Frequency Detection of Weak Signal in Narrowband Noise Based on Duffing Oscillator

Shuo Shi, Qianyao Ren(✉), Dezhi Li, and Xuemai Gu

Communication Research Center, Harbin Institute of Technology, Harbin, China
{crcss,lidezhi,guxuemai}@hit.edu.cn,
renqianyao@163.com

Abstract. Duffing oscillator is used to detect weak signal in strong noise because traditional linear methods cannot work correctly in this situation. Normal Duffing oscillator is used under broadband noise because it is immune to broadband noise. But it is not suitable in narrowband noise because zero expectation of noise is damaged in narrowband noise. In this paper, the difference influence to Duffing oscillator between broadband noise and narrowband noise is analyzed and the resistance of Duffing oscillator to narrowband noise is proved. Then a new frequency detection method based on higher initial driving force amplitude and duration of cycle state is developed. Finally, the appropriate initial amplitude needed in this method is confirmed and the method is verified that it can detect frequency in narrowband noise by simulation.

Keywords: Duffing oscillator · Narrowband noise · Signal frequency detection · Initial driving force

1 Introduction

In modern communication, weak signal detection is more and more important because weak signal is usually used or emergent in communication to economize transmitting power or just be restricted by channel which is normal in both signal reception and signal detection. The comprehension of weak signal can be divided into two parts. On the one hand, signal transmitting power is weak, and noise maybe not very strong or even can be ignored. Due to low signal power, the SNR is very low. On the other hand, signal power is not weak, but noise is strong enough to submerge signal. So the SNR is also very low. In other words, one cause is low signal power, the other cause is strong noise.

In complex channel with strong noise, weak signal mostly refers to the second kind that signal power is not very weak but noise is really strong. The traditional linear methods such as coherent method cannot detect weak signal when SNR is very low because strong noise can make judgment threshold value inaccurate. So the nonlinear method such as Duffing oscillator has been put forward to detect this kind of weak signal. Duffing oscillator is a kind of chaos system that is immune to noise and sensitive to signal [1]. Since Holmes found out that Duffing function contains strange

Q. Chen et al. (Eds.): ChinaCom 2016, Part I, LNICST 209, pp. 480–488, 2018.
DOI: 10.1007/978-3-319-66625-9_47

attractor and seeming random process can be generated by deterministic Duffing oscillator [2], many signal detection methods have been provided such as Lyapunov exponent [3] and intermittent chaos theory [4].

However, these methods to detect weak signal are almost studied in broadband noise because noise band is extended by scale transformation when it turns to communication signal with high frequency from original mathematic model. The band of noise in these methods is much wider than signal band, so noise that can really influence signal is weaker than the value used in calculating SNR. Therefore, the detection of weak signal in communication field should be studied under narrowband noise that is suitable for more communication environments.

Frequency detection is the first step of signal detection with which other signal parameters can be detected. Therefore, in this paper, frequency detection of weak signal in narrowband noise is studied and analyzed. First, the basic property of Duffing oscillator and method to detect signal frequency based on Duffing oscillator are introduced. Then, the influence of narrowband noise to Duffing oscillator is analyzed and compared with the influence in broadband noise. And an improved method based on the analysis is raised to detect signal frequency in narrowband noise. Finally, the method is verified and its parameter is set through simulation.

2 Basic Property of Duffing Oscillator

Duffing oscillator is the application of Duffing function. In many kinds of Duffing oscillators, the most common one is Holmes-Duffing oscillator whose function is:

$$\ddot{x} + k\dot{x} - x + x^3 = \gamma \cos(t) \tag{1}$$

where, γ cos (t) is driving force, γ is its amplitude and k is damping factor. There are two important characteristics of Duffing oscillator, one is the sensibility to initial value and the other one is immunity to noise.

2.1 Sensibility to Initial Value and Frequency Detection Method

There are two main states of Duffing oscillator that one is large scale cycle state and the other is chaos state and they are decided by initial value of driving force amplitude. To driving force γ cos(t), there is a critical amplitude of driving force γ_c. The state of oscillator will change with increasing of actual amplitude of driving force. It stays in chaos state when $\gamma < \gamma_c$ and turns into large scale cycle when $\gamma > \gamma_c$ and there is explicit difference between the two states, so the small change of driving force amplitude is converted to explicit change of oscillator state, that make Duffing oscillator is sensitive to initial value.

Therefore, signal frequency can be detected by Duffing oscillator array with difference frequency. The initial driving force value of each oscillator is set as critical value γ_c. When there is signal with the same frequency as some oscillator input, the corresponding oscillator will turn into large scale cycle state and other oscillators will keep in chaos state.

