

Coverage Probability and Data Rate of D2D Communication Under Cellular Networks by Sharing Uplink Channel

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Abstract. The device-to-device (D2D) communication has been regarded as a promising technique to effectively upgrade the existing cellular network. Despite the intriguing perspectives of D2D technique, the performance of D2D-aided cellular network may degrade owing to the severe interference imposed by newly introduced D2D links. With a motive to deal with this problem, the key performance parameters of D2D-aided underlying cellular networks, including the coverage probability and total data rate have been investigated. Firstly, we analyze the expressions of coverage probability for both the conventional cellular links and the D2D links, and then we give out the approximate expression of the ergodic rate for both individual links and that of the whole underlying system. After that, in order to optimize the performance of the system in terms of throughput, more parameters closely related to the channel capacity are studied. Finally, the simulation results revealed that the best values for key parameters (e.g. the density of D2D users) are attainable, and the total data rate can be greatly improved according to our proposed strategies.

1 Introduction

With the booming of smart devices, the mobile traffics have been through a tremendous growing trend. Since the existing standards and mechanisms can no longer efficiently support the ever-growing traffic demands of customers, buttons for effective countermeasures need to be pressing imminently. Meanwhile, heavy burden of base stations (BSs) should be relieved substantially by offloading the mobile traffics from BS side to the terminal side [1, 2].

Device-to-device (D2D) communication has been regarded as a newly introduced supplementary technique for cellular communication owing to its great capabilities of significantly improving the spectral efficiency of wireless networks [3, 4]. Since a so called “D2D pair” has plenty of merits over the conventional cellular networks (CNs), including extending the radio coverage and supporting a variety of proximity mobile services [5, 6]. However, since the same licensed spectrum is shared by the D2D users (DUs) and conventional Cellular Users (CUs) during D2D transmission, a severe interference to the CUs may affect the transmitting performance when the D2D transmitter (DT) and CUs are coexisting [7, 8].

To address the above-mentioned issues, interference-management schemes and power control policy should be studied [8–10]. In particular, it is also necessary to take the other important factors (for example, the location and number of D2D pairs) into account when we implement D2D-aided underlying CNs.

In [11], the author proposed an expression for the coverage probability of the CUs based on the Poisson point process (PPP). Besides, in [12], the authors introduced a model concerning signal-to-interference-plus-noise ratio (SINR) in multi-cell systems. However, both the authors of [11, 12] did not analyze the impact on the distribution of co-cell CUs, which is critical for a constantly changing network.

In this paper, we studied the coverage probability and the total data rate of the D2D aided cellular systems. We assume that K D2D pairs share the same radio resource with M CUs according to PPP model. The main contributions of this paper include the following: (1) analyzing the approximate expressions for the average coverage probabilities and the total data rate of both the CUs and the D2D pairs; (2) investigating the best performance of the hybrid network; and (3) optimizing the total data rate of the D2D-aided underlying CNs through simulations by fine-tuning the crucial parameters, including the density of DUs, the scaling factor (between D2D and cellular links) and the total number of users and the maximum number of users (including CUs and DUs).

The framework of the paper is as follows. Section 2 introduces the system model of D2D aided cellular system. In Sect. 3, we derived the expressions of coverage probability for the cellular and D2D links. And the total data rate is studied in Sect. 4. And then, the numerical results are analyzed in Sect. 5. Finally, Sect. 6 draws the conclusion of the paper.

2 System Model

In Fig. 1, we consider a single cell in which the BS located at the center with a radius of R . We assume that there are M CUs and K D2D pairs in the system model, and D2D pairs will share the same licensed uplink spectrum resources with CUs. Moreover, within the cell coverage, we assume that all the M CUs are distributed according to a homogeneous PPP Φ_1 model with the density of λ_c . Similarly, all the K DTs are set to be placed according to a homogeneous PPP Φ_2 model with the density of λ_d in the cell. In addition, the DT-DR interval (i.e. d_{TR}) is set to be time-invariant. Finally, each D2D pair is perceived as a single point under a large-cellular-coverage environment. The number of CUs (or DTs) in a cell can be modelled as a Poisson distributed random variable, and with the mean of $\mathbb{E}[K] = \lambda_d \pi R^2$ ($\mathbb{E}[M] = \lambda_c \pi R^2$).

