# Two-Stage Precoding Based Interference Alignment for Multi-cell Massive MIMO Communication

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**Abstract.** The two-stage precoding scheme was proposed to reduce the high overhead of channel estimation and channel state information (CSI) feedback for frequency division duplexing (FDD) massive MIMO systems. However, most of recent works focus only on single cell scenery. In this paper, we examine two-stage precoding for multi-cell FDD massive MIMO system. To manage the inter-cell interference and improve the data rate for cell-edge users, we propose a two-stage precoding based Interference alignment (IA) scheme for multi-cell massive MIMO system. Both theoretical analysis and numerical results show that proposed two-stage precoding based IA scheme can sufficiently improve the data rate for cell-edge users.

**Keywords:** Interference alignment · Two-stage precoding · Massive MIMO · Channel covariance matrix

## 1 Introduction

Massive multiple-input multiple-output (MIMO) technology has been widely considered as a promising technology to achieve the capacity requirement in 5G system [1–4]. The main idea of massive MIMO is that the base station (BS) employs a large number of antennas, which make the independently distributed channel vectors for different users become pairwise orthogonal. In this case, simple linear precodings, such as the matched-filtering (MF) and zero-forcing (ZF), become nearly optimal [5].

Channel state information (CSI) at BS side plays a principal role for downlink precoding and uplink detection in MIMO system. In time-division duplex (TDD) systems, the CSI at BS side can be obtained through uplink training

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by uplink-downlink reciprocity. Under this situation, the overhead of training is proportional to the total number of user antennas. However, in FDD system where uplink-downlink reciprocity does not exist, the CSI at BS side should be acquired by downlink training, channel estimation at user side, and CSI feedback. In this case, both the number of orthogonal training symbols and the amount of CSI feedback are in scale with the number of BS antennas. Therefore, the large amount of BS antennas in FDD massive MIMO system will lead to unacceptable overhead.

However, FDD dominant current wireless cellular system. To overcome this difficulty and make massive MIMO practical in FDD system, a two-stage precoding scheme called "Joint Spatial Division and Multiplexing (JSDM)" was first proposed in [6]. The concept behind two-stage precoding is: (i) users are grouped into different clusters, and the users in the same cluster have almost similar covariance matrices; the covariance matrices of different cluster are independent and occupy different subspaces; (ii) the downlink precoding is divided into two stages: a prebeamforming stage and an inner precoding stage. The prebeamforming, which only depends on channel covariance matrices, is used to eliminate inter-cluster interference. After the prebeamforming stage, the high dimensional massive MIMO channel links for different clusters are partitioned into several independently equivalent channels of reduced dimensions. Then each cluster separately performs the inner precoding to eliminate the intra cluster interferences.

Many researchers followed the two-stage precoding for FDD massive MIMO. In [7], The authors developed a low complexity online iterative algorithm to track the prebeamforming matrix. The iterative algorithm minimizes the total interference power minus weighted total desired power step by step, and converge to global optimal solution under static channels. In [8], the signal-to-leakage-plus-noise ratio (SLNR) is considered to design prebeamforming matrix, an iterative algorithm was proposed to design prebeamforming matrix by maximizing the SLNR. In [9], the JSDM algorithm was adopted in mm-Wave communication.

Almost all above mentioned works [6–9] only considered single cell scenario. To the best of our knowledge, the two-stage precoding in multi-cell scenario is only considered in [7]. In this paper, we examine the two-stage precoding for multi-cell systems. We focus on inter-cell interference management and improving data rate for cell-edge users. Interference alignment (IA) is a promising interference management scheme for multi-cell cellular system [10]. However, BS need to know global CSI to preform IA, which lead to unaffordable signaling overhead in massive MIMO system. Fortunately, two-stage precoding can efficiently reduce the channel dimension, which makes it possible to perform IA in massive MIMO system. Therefore, IA can eliminates interference and improve data rate for users under two-stage precoding, and two-stage precoding can in turn reduces signaling overhead for IA. In this paper we combine the two technologies together and propose a two-stage precoding based IA scheme for multi-cell massive MIMO system. Both theoretical analysis and numerical results show that proposed scheme can sufficiently improve the data rate for cell-edge users.

The rest of this paper is organized as follows. The system model is described in Sect. 2. Section 3 illustrates proposed scheme and analyzes its performance. Numerical results and conclusion are given in Sects. 4 and 5 separately.