2.2 Immunity to Gaussian White Noise

When there is noise in Duffing oscillator, the variable x of Duffing oscillator will be interfered by Δx. So the function will turn into:

$$(\ddot{x}+\Delta\ddot{x})+k(\dot{x}+\Delta\dot{x})-(x+\Delta x)+(x+\Delta x)^3=\gamma\cos(t)+n(t) \tag{2}$$

The result of subtraction between (1) and (2) is (3), where the higher order of Δx is ignored because Δx is very small.

$$\Delta\ddot{x}+k\Delta\dot{x}-\Delta x+3x^2\Delta x=\mathrm{n}(t) \tag{3}$$

The vector term of (3) is shown as (4)

$$\dot{X}(t)=A(t)X(t)+N(t) \tag{4}$$

where $X(t)=\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}\Delta x(t)\\\Delta\dot{x}(t)\end{bmatrix}$, $A(\mathrm{t})=\begin{bmatrix}0 & 1\\1-3x^2 & -k\end{bmatrix}$, $N(t)=\begin{bmatrix}0\\n(t)\end{bmatrix}$

The result of (4) is

$$X(t)=\phi(t,t_0)X_0+\int\phi(t,u)N(u)du \tag{5}$$

It can be considered as $X(t)=\int\phi(t,u)N(u)du$ because $\phi(t,\ t_0)\ X_0$ is transient solution. The expectation of $X(t)$ is $E\{X(t)\}=\int\phi(t,u)E\{N(u)\}du=0$.

According to the analysis above, the expectation of Δx and its derivative is zero. Therefore, Duffing oscillator will be immune to noise as long as the expectation of noise or interference is zero. Noise will not change the state of Duffing oscillator but make its image rough.

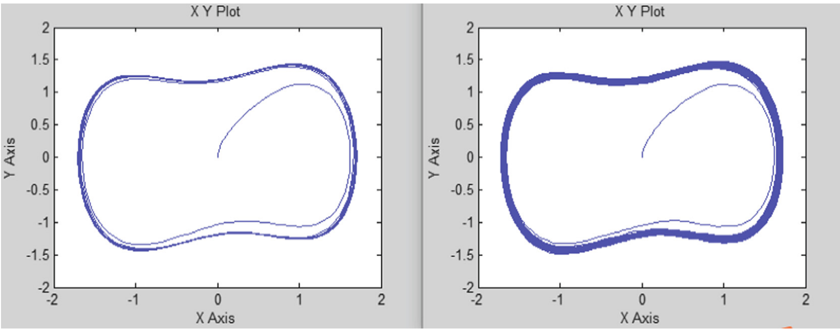
3 Frequency Detection Method in Narrowband Noise

3.1 The Difference of Duffing Oscillator in Narrowband and Broadband Noise

According the analysis above, the immunity of Duffing oscillator to noise is on the base of Gaussian white noise whose expectation is zero. When noise band is not infinity, noise expectation is nonzero and the expectation of Δx is nonzero either. So Duffing oscillator is not completely immune to narrowband noise.

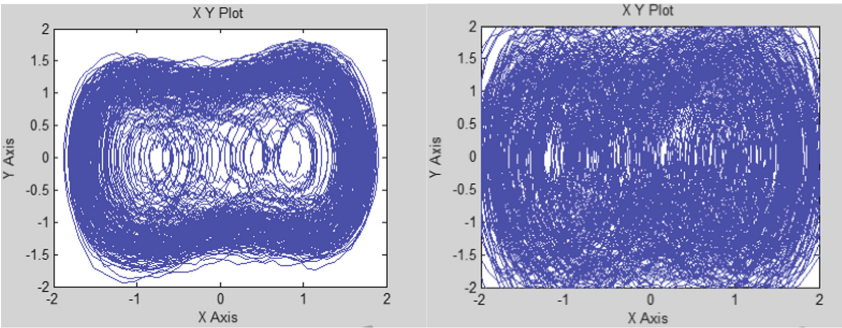
This conclusion can also be raised through analysis of noise frequency spectrum. Duffing oscillator is firstly applied in dynamics system where driving force frequency is 1–10 Hz. When it is used in communication system where driving force frequency is much more than the one in dynamics system, scale transformation has to be used. But noise band will be extended by scale transformation at the same time. So the noise power is very strong while real noise whose frequency range is similar to signal that can influence signal is not strong. So the conclusion of Duffing oscillator that it is immune to noise is suitable in broadband noise but not in narrowband noise.

