Moreover, the corresponding gradient L_u_i and L_v_i updating function is given by

$$L\_u_i = \sum_i (\bar{g}_i + \varepsilon_i)p_i - I_{th}, \ L\_v_i = p_i - p_i^{\max}, \ \forall i \in \mathbf{K}. \tag{18}$$

To achieve the optimal solution of each SU in the robust formulation, the optimal solution p_i^* for (14) by considering the Karush-Kuhn-Tucker (KKT) conditions can be calculated through the following equality

$$\frac{\partial L(\{p_i\}, u_i, v_i)}{\partial p_i} = 0, \ \forall i \in \mathbf{K}. \tag{19}$$

Therefore, we can get the optimal solution p_i^* for each SU as follows

$$p_i^* = \frac{1}{(u_i \sum_i (\bar{g}_i + \varepsilon_i)/I_{th} + v_i/p_i^{\max}) \ln 2} - (\sum_{j \neq i} \bar{\alpha}_{ij} p_j + \sigma_i/h_{ii} + \varepsilon_0 \sqrt{\sum_{j \neq i} p_j^2}). \tag{20}$$

Different form the traditional water filling solution, the cognitive radio system will strictly converge to the optimal solution in (20). Hence, the robust distributed algorithm can tackle above formulation [10].

4 Performance Evaluation

4.1 Simulation Settings

In this section, we describe the detail of parameters setting and the channel model for SUs and PUs in our simulations. Firstly, the related parameters and their typical value are provided in Table 1. And, the Euclidean norm is adopted for all parameters' uncertainty regions in our simulations.

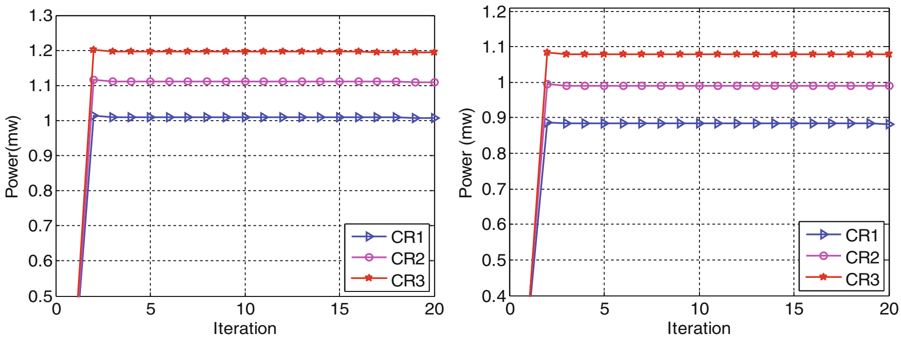
4.2 Simulation Results

Simulation results are provided to compare the performance of robust algorithm with the non-robust algorithm under the underlay network scenario. Here, the non-robust method refers to the algorithm that does not take channel uncertainties into account and directly utilizes the parameters all channels as if they were perfect. However, we consider the channel parameter uncertainties in robust algorithm.

Table 1. Simulation parameters setting

Related parameters	Typical valves
Number of PUs	1
Number of SUs	3
PU maximum interference threshold	3.5×10^{-10} mw
Perturbation rang of uncertainty parameters ε_0	10%
Perturbation rang of uncertainty parameters ε_i	10%
SU maximum transmit power P	$[1.1, 1.2, 1.3]$ mw
Average additive noise power σ_i at SU receiver	0.0001 mw

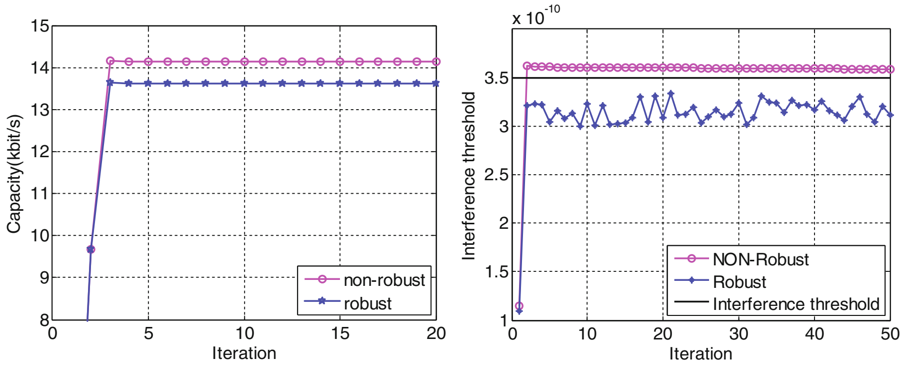
In Fig. 1, we compare the power allocation algorithms with iteration times in the non-robust scheme and the robust scheme. With the increasing number of iterations, robust algorithm and non-robust algorithm quickly tend to a stable value that not exceed its power budget, i.e., establish a balance. Simulation results show that the optimal power of robust algorithm is slightly less than the non-robust algorithm. That's because the robust algorithm guarantees an acceptable level of performance under worst case conditions.