In the following, a normal situation is considered, i.e. both the inter-CU and inter-DU interference exist, with non-zero the interference of CU-DU observed at the same time. All the above-mentioned interferences are illustrated in Fig. 1. Under this circumstance, three primary interference sources exist, including the interference applied on the DR by CUs, the interference applied on the DR by the geographically close-by DTs, and the interference applied on the BS by the DTs.

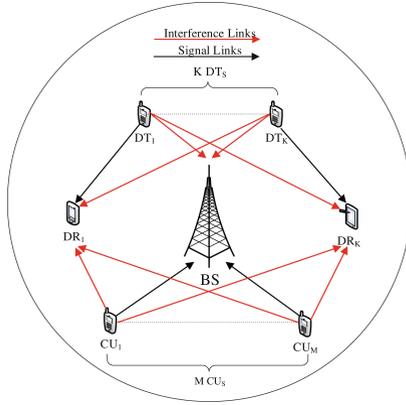


Fig. 1. The system model of the D2D aided cellular system, and each cell provided the services to all users (i.e. CUs and DUs), and DUs share the same uplink resource with CUs.

For a given cell, the received signal at the BS can be rewritten as

$$\begin{aligned}
 y_{BS_0} = & \sum_{k=1}^M \sqrt{cd_{C_k,BS}^{-\alpha} P_{C_k}} \cdot h_{C_k,BS} \cdot s_{C_k} \\
 & + \sum_{i=1}^K \sqrt{cd_{DT_i,BS}^{-\alpha} P_{DT_i}} \cdot h_{DT_i,BS} \cdot s_{DT_i} + N, \tag{1}
 \end{aligned}$$

where we let P_{C_k} denote the transmit power of the k -th CU, and P_{DT_i} denote the transmit power of the i -th DT, and $d_{C_k,BS}$ ($d_{DT_i,BS}$) represents the interval between the k -th CU (the i -th DT) and the BS. Furthermore, $h_{C_k,BS}$ ($h_{DT_i,BS}$) denotes the channel coefficient of the k -th CU (the i -th DT) to the BS with a distribution of $\mathcal{CN}(0, \mu)$. Besides, c and α denote the path loss constant and the path loss exponent, respectively, and s_{C_k} and s_{DT_i} denote the transmit signals of the k -th CU and the i -th DT, respectively. Moreover, $E\{|s|^2\} = 1$ is satisfied, and, N denotes the variance of the additive white Gaussian noise (AWGN) at the receiver with a distribution of $\mathcal{CN}(0, \sigma^2)$.

As a result, the received signals at the j -DR can be rewritten as

$$\begin{aligned}
 y_{DR_j} = & \sum_{k=1}^M \sqrt{cd_{C_k,DR_j}^{-\alpha} P_{C_k}} \cdot h_{C_k,DR_j} \cdot s_{C_k} \\
 & + \sum_{i=1}^K \sqrt{cd_{DT_i,DR_j}^{-\alpha} P_{DT_i}} \cdot h_{DT_i,DR_j} \cdot s_{DT_i} + N, \tag{2}
 \end{aligned}$$

where d_{C_k,DR_j} denotes the interval between the j -th DR and the k -th CU, d_{DT_i,DR_j} denotes the interval between the j -th DR and the i -th DT, respectively.

Moreover, h_{C_k, DR_j} represents the channel coefficients of the k -th CU to the j -th DR, h_{DT_i, DR_j} represents the channel coefficients of the j -th DR to the i -th DT, respectively, under the distribution following $\mathcal{CN}(0, \mu)$.