**Notations:** We use lowercase (uppercase) boldface to donates column vector (matrix). $(\cdot)^H$ denotes the complex conjugate transpose operation, and $(\cdot)^T$ denotes the transpose operation. $\mathbf{I}_N$ donates a $N \times N$ identity matrix. $\mathbb{C}^{N\times M}$ is the $N \times M$ complex number space. $\mathbb{E}\{\cdot\}$ means expectation operator. We use $\det\{\cdot\}$ and $rank\{\cdot\}$ to donate determinant and rank of a matrix. $\mathbf{n} \sim \mathcal{CN}(0, \mathbf{I}_N)$ means $\mathbf{n}$ is complex circularly-symmetric Gaussian distributed with zero mean and covariance $\mathbf{I}_N$.

## 2 System Model

We consider the typical three-cell FDD system to implement full spectrum reuse, where each cell consists of one BS at the geometric center position. Each BS is equipped with $N_t \gg 1$ antennas in the form of uniform linear array (ULA). The corresponding BSs are separately donated as $BS_1$, $BS_2$, $BS_3$. Thus, as illustrated in Fig. 1, only the three adjacent sectors with mutual interference are analyzed for simplicity. Users, each with $N_r$-antennas, are randomly distributed. According to geographic information, we can partition the users into $G$ groups, and the users in one specific cluster are almost co-located. The $k$-th user in group $j$ can be donated as $(g, k)$, $k = 1, 2, \ldots, K_j$, and $j = 1, 2, \ldots, G$, where $K_j$ is the number of users in group $j$.

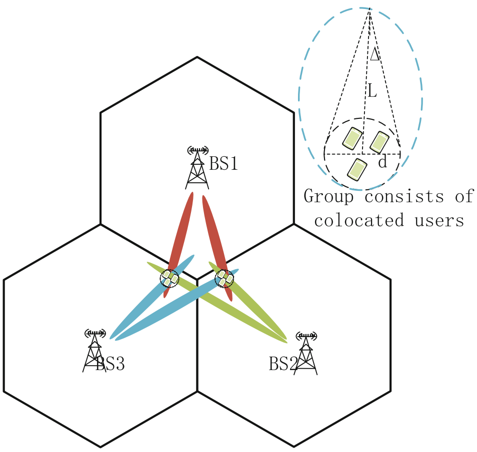


**Fig. 1.** A three cell massive MIMO cellular system. Users are partitioned into groups and three BSs jointly transmit data for each group.

We suppose that BSs antennas are elevated at a very high amplitude, such that there is not enough local scattering around the BS antennas. In this case, antenna correlation at BS side should be considered. Since users in same group are almost co-located, such that it is reasonable to assume that user in group $g$ have the same channel covariance matrix $\mathbf{R}_g$. Then, the channel between BS $i$ and user $k$ in group $g$ is given by

$$\mathbf{H}_{g,k} = \mathbf{W}^i_{g,k}\{\mathbf{R}^i_g\}^{\frac{1}{2}} = \mathbf{W}^i_{g,k}\{\boldsymbol{\Lambda}^i_g\}^{\frac{1}{2}}\{\mathbf{E}^i_g\}^H \tag{1}$$

where $\mathbf{W}^i_{g,k} \in \mathbb{C}^{N_r \times r^i_g} \sim \mathcal{CN}(0, \mathbf{I})$, $\boldsymbol{\Lambda}^i_g$ is an diagonal matrix whose elements are nonzero eigenvalues of $\mathbf{R}^i_g$, $\mathbf{E}^i_g \in \mathbb{C}^{N_t \times r^i_g}$ is the tall unitary matrix of the eigenvectors of $\mathbf{R}^i_g$ corresponding to the nonzero eigenvalues, and $r^i_g$ is rank of $\mathbf{R}^i_g$. We adopt one ring channel model where users group is surrounded by a ring of scatters with radius $d$ [11]. Under this model, the $(m,n)$th entry of $\mathbf{R}^i_g$ is given by

$$\left[\mathbf{R}^i_g\right]_{m,n} = \frac{1}{2\Delta^i_g}\int_{\theta^i_g-\Delta^i_g}^{\theta^i_g+\Delta^i_g} \exp\left[\frac{-2i\pi(p-q)\sin(\alpha)\tau}{\lambda}\right] d\alpha \tag{2}$$

where $\alpha$ is the angle of departure (AoD) of one path from BS to scattering ring with interval $[\theta^i_g - \Delta^i_g, \theta^i_g + \Delta^i_g]$, $\theta^i_g$ is the azimuth angle of central point of scattering ring, $\tau$ is the antenna element spacing, and $\lambda$ is the carrier wavelength. $\Delta^i_g \approx \arctan(d/L^i_g)$ is the angular spread (AS) and $L^i_g$ is the distance between BS $i$ and group $g$.