Due to the influence of strong narrowband noise, phase diagram of Duffing oscillator is so rough that cycle and chaos state cannot be distinguished. Taking large scale cycle state as an example, its phase diagram in broadband noise and narrowband noise is different as shown in Figs. 1 and 2.



(a)no noise (b)noise band 50GHz

Fig. 1. Phase diagram of Duffing oscillator when there is no noise and there is noise whose band is 50 GHz.



(a)noise band 500MHz (b)noise band 50MHz

Fig. 2. Phase diagram of Duffing oscillator when noise band is 500 MHz and 50 MHz.

In Figs. 1 and 2, initial amplitude of driving force is 0.8261, and signal amplitude is 0.1. SNR is −10 dB and noise band is different in each figure. According to the result. The large-scale cycle state is clear when noise band is 50 GHz that it is similar to phase diagram without noise. With the reduce of noise band, phase diagram of Duffing oscillator does not show the proper state that it displays like phase diagram of chaos.

That is to say, under strong narrowband noise, oscillator will not be periodic or chaotic as usual, and the conclusion that is obtained without noise is incorrect.

3.2 Improvement of Duffing Oscillator's Resistance to Narrowband Noise

According to the analysis above, under strong narrowband noise, Duffing oscillator is not immune to noise, but it can still resist the influence of noise.

Duffing function with narrowband noise can be expressed as:

$$(\ddot{x}' + \Delta\ddot{x}') + k(\dot{x}' + \Delta\dot{x}') - (x' + \Delta x') + (x' + \Delta x')^3 = (\gamma + \frac{n(t)}{\cos(t)})\cos(t) \tag{6}$$

Equation (6) is another expression of (2) with the same style as (1). The actual total amplitude of driving force is $\gamma' = \gamma + \frac{n(t)}{\cos(t)}$ where γ is initial amplitude of Duffing oscillator. According to the property of Duffing oscillator, when $\gamma' > \gamma_c$, the interference of Duffing oscillator in cycle state can be ignored which means $\Delta x' = 0$. So, as long as the instantaneous maximum value of $\frac{n(t)}{\cos(t)}$ is smaller than the difference value of initial driving force amplitude γ and critical value γ_c, Duffing oscillator will not enter chaos state. That is to say, the higher initial driving force amplitude is, the harder Duffing oscillator enter chaos state.

If $\gamma' < \gamma_c$, Duffing oscillator will be influenced by narrowband noise seriously. Assuming $\cos(t) > 0$, because $\gamma > \gamma_c$, the instantaneous value of noise is lower than zero. Noise can be divided into two parts as $n(t) = n_1(t) + n_2(t)$, where $n_1(t) < 0$, $n_2(t) < 0$, let $\gamma + \frac{n_1(t)}{\cos(t)} = \gamma_c$, so the Duffing function turn into:

$$\begin{aligned}(\ddot{x}' + \Delta\ddot{x}') + k(\dot{x}' + \Delta\dot{x}') - (x' + \Delta x') + (x' + \Delta x')^3 &= \left[\gamma + \frac{n_1(t)}{\cos(t)} + \frac{n_2(t)}{\cos(t)}\right]\cos(t) \\ &= \left[\gamma_c + \frac{n_2(t)}{\cos(t)}\right]\cos(t) \\ &= \gamma_c\cos(t) + n_2(t)\end{aligned} \tag{7}$$

And the perturbation equation correspondingly is

$$\Delta\ddot{x} + k\Delta\dot{x} - \Delta x + (x + \Delta x)^3 - x^3 = \mathrm{n}_2(\mathrm{t}) \tag{8}$$

$\mathrm{n}_2(t)$ is divided from $\mathrm{n}(t)$, and they have the same sign, so the instantaneous absolute value of $\mathrm{n}_2(t)$ is smaller than the one of $\mathrm{n}(t)$, and the same result can be reached when $\cos(t) < 0$. So the influence of $\mathrm{n}_2(t)$ is less than $\mathrm{n}(t)$. That is to say, narrowband noise to Duffing oscillator has been reduced.

Above all, when the total amplitude of driving force γ' that is the combination of initial amplitude γ and amplitude of signal input is higher than instantaneous value of $\gamma_c + \frac{n(t)}{\cos(t)}$, Duffing oscillator will always be in large scale cycle state, which is similar to the state without noise. And the difference is the size of cycle orbit. When the total amplitude of driving force γ' is lower than the instantaneous value of $\gamma_c + \frac{n(t)}{\cos(t)}$, the influence of narrowband noise still exists. The higher the total amplitude of driving force is, the smaller the influence is. Therefore, the increasing of initial driving force γ can improve the resistance of Duffing oscillator to narrow noise.