We apply subscript 0 and $k(l)$ represent the BS and the $k(l)$ -th CU, and we apply subscript $i(j)$ denote the i -th DT (the j -th DR). For instance, we let $d_{i,j}$ instead of d_{DT_i, DR_j} denote the interval between the i -th DT and the j -th DR. According to the above-mentioned models, the SINR of the CU and the j -th DR can be expressed as

$$\text{SINR}_k = \frac{P_k d_{k,0}^{-\alpha} |h_{k,0}|^2}{\sum_{i=1}^K P_i d_{i,0}^{-\alpha} |h_{i,0}|^2 + \sum_{\substack{l=1 \\ l \neq k}}^M P_l d_{l,0}^{-\alpha} |h_{l,0}|^2 + \sigma^2},$$

at the BS (3a)

$$\text{SINR}_j = \frac{P_i d_{i,j}^{-\alpha} |h_{i,j}|^2}{\sum_{k=1}^M P_k d_{k,j}^{-\alpha} |h_{k,j}|^2 + \sum_{\substack{i=1 \\ i \neq j}}^K P_i d_{i,j}^{-\alpha} |h_{i,j}|^2 + \sigma^2},$$

at the DR. (3b)

3 Analysis of Coverage Probability

In this section, we will discuss the coverage probability of the cellular links and the D2D links, and derive the approximate expression of each link, where all CUs and D2D pairs are functioning synchronously. Without loss of generality, the DT-DR interval (i.e. d_{TR}) is set to be time-invariant.

3.1 Coverage Probability for Cellular Links

According to the aforementioned assumptions and the realizations of the PPP Φ_1 and Φ_2 , for a established SINR threshold (at BS) at the k -th CU (i.e. β_c), the average uplink coverage probability can then be written as

$$P_{\text{cov}}^{\text{CU}}(\beta_c, \lambda, \alpha) = \mathbb{E}[\mathbb{P}\{\text{SINR}_k > \beta_c\}], \quad (4)$$

with

$$\begin{aligned} & \mathbb{P}\{d_{TR} > \beta_c\} \\ &= \mathbb{P}\left\{ \frac{P_k d_{k,0}^{-\alpha} |h_{k,0}|^2}{\sum_{i \in \phi_2} P_i d_{i,0}^{-\alpha} |h_{i,0}|^2 + \sum_{l \in \phi_1 \setminus \{k\}} P_l d_{l,0}^{-\alpha} |h_{l,0}|^2 + \sigma^2} > \beta_c \right\} \\ &= \exp\left(-\frac{\mu \beta_c d_{k,0}^\alpha \sigma^2}{P_k}\right) L_{I_d}\left(\frac{\mu \beta_c d_{k,0}^\alpha}{P_k}\right) L_{I_c}\left(\frac{\mu \beta_c d_{k,0}^\alpha}{P_k}\right), \end{aligned} \quad (5)$$

where $I_d = \sum_{i \in \phi_2} P_i d_{i,0}^{-\alpha} |h_{i,0}|^2$, $I_c = \sum_{l \in \phi_1 \setminus \{k\}}^M P_l d_{l,0}^{-\alpha} |h_{l,0}|^2$, $s = \frac{\mu \beta_c d_{k,0}^\alpha}{P_k}$, and $\mathcal{L}_{I_d}(s)$

and $\mathcal{L}_{I_c}(s)$ denote the Laplace transformation of random variables(i.e. I_d and I_c evaluated at s , respectively). Moreover, the expression $\mathcal{L}_{I_d}(s)$ is shown to be

$$\begin{aligned} \mathcal{L}_{I_d}(s) &= \mathbb{E} \left[\exp \left(-s \sum_{i \in \phi_2} P_i d_{i,0}^{-\alpha} |h_{i,0}|^2 \right) \right] \\ &= \exp \left[-\frac{2\pi\lambda_d}{\alpha} \left(\frac{\mu\beta_c P_i}{P_k} \right)^{\frac{2}{\alpha}} \Gamma \left(\frac{2}{\alpha} \right) \Gamma \left(1 - \frac{2}{\alpha} \right) d_{k,0}^2 \right], \end{aligned} \tag{6}$$

where $\Gamma(x)$ denotes the Gamma function with $\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt$.