Interestingly, in the massive MIMO system, the toeplitz matrix $\mathbf{R}^i_j$ asymptotically approaches to a circulant matrix, and $\mathbf{E}^i_j$ can be constructed by $r^i_j$ columns of the $N_t \times N_t$ unitary discrete Fourier transform (DFT) matrix $\mathbf{F}_{N_t}$ as [6]

$$\mathbf{E}^i_g = \left[\mathbf{f}_n : n \in \mathcal{I}^i_g\right] \tag{3}$$

where $\mathbf{f}_n$ represents $n$-th column of $\mathbf{F}_{N_t}$, and the index set $\mathcal{I}^i_g$ is defined as

$$\begin{aligned}\mathcal{I}^i_j = &\Big\{n : 2n/N_t - 1 \in \big[\frac{\tau}{\lambda}\sin(\theta^i_g + \Delta^i_g).\\ &\qquad \frac{\tau}{\lambda}\sin(\theta^i_g - \Delta^i_g)\big], n = 0, 1, \cdots, N_t - 1\Big\}.\end{aligned} \tag{4}$$

Moreover, we have $r^i_g = \left\lfloor N_t \min\left\{1, \rho^i_g\right\}\right\rfloor$, where

$$\begin{aligned}\rho^i_g = &\left|\frac{\tau}{\lambda}\sin(\theta^i_g + \Delta^i_g) - \frac{\tau}{\lambda}\sin(\theta^i_g - \Delta^i_g)\right|\\ = &\,2\frac{\tau}{\lambda}\left|\cos(\theta^i_g)\right|\sin(\Delta^i_g).\end{aligned} \tag{5}$$

Since the AS $\Delta^i_g$ is relatively small, $\mathbf{R}^i_g$ possesses low rank property, i.e., $r^i_g \ll N_t$.

# 3 Two-Stage Precoding Based IA Scheme

## 3.1 Proposed Transmission Scheme

To improve the data rate for cell-edge users, we let three BSs to jointly transmit data for each users group. Then the received signal at group $g$ is given by,

$$\mathbf{y}_g = \sum_{i=1}^{3} \mathbf{H}_g^i \mathbf{P}_g^i \mathbf{x}_g^i + \sum_{i=1}^{3} \sum_{g'=1, g' \neq g}^{G} \mathbf{H}_{g'}^i \mathbf{P}_{g'}^i \mathbf{x}_{g'}^i + \mathbf{n}_g \tag{6}$$

where $\mathbf{H}_g^i = \left[ \{\mathbf{H}_{g,1}^i\}^T \{\mathbf{H}_{g,1}^i\}^T \cdots \{\mathbf{H}_{g,K_g}^i\}^T \right]^T \in \mathbb{C}^{K_g N_t \times N_r}$ is the channel matrix associates with the BS $i$ and group $g$, $\mathbf{P}_g^i \in \mathbb{C}^{N_r \times S_g^i}$ is the precoding matrix, $\mathbf{x}_g^i \in \mathbb{C}^{S_g^i \times 1}$ is the data vector transmitted by BS $i$ to group $g$, and $\mathbf{n}_g \sim \mathcal{CN}(0, \mathbf{I}_{N_t})$ is the additive complex Gaussian noise. In this paper, we adopt the two-stage precoding framework, where the precoding process can be divided into two stages as

$$\mathbf{P}_g^i = \mathbf{B}_g^i \mathbf{V}_g^i, \tag{7}$$

where the prebeamforming matrix $\mathbf{B}_g^i$, related to spatial correlation matrices, is utilized to eliminate the inter-cluster interferences; the $M_g^i \times S_g^i$ matrix $\mathbf{V}_g^i$ denotes the inner precoder dealing with the intra-cluster interferences, which depends on $K_g N_r \times M_g^i$ effective equivalent channel matrix $\overline{\mathbf{H}}_g^i = \mathbf{H}_g^i \mathbf{B}_g^i$; $M_g^i$ is the rank of $\overline{\mathbf{H}}_g^i$ seen by the inner precoder, which satisfies $S_g^i \leq M_g^i \leq r_g^i$. It can be found that $\overline{\mathbf{H}}_g^i$ possesses a much smaller number of unknown parameters than the original channel matrix $\mathbf{H}_g^i$.