3.3 New Method to Detect Signal Frequency Based on the Improvement

According to analysis above, increase of initial driving force amplitude can improve the resistance of Duffing oscillator to noise, but the influence of narrowband noise still exists.

The zero expectation of noise is used in proof of immunity to noise, so it will need enough time to reflect that the mean value of noise is zero. That is to say, if Duffing oscillator can run for a long time, it will reach the state that is immune to noise in broadband noise. It is also suitable to narrowband noise that the resistance of Duffing oscillator will be displayed to maximum when Duffing oscillator runs for a longer time. When the time is not enough long, the roughness of phase diagram or sequence diagram will make the state of Duffing oscillator not clear which will influence the judgment of state.

Therefore, the state of Duffing oscillator can be reached by detecting the total duration of cycle state in narrowband noise. Even if there is still influence of narrowband noise in sequence diagram, total duration of cycle state is longer when oscillator is in cycle state theoretically than the one when oscillator is in chaos state theoretically. So the relationship between the frequency of signal and Duffing oscillator can be reached by detecting the total duration of cycle state. When the frequency of signal is the same as Duffing oscillator, it is the equivalent to add the total amplitude of driving force. That is to say, its duration of cycle state will be longer than the one without signal.

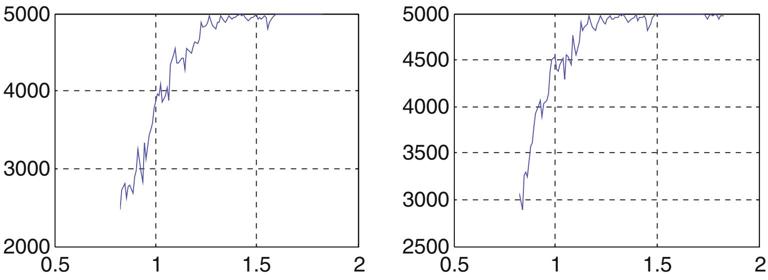
Above all, there are two important steps in frequency detection under strong narrowband noise. The first step is increasing initial driving force amplitude to an appropriate value and the second one is judging Duffing oscillator state by duration of cycle state instead of choice just between chaos state and cycle state.

4 Simulation

4.1 The Appropriate Initial Driving Force Amplitude

According to analysis above, initial driving force amplitude is critical value under broadband noise, while it is higher than critical value under narrowband noise. So it should be confirmed first before signal frequency detection.

Variation tendency of cycle time duration and driving force amplitude is reached to confirmed appropriate initial driving force amplitude. It is shown as Fig. 3 and the driving force amplitude whose duration of cycle state is long and next variation tendency is increasing should be the appropriate one.



(a) only noise input (b) signal and noise input

Fig. 3. The variation tendency of cycle time duration and driving force amplitude. The abscissa is driving force amplitude and ordinate is number of cycle point in simulation.

According to Fig. 3, the variation tendency is the same as analysis result above. To find out the appropriate initial value, the difference value is calculated and shown in Fig. 4. The smallest difference that is not changing slowly is corresponding to the appropriate value. From Fig. 4, it can be found that the appropriate value is between 1.0 and 1.2 in driving force amplitude. The accurate value of this part is shown in Table 1. So the critical value of driving force in narrowband noise can be set as 1.0961 according to the result.

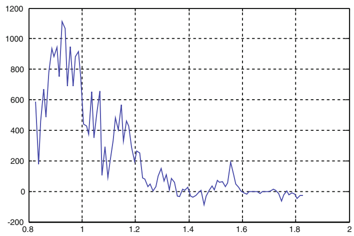


Fig. 4. The partial difference of two variation tendency. The abscissa is driving force amplitude and ordinate is difference number of cycle point in simulation between two situations above.

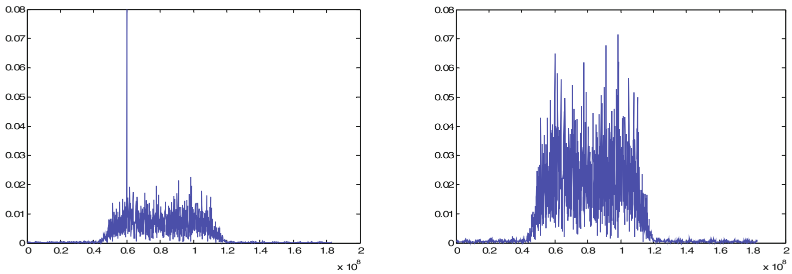
4.2 Verification of the New Method

Let the band of narrowband noise is 50 MHz, and the frequency of driving force is 60 MHz, and sample frequency is 6 GHz, so SNR = 0 dB means $N_0 = 10^{-10}$, SNR = −10 dB means $N_0 = 10^{-9}$, where the amplitude of signal is 0.1. The frequency spectrum of signal and noise with this two SNR is shown as Fig. 5.