Similarly, we can derive

$$\mathcal{L}_{I_c}(s) = \exp \left[-\frac{2\pi\lambda_c}{\alpha} \left(\frac{\mu\beta_c P_l}{P_k} \right)^{\frac{2}{\alpha}} \Gamma \left(\frac{2}{\alpha} \right) \Gamma \left(1 - \frac{2}{\alpha} \right) d_{k,0}^2 \right]. \tag{7}$$

By substituting (6) and (7) into (5) and using the Euler’s Formula $\Gamma(1-x)\Gamma(x) = \frac{\pi}{\sin \pi x}$, the cellular links’ coverage probability can be written as

$$P_{\text{cov}}^{\text{CUE}}(\beta_c, \lambda, \alpha) = \int_0^R e^{-ar^\alpha - br^2 - cr^2} f_r(r) dr, \tag{8}$$

where $a = \frac{\beta_c \sigma^2}{P_k}$, $b = \frac{2\pi^2 \lambda_d}{\alpha \sin(2\pi/\alpha)} \left(\frac{\mu\beta_c P_i}{P_k} \right)^{\frac{2}{\alpha}}$, $c = \frac{2\pi^2 \lambda_c}{\alpha \sin(2\pi/\alpha)} \left(\frac{\mu\beta_c P_l}{P_k} \right)^{\frac{2}{\alpha}}$ and $r = d_{k,0}$.

3.2 Coverage Probability for D2D Links

Now we consider a D2D pair and assume that its DR is located at the origin place. We can define that the threshold at the j -th DR is β_d , and then the average coverage probability over the plane can be written as

$$P_{\text{cov}}^{\text{D2D}}(\beta_d, \lambda, \alpha) = \mathbb{E} [\mathbb{P} \{ \text{SINR}_j > \beta_d \}], \tag{9}$$

where

$$\begin{aligned} \mathbb{P} \{ \text{SINR}_j > \beta_d \} &= \mathbb{P} \left\{ \frac{P_j d_{TR}^{-\alpha} |h_{i,i}|^2}{I_c + I'_d + \sigma^2} > \beta_d \right\} \\ &= \exp \left(-\frac{\mu\beta_d d_{TR}^\alpha \sigma^2}{P_j} \right) L_{I'_c}(s') L_{I'_d}(s'), \end{aligned} \tag{10}$$

with $I'_c = \sum_{k \in \phi_1} P_k d_{k,j}^{-\alpha} |h_{k,j}|^2$, $I'_d = \sum_{i \in \phi_2 \setminus \{j\}}^N P_i d_{i,j}^{-\alpha} |h_{i,j}|^2$ and $s' = \frac{\mu \beta_d d_{TR}^\alpha}{P_j}$.

Similar to (6), the expressions of $\mathcal{L}_{I'_c}(s')$ and $\mathcal{L}_{I'_d}(s')$ can be given by

$$\begin{aligned} \mathcal{L}_{I'_c}(s') &= \exp \left[-\frac{2\pi\lambda_c}{\alpha} \left(\frac{\mu\beta_d d_{TR}^\alpha P_k}{P_j} \right)^{\frac{2}{\alpha}} \Gamma\left(\frac{2}{\alpha}\right) \Gamma\left(1 - \frac{2}{\alpha}\right) \right] \\ &= \exp \left[\frac{2\mu\pi^2\lambda_c}{\alpha \sin(2\pi/\alpha)} \left(\frac{\beta_d P_k}{P_i} \right)^{\frac{2}{\alpha}} \right] \end{aligned} \tag{11}$$

and

$$\begin{aligned} \mathcal{L}_{I'_d}(s') &= \exp \left[-\frac{2\pi\lambda_d}{\alpha} \left(\frac{\mu\beta_d d_{TR}^\alpha P_i}{P_j} \right)^{\frac{2}{\alpha}} \Gamma\left(\frac{2}{\alpha}\right) \Gamma\left(1 - \frac{2}{\alpha}\right) \right] \\ &= \exp \left[\frac{2\mu\pi^2\lambda_d}{\alpha \sin(2\pi/\alpha)} (\beta_d)^{\frac{2}{\alpha}} \right] \end{aligned} \tag{12}$$

where P_j denotes the transmit power of the j -th DT.