The designing of the prebeamforming matrix $\mathbf{B}_g^i$ is the key task of two-stage precoding and has been examined in [6,8] for single-cell system.

Clearly, treating all groups in the whole system as one big group, we can directly calculate the $\mathbf{B}_g^i$ with the methods for the single-cell system. Without loss of generality, we adopt the DFT based prebeamforming, and achieve the prebemforming matrices through concentrating the subspace span$\{\mathbf{B}_g^i\}$ into the null-space of span$\{\mathbf{\Xi}_g^i\}$, where $\mathbf{\Xi}_g^i$ is constructed by $\mathbf{E}_{g'}^i$ of all but the group $g$ in the system as

$$\mathbf{\Xi}_g^i = \left[ \mathbf{f}_n : n \in \bigcup_{g'=1, g' \neq g}^{G} \mathcal{I}_{g'}^i \right]. \tag{8}$$

Utilizing the orthogonality of columns of DFT matrix, $\mathbf{B}_g^i$ for group $g$ is given by

$$\mathbf{B}_g^i = \left[ \mathbf{f}_n : n \in \left( \mathcal{I}_g^i - \bigcup_{g'=1, g' \neq g}^{G} \mathcal{I}_{g'}^i \right) \right]. \tag{9}$$

where the set $\mathcal{A} - \mathcal{B}$ contains all the elements that are in set $\mathcal{A}$ but not in set $\mathcal{B}$, i.e., $\mathcal{A} - \mathcal{B} = \{x : x \in \mathcal{A} \text{ and } x \notin \mathcal{B}\}$. It can be learned from the computation of

$\mathbf{B}_g^i$ that $M_g^i$ equals the number of columns of $\mathbf{E}_g^i$ that linearly independent with columns of $\mathbf{\Xi}_g^i$, namely,

$$M_g^i = \text{rank}\{\mathbf{B}_g^i\} = \left| \mathcal{I}_g^i - \bigcup_{g'=1,g'\neq g}^{J} \mathcal{I}_{g'}^i \right|, \tag{10}$$

where $|\mathcal{A}|$ donates the number of elements in set $\mathcal{A}$. The resultant prebeamforming matrices $\mathbf{B}_g^i$ satisfies the following constraint:

$$\left\{\mathbf{E}_{g'}^i\right\}^T \mathbf{B}_g^i = 0, \forall g' \neq g, \tag{11}$$

which means that the transmitted signal to group $g$ will not cause interference to other groups. Then the inter-group interference terms in (6) are eliminated, and the received signals can be simplified as

$$\mathbf{y}_g = \sum_{i=1}^{3} \overline{\mathbf{H}}_g^i \mathbf{V}_g^i \mathbf{x}_g^i + \mathbf{n}_g \tag{12}$$

where $\overline{\mathbf{H}}_g = \mathbf{H}_g \mathbf{B}_g$, of dimension $K_g N \times D_g^i$, is the reduced dimensional effective channel between BS $i$ and group $g$. As 3 BSs jointly server one group, the received signal is superposition of signal transmitted by 3 BSs. For simplicity of expression, we assume that the number of user in each group is same as number of BSs. But the scheme can be easily extended to the general case. In this case, each BS serves one specific user in each group. Thus we can rewrite the received signal in (12) separately for each user.

$$\mathbf{y}_{g,1} = \overline{\mathbf{H}}_{g,1}^1 \mathbf{V}_g^1 \mathbf{x}_g^1 + \sum_{i=2,3} \overline{\mathbf{H}}_{g,1}^i \mathbf{V}_g^i \mathbf{x}_g^i + \mathbf{n}_{g,1} \tag{13}$$

$$\mathbf{y}_{g,2} = \overline{\mathbf{H}}_{g,2}^2 \mathbf{V}_g^2 \mathbf{x}_g^2 + \sum_{i=1,3} \overline{\mathbf{H}}_{g,2}^i \mathbf{V}_g^i \mathbf{x}_g^i + \mathbf{n}_{g,2} \tag{14}$$

$$\mathbf{y}_{g,3} = \overline{\mathbf{H}}_{g,3}^3 \mathbf{V}_g^3 \mathbf{x}_g^3 + \sum_{i=1,2} \overline{\mathbf{H}}_{g,3}^i \mathbf{V}_g^i \mathbf{x}_g^i + \mathbf{n}_{g,3} \tag{15}$$