As what is shown in Fig. 5, when SNR is 0 dB, signal is obvious in noise which is the same as in broadband noise. but when SNR is −10 dB, signal is covered by noise even if it is still obvious in broadband noise. The result of this two situation is shown in Fig. 6.

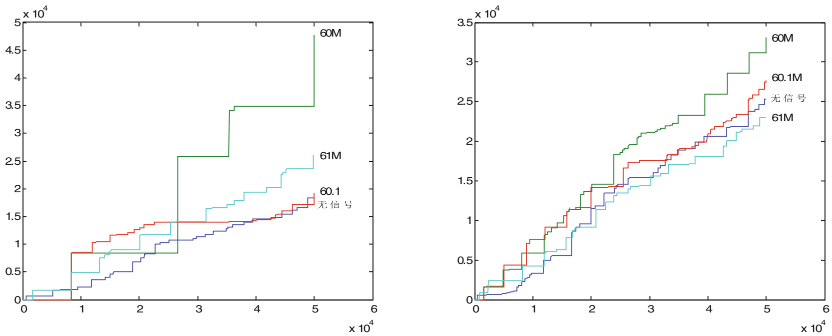
Table 1. The difference of two variation tendency.

Amplitude	Noise	Signal	Difference
1.0661	3881	4538	657
1.0761	4341	4447	106
1.0861	4473	4766	293
1.0961	4542	4633	91
1.1061	4350	4550	200
1.1161	4359	4693	334
1.1261	4420	4900	480
1.1361	4420	4825	405
1.1461	4283	4850	576
1.1561	4558	4887	329
1.1661	4513	4973	460
1.1761	4479	4907	428
1.1861	4561	4851	290
1.1961	4624	4820	196



(a) SNR=0dB (b) SNR=-10dB

Fig. 5. The frequency spectrum of signal and noise with two SNR. The abscissa is time and ordinate is amplitude.



(a) SNR=0dB (b) SNR=-10dB

Fig. 6. The result of frequency detection mentioned in this paper. The abscissa is time and ordinate is number of cycle point.

According to Fig. 6, number of cycle point is on behalf of cycle state duration. When the frequency of signal and oscillator is the same, the duration of cycle state is obviously longer than others including the signal in wrong frequency and noise only. When the SNR is 0 dB, the disparity is very obvious. With the reduction of SNR, the disparity is reducing too. But the result is still able to be detected.

5 Conclusion

In the paper, the difference of Duffing oscillator in broadband noise and narrowband noise is analyzed. It is raised that Duffing oscillator is not immune to noise in narrowband noise like in broadband noise. But the resistance of Duffing oscillator to narrowband noise can be improved by increasing initial driving force amplitude to an appropriate value and the judgment of oscillator state can be more accurate by calculating and comparing total duration of cycle time instead of choice between chaos and cycle state. With this two ways, a new method to detect signal frequency is put forward. Its appropriate driving force value is found out by variation tendency of cycle time duration and driving force amplitude and its correctness is verified by simulation.

References

1. Wang, G., He, S.: A quantitative study on detection and estimation of weak signals by using chaotic duffing oscillators. Trans. Circ. Syst. **50**(7), 945–953 (2003)
2. Holmes, P.: A nonlinear oscillator with a strange attractor. Philos. Trans. Roy. Soc. B Biol. Sci. **292**(1394), 419–448 (1979)
3. Birx, D.L., Pipenberg, S.J.: Chaotic oscillators and complex mapping feed forward networks (CMFFNS) for signal detection in noisy environments. In: International Joint Conference on Neural Networks, IJCNN, pp. 881–888, (1992)
4. Haykin, S., Puthusserypady, S.: Chaotic dynamics of sea clutter. Chaos: Interdiscipl. J. Nonlinear Sci. **7**(4), 777–802 (1997)
5. Vahedi, H., Gharehpetian, G.B., Karrari, M.: Application of duffing oscillators for passive islanding detection of inverter-based distributed generation units. Trans. Power Deliv. **27**(4), 1973–1983 (2012)
6. Yi, W., Song, J.: The analysis of Melnikov process of duffing oscillator in non-Gaussian color noise environment. In: 4th International Congress on Image and Signal Processing (2011)
7. Wang, G., Chen, D., Lin, J., Chen, X.: The application of chaotic oscillators to weak signal detection. Trans. Ind. Electron. **46**(2), 440–444 (1999)