(a) Non-Robust Algorithm (b) Robust Algorithm

Fig. 1. Convergence of robust algorithm and non-robust algorithm

In Fig. 2a, we show the performance of channel capacity for the non-robust scheme and the robust scheme, which characterize the trade-off between uncertainty and networks throughout. Under the worst-case, the maximum channel capacity of robust algorithm is almost equal to the non-robust algorithm, which shows the superiority of the robust algorithm. In Fig. 2b, we compare total interference of various power allocation algorithms. The straight line is the permissible interference power level. It can be observed that the interference caused by SUs transmitter to PU receiver under robust algorithm is always below the permissible threshold while the interference under non-robust algorithm exceed permissible threshold. The non-robust algorithm is not as successful as the robust algorithm at preventing violations of the permissible



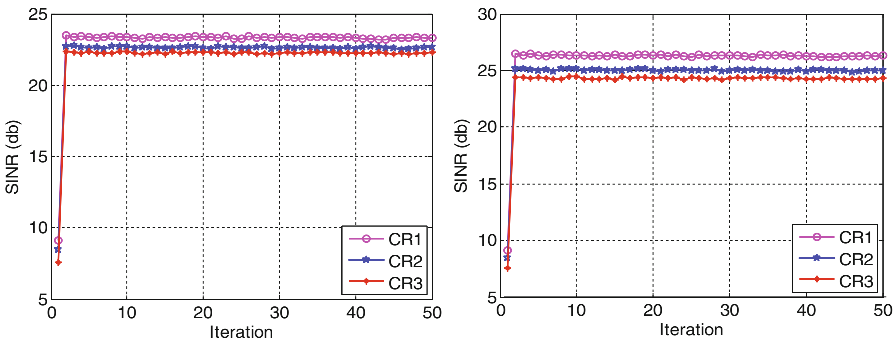
(a) Capacity comparison (b) Interference power comparison

Fig. 2. Performance comparison for robust scheme and non-robust scheme

interference power level. Therefore, we can summarize that robust algorithm always guarantee the quality of services for PU.

To compare the SINR of SUs receiver, we plot the SUs' SINR for the two power allocation algorithms in Fig. 3. Simulation results show that the SINR of robust algorithm is slightly less than non-robust algorithm because of considering parameter uncertainty under the worst-case. Moreover, the non-robust algorithm violated the permissible interference power level as in Fig. 2b. Therefore, the robust algorithm for cognitive radio wireless ad-hoc networks can attain a good trade-off between uncertainty and capacity.

In Fig. 4, we depict power convergence properties of three users with iteration times for robust algorithm and non-robust algorithm. As seen in Fig. 4, both the proposed scheme and non-robust scheme quickly converge to a certain value which does not exceed the maximum power upper threshold. However, the converge value of



(a) Robust Algorithm (b) Non-Robust Algorithm

Fig. 3. SU SINR comparison between under robust algorithm and under non-robust algorithm

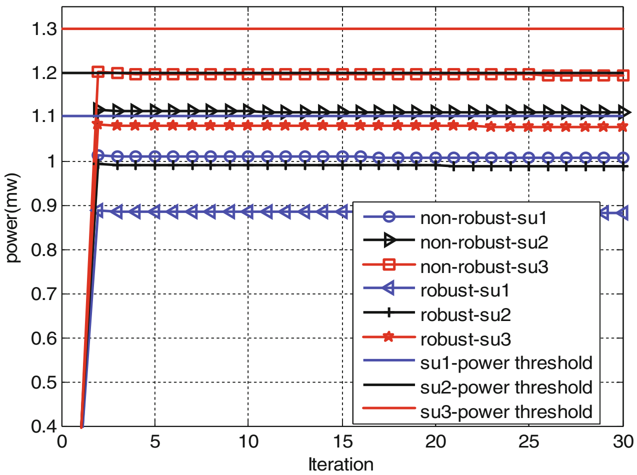


Fig. 4. SU power comparison between under robust algorithm and under non-robust algorithm

the proposed scheme is less than that of the non-robust method, it is due to the proposed scheme that considers parameter uncertainty is consider to sacrifice one's own power. These results also demonstrate that the more robustness the system is, the more efficient the robust algorithm is.

5 Conclusions

In this paper, we have studied power allocation with parameter uncertainty in underlay CRNs. To maximize the channel capacity under the constraints SU transmit power and interference thresholds, we proposed robust channel capacity maximization algorithm that ensure power threshold and PUs' quality of service requirement. We first describe ellipsoid set model for parameter uncertainty, and formulate it as a standard SOCP problem by considering the worst-case. Then, we apply dual decomposition theory to tackle SOCP problem. Simulation results show that robust algorithm can achieve almost the same maximum channel capacity as the non-robust algorithm.

The study of distributed robust optimization, in general, remains wide open, with many challenging issues and possible applications where robustness to uncertainty is as important as optimality in the nominal model. In the future, we will plan to extend our work to a multiuser system to further study capacity characteristics.

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