Blind Spectrum Sensing Based on Unilateral Goodness of Fit Testing for Multi-antenna Cognitive Radio System

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Abstract. Goodness of fit tests have been used to find available spectrum in cognitive radio system. In this paper, a unilateral Right-tail Anderson-Darling (URAD) criterion, one of goodness of fit test, is introduced and a blind spectrum sensing scheme based on URAD criterion by using Student's distribution is proposed for multiple antennas cognitive radio system. The spectrum sensing is reformulated as a unilateral Student's testing problem, and the URAD criterion is employed to sense the available spectrum. Numerical simulations verify that the proposed spectrum scheme is robust to noise uncertainty, and greatly outperforms five classical spectrum sensing schemes.

Keywords: Cognitive radio · Blind spectrum sensing · A unilateral Right-tail Anderson-Darling criterion · Noise uncertainty · Multiple antennas

1 Introduction

In wireless communication, Cognitive Radio (CR) is a promising technology to solve the problem of spectrum scarcity owing to the increase of wireless applications and services. The main purpose of CR system is to detect the presence of primary user (PU) within the desired frequency band and then enable secondary users (SU) to access the vacant channel rapidly without causing interference to PU [1]. Therefore, spectrum sensing is a fundamental task in CR.

Recently, goodness of fit (GOF) test is utilized in spectrum sensing and several spectrum sensing schemes based on GOF test, with different criteria and different statistics, are proposed in [2–6,8]. For examples, an Anderson-Darling (AD) sensing in [2] using AD criterion is proposed; spectrum sensing based on Order-Statistics is illustrated in [3]. Both [2,3] show that the spectrum sensing scheme based on GOF test is superior to energy detection (ED) scheme in Additive White Gaussian Noise (AWGN) Channels; however, those schemes also

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suffer from the noise uncertainty and the noise variance must be known as prior information. Subsequently, to circumvent the weakness in [2,3], several schemes are given in [2]. In [4], a multiple antennas assisted and empirical characteristic function (MECF) based blind spectrum sensing is proposed via calculating the distance between the empirical characteristic function and assumed characteristic function. In [5], a new statistics is constructed via sample feature and the AD criterion is used. In [6], a censored AD (CAD) criterion is given. All proposed schemes in [4–6] are better than ED.

However, all schemes in [2–6] are only effective for static PU signal, which means the PU signal are unchangeable during the sensing period, according to [7], and this is not common situation in CR system. Meanwhile, [7] shows that the performance of AD sensing [3] is worse than ED scheme when the PU signal is dynamic during the sensing period (the detailed simulation can be found in [7]). The essential reason is that AD criterion is sensitive to mean rather than variance. To apply the GOF tests into spectrum sensing for dynamic PU signal and improve the performance of AD sensing, Jin in [8] propose a spectrum sensing scheme based on a modified AD criterion using chi2-distribution (MADC). Even though MADC scheme is effective, the work [8] did not fully exploit the inherent signal property, especially, in multi-antenna system.

It is widely known that multiple antennas can offer extra space-dimension information which can be employed to improve the spectrum sensing performance and beneficial to achieve blind spectrum sensing. For example, Zeng utilizes the multiple antennas and proposes two famous spectrum sensing methods including Covariance Absolute Value (CAV) detection and Maximum-Minimum Eigenvalue (MME) detection. However, the existing methods has poor detection performance with small samples.

For improving detection performance with small samples, in this paper, we apply the GOF tests into the multi-antenna CR scenarios. Firstly, we introduce a new kind of statistic via utilizing the dimension information and reformulate the spectrum sensing problem as a unilateral GOF test problem. To examine the above GOF test, we deduce a new GOF criterion called unilateral Right-tail Anderson-Darling criterion (URAD); and then a new blind spectrum sensing scheme based on URAD criterion using this statistic is proposed, which is called URAD sensing. Our analyses and simulations show that the URAD sensing does not need noise variance and be free of noise uncertainty. Moreover, the URAD sensing is superior to ED no matter when there is noise uncertainty or not.

The rest of this paper is organized as follows: URAD sensing is introduced in Sect. 2; the simulation results are given in Sect. 3; finally, the conclusion is drawn.

2 System Model

Suppose that each of P antennas in SU receives N samples during the sensing period. The received sample for the p_{th} antenna at n instance is denoted as $X_p(n)$ $(p = 1, 2, \cdots, P; n = 1, 2, \cdots, N)$. For spectrum sensing, there are two hypothesizes H_0 and H_1, where H_0 denotes the PU is absent and H_1 denotes the PU is present. Therefore, the spectrum sensing problem can be formulated as a binary hypothesis test [1], such that

$$X_p(n) = \begin{cases} W_p(n) & , H_0 \\ S_p(n) + W_p(n) & , H_1 \end{cases} \tag{1}$$

where $S_p(n)$ and $W_p(n)$ are the samples of the transmitted PU signal and the Gaussian noise, respectively. Without loss of generality and for simplify, we assume that $X_p(n)$ and $W_p(n)$ are completely independent; at the same time, we also assume $X_p(n)$ is real-valued; otherwise, simply replace $X_p(n)$ by its real or imaginary parts.