Obviously, with prebeamforming and joint transmission, the effective channel of each group becomes a 3 BS and 3 users MIMO X channel [10]. A promising interference management scheme named interference alignment (IA) has been proposed to efficiently achieve multiple signaling dimensions under MIMO X channel [10]. The inner precoder $\mathbf{V}_g^1, \mathbf{V}_g^2$ and $\mathbf{V}_g^3$ are carefully chosen to consolidate the interference at each receiver into a reduced-dimensional subspace space, while keep the desired signals separable from interference. An $N \times S_g^k$ suppression matrix $\mathbf{U}_{g,k}$ whose columns are orthonormal to the interference subspace is used at each user to eliminate interference. When IA is feasible [10], the following conditions are met

$$rank\{\mathbf{U}_{g,k}^H \overline{\mathbf{H}}_{g,k}^k \mathbf{V}_g^k\} = S_g^k, \tag{16}$$

$$\mathbf{U}_{g,k}^H \overline{\mathbf{H}}_{g,k}^i \mathbf{V}_g^i = \mathbf{0}, \forall k \neq i. \tag{17}$$

Thus, the intra-group interference is completely eliminated and the desired signal of user $k$ in group $g$ can be rewrote as

$$\underline{\mathbf{y}}_{g,k} = \underline{\mathbf{H}}^k_{g,k}\mathbf{x}^k_g + \underline{\mathbf{n}}_{g,k} \tag{18}$$

where $\underline{\mathbf{y}}_{g,k} = \mathbf{U}^H_{g,k}\mathbf{y}_{g,k}$, $\underline{\mathbf{H}}^k_{g,k} = \mathbf{U}^H_{g,k}\overline{\mathbf{H}}^k_{g,k}\mathbf{V}^k_g$ is the $S^k_g \times S^k_g$ full rank effective channel between $k$th BS and user $k$ in group $g$, $\underline{\mathbf{n}}_{g,k} = \underline{\mathbf{U}}^H_{g,k}\mathbf{n}_{g,k}$ is the effective Gaussian noise with distribution $\mathcal{CN}(0, \mathbf{I}_{S^k_g})$.

### 3.2 Performance Analysis

In this subsection, we provide performance analysis of proposed transmission scheme in term of number of data streams, achievable rate and signaling overhead. For simplicity of expression, we consider a symmetric scenario with same rank $r^i_g = r$ of the channel covariance matrix, same dimension $M^i_g = M$ of the prebeamforming matrix, and same number $S^i_g = S$ of data streams per group. However, the analysis can be easily extended to the general case.

To meet the IA feasible condition (16) and (17), the number of data streams should satisfy the follow constrain,

$$S \leq \frac{M+N}{K_g+1}. \tag{19}$$

As 3 BSs transmit data for one user group simultaneously, the total number of data streams for each group is $3S$.

We assume that the intra-group interference is exactly eliminated by IA. By carefully considering the leaked inter-group interference, the achievable rate of group $g$ is given by

$$R_g = \sum_{k=1}^{3} \log\det\left\{\mathbf{I}_S + \mathbf{K}^{-1}_{g,k}\underline{\mathbf{H}}^k_{g,k}\mathbf{Q}^k_g\left(\underline{\mathbf{H}}^k_{g,k}\right)^H\right\}. \tag{20}$$

where

$$\mathbf{K}_{g,k} = \mathbf{I}_S + \sum_{i=1}^{3}\sum_{g'=1,g'\neq g}^{G} \mathbf{H}^i_{g,k}\mathbf{P}^i_{g'}\mathbf{Q}^i_{g'}\left(\mathbf{H}^i_{g,k}\mathbf{P}^i_{g'}\right)^H \tag{21}$$

is the covariance matrix of inter-group interference pluse noise, and $\mathbf{Q}^k_g = \mathbb{E}\{\mathbf{x}^i_g[\mathbf{x}^i_g]^H\}$ is the covariance matrix of data vector.

In the prebeamforming stage, the dominant eigenmodes $\{\mathbf{E}^i_g\}$ is necessary. It is fact the channel statistical informations remain unchanged within a long time, such that the acquisition of $\{\mathbf{E}^i_g\}$ has low overhead. We focus on the analysis for acquisition of the effective channel $\{\overline{\mathbf{H}}^i_g\}$, which are essential to the design of inner precoding. Thanks to the prebeamforming, the effective channel of different groups are independent. Training symbols can be reuse between groups. But it should be note that 3 BSs transmit data to each group simultaneously. Therefore, the training symbols of different BSs for a specific group should be orthogonal. In this case, we need $3M$ orthogonal dimensions to train the effective channels. To perform IA, the BSs need to know not only the intended channels but also the interference channels. Therefore, each group need to feedback $9 \times M \times N$ complex channel coefficients, which is 3 times of that without IA. We assume that each complex channel coefficient is quantized into $Q$ bits, the channel coherent block length is $T$, and the rate of the feedback channel is $F$ bits per symbol. Taking into consideration of overhead of training and feedback, the effective achievable rate of group $g$ is given by

$$R_{g,ohd} = \max\left\{1 - \frac{3M}{T} - \frac{9MNQ}{FT}, 0\right\} R_g. \tag{22}$$