3 URAD Sensing

3.1 Spectrum Sensing as a Unilateral GOF Test Problem

Denote the correlation coefficient between $X_p(n)$ and $X_q(n)$ $(p \neq q)$ as following

$$\rho_{p,q} = \frac{\sum_{n=1}^{N} X_p(n)X_q(n) - N\bar{X}_p\bar{X}_q}{\sqrt{\sum_{n=1}^{N} X_p^2(n) - N\bar{X}_p^2}\sqrt{\sum_{n=1}^{N} X_q^2(n) - N\bar{X}_q^2}} \tag{2}$$

where $\bar{X}_p = \frac{1}{N}\sum_{n=1}^{N} X_p(n)$.

In terms of (2), it is easily to find that $\rho_{p,q} = \rho_{q,p}$, thus, for a given P antennas, we can get $M = P(P-1)/2$ different correlation coefficients. For simplicity, let ρ_m $(m = 1, 2, \ldots, M)$ denote the M_{th} different correlation coefficients. Define variable η_m as

$$\eta_m \stackrel{\Delta}{=} \sqrt{N-2}\rho_m \Big/ \sqrt{1-\rho_m^2} \tag{3}$$

When the PU is absent, $X_p(n)$ is the Gaussian noise $W_p(n)$, the received signal between p_{th} and q_{th} are independent and identically distributed, thus, the ρ_m and η_m are equal to zero. Actually, since the number of samples is limited in real situation, η_m is not always equal to zero and obeys a certain distribution. According to [10] in page 121, in this case, the variable η_m obeys a Student's distribution with $N-2$ degrees and its cumulative distribution function (CDF) is denoted as $F_0(\eta)$ in this paper. Let $F_M(\eta)$ denote the empirical CDF of the variable η_m, that is,

$$F_M(\eta) \stackrel{\Delta}{=} \left|\{m : \eta_m < \eta, 1 \leq m \leq M\}\right| / M \tag{4}$$

where $|\bullet|$ is the cardinality function. According to the Glivenko-Cantelli theorem, in H_0 case, $F_0(\eta) = F_M(\eta)$.

When the PU is present, the received signal between p_{th} and q_{th} are correlated. ρ_m is the positive correlation coefficient ($0 < \rho_m < 1$) and is increased with the growth of signal to noise ratio (SNR). In terms of (2), it can be found that η_m also increases as ρ_m grows when $0 < \rho_m < 1$. In this situation, the probability density function (PDF) of η_m deviates rightward from the PDF of the Student's distribution with $N-2$ degrees, leading to $F_0(\eta) > F_M(\eta)$.

Hence, the spectrum sensing problem can be reformulated as a unilateral Student's distribution testing problem, that is,

$$\begin{cases} F_0(\eta) = F_M(\eta), \ H_0 \\ F_0(\eta) > F_M(\eta), \ H_1 \end{cases} \tag{5}$$

3.2 URAD Criterion

Based on the above analysis, we employ η_m as a new statistic for the proposed scheme. In the following, different from existing works [2–6], we propose a new GOF criterion and apply it to test formula (5).

Two modified AD criteria are introduced in [11] such as Right-tail Anderson-Darling (RAD) criterion and Left-tail Anderson-Darling (LAD) criterion, respectively, which emphasizes the right-tail test and the left-tail test, respectively. The test statistics of RAD criterion can be defined as [11]

$$T_{RAD} = M\int_{-\infty}^{+\infty} [F_0(\eta) - F_M(\eta)]^2 \frac{dF_0(\eta)}{1 - F_0(\eta)} \tag{6}$$

For testing the unilateral alternative hypothesis, the square of $F_0(\eta) - F_M(\eta)$ should not be considered because $F_0(\eta) - F_M(\eta) \geq 0$ is always contented (see (5)). Therefore, based on RAD criterion, we propose a unilateral RAD (URAD) criterion, and then the test statistic of URAD criterion is defined as

$$T_{URAD} = M\int_{-\infty}^{+\infty} [F_0(\eta) - F_M(\eta)] \frac{dF_0(\eta)}{1 - F_0(\eta)} \tag{7}$$