## 4 Numerical Result

In this section, we evaluate the proposed two-stage precoding based IA scheme for multi-cell massive MIMO communication through numerical simulation. We consider a three cell cellular system with 3 groups in cell-edge. The number of users in each group is 3. The radius of the cell is 1 km. The distance between user groups and BS is 800 m. Each BS is equipped with a ULA with $M = 128$ antennas, and each user is equipped with $N = 2$ antennas. The BS antenna elements spacing equal to half wavelength. The carrier frequency is 2 GHz. We generate massive MIMO channel according to (1) and (2). The variance of the noise is 1. A total number of $10^4$ Monte Carlo runs are used to numerically average the achievable rate.

Firstly, we compare the performance of two-stage precoding without IA (each user group is served by only on BS) and the proposed two-stage precoding based IA transmission scheme. In this example, the rank $D$ of effective channel is 2, such that if IA is not applied, each group can only receive 2 data streams, which is less than the number of users in group. Whereas, the total number of streams for each group with IA is 3. In this case, the proposed scheme achieves an improvement factor of 3/2 in number of data streams. Figure 2 compares the sum rate per group of the two-stage precoding transmission with IA against the two-stage precoding transmission without IA. The simulation results show that the sum rate is obviously improved by IA.

To uncover the impact of training and CSI feedback overhead on the performance, we consider a numerical example of effective sum rate in (22), where $\mathrm{SNR} = 30\,\mathrm{dB}$, $F = 4$ and $Q = 16$. Figure 3 shows the effective sum rate per group versus channel coherent block length $T$. From Fig. 3 we can observe that the proposed two-stage precoding with IA is still efficient when overhead is considered. But the gaps between transmission with IA and that without IA become smaller when $T$ decreases. The efficient sum rate of transmission with IA become lower than that without IA when $T$ is extremely small. The reasons behind this is that BS need to know more CSI to perform interference alignment, which increases the overhead of channel estimation and feedback.

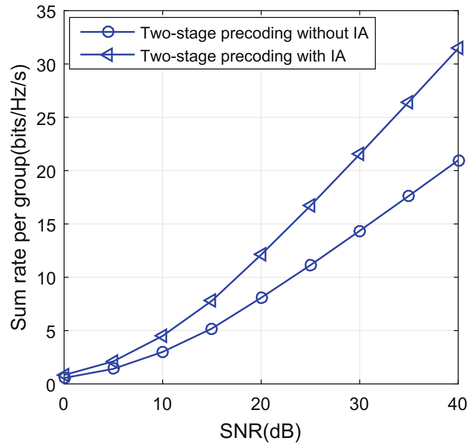


**Fig. 2.** Sum rate per group over two-stage precoding transmission with IA and that without IA versus SNR.

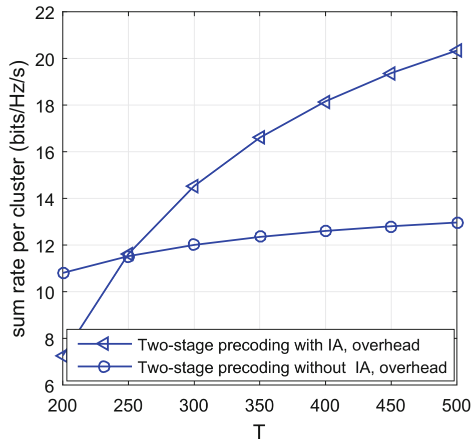


**Fig. 3.** Effective sum rate per group versus channel coherent block length T (SNR = 30 dB, $F = 4, Q = 16$).

## 5 Conclusion

In this paper, we investigated two-stage precoding for multi-cell FDD massive MIMO systems, where multiple antennas at user side is considered. We proposed a two-stage precoding based interference alignment scheme. The performance of proposed transmission scheme was evaluated by theoretical analysis. Numerical simulation showed that the proposed scheme can efficiently improve the data rate of cell-edge users with affordable overhead.

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