For given η_m $(m = 1, 2, \ldots, M)$, we assume $\eta_1 \leq \eta_2 \leq \cdots \leq \eta_M$, then formula (6) can be rewritten as

$$\begin{aligned} T_{URAD} &= M\textstyle\int_{-\infty}^{+\infty} [F_0(\eta) - F_M(\eta)] \frac{dF_0(\eta)}{1-F_0(\eta)} \\ &= M\textstyle\int_{-\infty}^{\eta_1} [F_0(\eta) - 0] \frac{dF_0(\eta)}{1-F_0(\eta)} \\ &+ M\textstyle\int_{\eta_1}^{\eta_2} \left[F_0(\eta) - \frac{1}{M}\right] \frac{dF_0(\eta)}{1-F_0(\eta)} \\ &+ \cdots \\ &+ M\textstyle\int_{\eta_M}^{+\infty} [F_0(\eta) - 1] \frac{dF_0(\eta)}{1-F_0(\eta)} \\ &= -\sum_{m=1}^{M} \ln(1 - F_0(\eta_m)) - M \\ &= -\sum_{m=1}^{M} \ln(1 - Z_m) - M \end{aligned} \tag{8}$$

where $Z_m = F_0(\eta_m)$. In the URAD criterion, the hypothesis H_0 is accepted if $T_{URAD} < \gamma$, where γ is a decision threshold; otherwise, the hypothesis H_0 is rejected. Denote the false alarm probability P_f and detection probability P_d, respectively, as

$$P_f = prob\{T_{URAD} \geq \gamma | H_0\} \tag{9}$$

$$P_d = prob\{T_{URAD} \geq \gamma | H_1\} \tag{10}$$

According to (9), the γ can be determined for the pre-given P_f through Monte Carlo simulations. Table 1 presents some simulation results using more than 10^5 Monte Carlo simulations. Note that the detection threshold keeps unchangeable when the $N \geq 50$.

Table 1. The threshold versus samples in urad sensing with $P_f = 0.05$

N	10	20	30	40	$N \geq 50$
$P = 4$	3.637	3.156	3.017	2.990	2.818

Hence, the proposed URAD sensing can be summarized in the following steps:

Step1: selecting a detection threshold γ for a given P_f;

Step2: calculating the M different correlation coefficients according to (2) using the received samples;

Step3: calculating the $\eta_m (m = 1, 2, \ldots, M)$ and calculating T_{URAD};

Step4: accepting H_0 if $T_{URAD} < \gamma$; otherwise, accept H_1.

Remark: In the real CR system, the noise uncertainty β always exists [9]. When the noise uncertainty is considered in spectrum sensing, the real noise variance is evenly distributed in an interval $[c^{-1}\sigma^2, c\sigma^2]$, where $c = 10^{(0.1\beta)}$ [9]. Hence, if spectrum sensing scheme needs noise variance, the spectrum sensing scheme must be affected by noise uncertainty. From (3), (8) and Table 1, it can be readily seen that the $\eta_m (m = 1, 2, \ldots, M)$, T_{URAD} and detection threshold are independent of the noise variance, which make the URAD sensing be free of the noise uncertainty. Meanwhile, all that the URAD sensing needs is the received samples, that is, no other prior information is required.

4 Simulation Results

In this section, the detailed detection performance comparisons among the URAD sensing, the ED scheme [1], AD scheme [2], MADC scheme [8], MME scheme and CAV scheme [9] are illustrated.

Suppose the noise variance $\sigma^2 = 1$ for the ED, AD and MADC schemes. Meanwhile the noise variance is assumed to be unknown for MME and URAD sensing. Two types of PU signal are illustrated as in [7]. One type of PU signal is Gaussian variable with zero mean and α^2 signal variance, which is utilized for all of radio frequency (RF), intermediate frequency (IF) and baseband sensing;

another type of PU signal is $S_p(n) = sin[(2\pi n)/K + \varphi]$, which is considered in RF/IF sensing (see [7]). For two types of PU signal, only simulations using Gaussian PU signal with α^2 are provided owing to the similar simulation results for the proposed scheme and the wider application fields for Gaussian PU signal.

Figure 1 presents the detection probabilities, P_d, of six schemes with respect to different SNR scenarios at $P_f = 0.05, N = 50, P = 4$ over AWGN channels. It can be seen that the proposed scheme is much better than other schemes for the same number of samples at different $SNRs$, i.e., the URAD sensing outperforms ED almost 2 dB. For example, at SNR = −7 dB, P_d of the URAD sensing, ED, MADC, MME, AD and CAV respectively, is 0.88, 0.57, 0.50, 0.36, 0.16, 0.53.

To further prove the performance of the proposed scheme, Fig. 2 shows Receiver Operating Characteristic (ROC) curves of five schemes at SNR = −6 dB, $N = 50, P = 4$. It is clear that the performance of the URAD sensing is superior to ED, MADC, MME and AD whatever the P_f is.

In the real CR system, it is necessary to verify the effect of noise uncertainty for the proposed scheme due to the fact that the noise uncertainty β always exists. Note that the noise uncertainty is normally below 1 dB to 2 dB. Figure 3 presents the ROC curves of five schemes over AWGN channels with β=1 dB, SNR = −6 dB, $N = 50$, $P = 4$.

An examination of the Figs. 2 and 3 reveals that the URAD sensing and MME are free of noise uncertainty, and the MADC, ED, AD are affected by noise uncertainty. Meanwhile, it is not hardly to find ED scheme is the most sensitive to noise uncertainty among five schemes. For example, when there is no noise uncertainty at $P_f = 0.05$, $SNR = -6$ dB, P_d of the URAD sensing, ED, MADC, MME, AD respectively, is 0.91, 0.73, 0.65, 0.49, 0.23 (see Figs. 1 or 2); when = 1 dB and $P_f = 0.05$, P_d of the URAD sensing, ED, MADC, MME, AD respectively, is 0.90, 0.41, 0.39, 0.49, 0.24 (see Fig. 3).

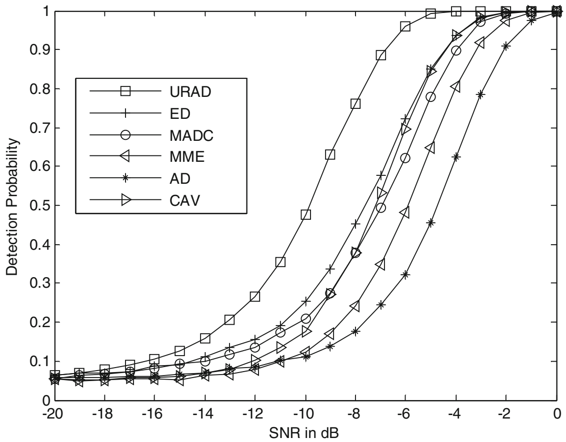

Fig. 1. P_d against $SNRs$ for six schemes with $P_f = 0.05$, $N = 50$, $P = 4$

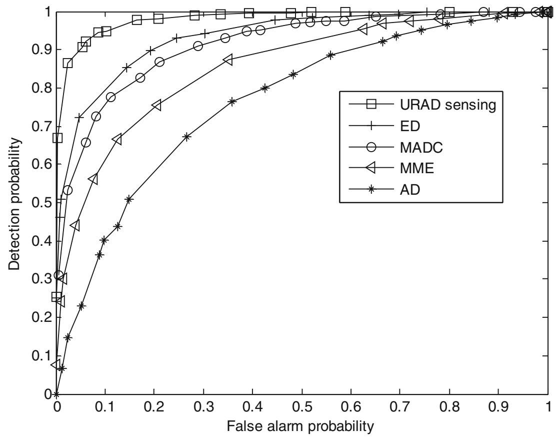

Fig. 2. ROC curves of five schemes when $SNR = -6\,\text{dB}$, $N = 50, P = 4$

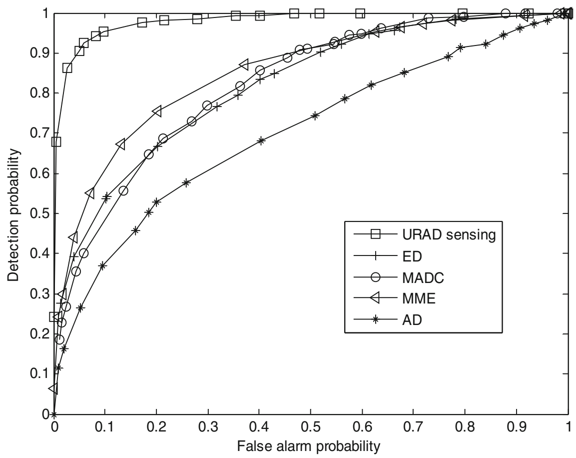

Fig. 3. ROC curves of five schemes when $\beta = 1\,\text{dB}$, $SNR = -6\,\text{dB}$, $N = 50, P = 4$

5 Conclusion

In this paper, multi-antenna assisted and URAD criterion based blind spectrum sensing scheme using Student's distribution is proposed. It does not need any prior information and be free of noise uncertainty. Both theoretical analysis and simulations show that the URAD sensing is more effective and greatly outperforms four existing schemes. For dynamic PU signal, it is worth noting that the URAD sensing is better than ED scheme no matter when there is noise uncertainty or not and no matter what the noise uncertainty is.

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