Let us consider the case in which a time-invariant transmit power is assumed in either the DTs (P_D) or the CUs (P_C), thus the expression of the coverage probability of the typical D2D links can then be derived as

$$\begin{aligned} P_{cov}^{D2D}(\beta_d, \lambda, \alpha) &= \mathbb{E} [\mathbb{P} \{ \text{SINR}_{DR} > \beta_d \}] \\ &= \exp(-a' d_{TR}^\alpha - b' d_{TR}^2 - c' d_{TR}^2), \end{aligned} \tag{13}$$

where $a' = \left(\frac{\beta_d \sigma^2}{P_D} \right)$, $b' = \frac{2\mu\pi^2\lambda_d}{\alpha \sin(2\pi/\alpha)} (\beta_d)^{\frac{2}{\alpha}}$, and $c' = \frac{2\mu\pi^2\lambda_c}{\alpha \sin(2\pi/\alpha)} \left(\frac{\beta_d P_k}{P_i} \right)^{\frac{2}{\alpha}}$.

4 Analysis of Sum Rate

Now we give out the ergodic rate of each part in the system. And then, the total rate of system is discussed. Finally, we propose an optimization constraint to enhance the system performance by fine-tuning one important index: scale factor for the D2D pairs and controlling the number of all users at the same time.

4.1 Ergodic Rate of D2D and Cellular Links

From (29) in [13], the ergodic rate of D2D links can then be derived relying on the SINR distribution of the typical D2D links:

$$\bar{R}^{D2D} = \int_0^\infty \log_2(1 + \beta_d) f_{\text{SINR}}(\beta_d) d\beta_d. \tag{14}$$

Similar to (14), the ergodic rate of cellular links can be given by

$$\bar{R}^{\text{CU}} = \frac{1}{\ln 2} \int_0^\infty \frac{P_{\text{cov}}^{\text{CU}}}{1 + \beta_c} d\beta_c. \quad (15)$$

4.2 Sum Rate of Hybrid Network

And according to the Shannon's theorem [14], the total rate of D2D links can be expressed as

$$\begin{aligned} R^{\text{D2D}} &= \mathbb{E} \left[\sum_{i=1}^K \log_2 (1 + \text{SINR}_{DR}) \right] \\ &= \lambda_d \pi R^2 \cdot \bar{R}^{\text{D2D}}. \end{aligned} \quad (16)$$

In the same approach, we get the total rate of cellular links as

$$R^{\text{CU}} = \lambda_c \pi R^2 \cdot \bar{R}^{\text{CU}}. \quad (17)$$

Based on (16) and (17), the total rate of Hybrid Network can be expressed as

$$R^{\text{SUM}} = \lambda_d \pi R^2 \bar{R}^{\text{D2D}} + \lambda_c \pi R^2 \bar{R}^{\text{CU}}. \quad (18)$$

4.3 Best Model of Allocation Schemes for Hybrid Network

Based on previous analysis, the system performance cannot keep getting better unlimitedly with the increase of the number of D2D links sharing the same resources with cellular links. In order to obtain the best ratio of D2D pairs compared to total users, we define a density factor λ for all the users sharing the same resource (i.e. there are totally N users, $N = \lambda \pi R^2$), and a scale factor η for all D2D users, indicating the proportion of D2D users out of total users (i.e. the density of D2D pairs can then be expressed as $\lambda_d = \eta \lambda$ and that of CUs can be expressed as $\lambda_c = (1 - \eta) \lambda$). In this case, the sum data rate can be rewritten as

$$R^{\text{SUM}} = \eta \lambda \pi R^2 \bar{R}^{\text{D2D}} + (1 - \eta) \lambda \pi R^2 \bar{R}^{\text{CU}}. \quad (19)$$

When the number of CUs or D2D pairs increases, the interference applied by these additional users will be increased consequently. Based on the previous analysis, the sum data rate of hybrid network is determined by the factors η and λ . Furthermore, in order to maximize the performance of hybrid system, the optimization functions can then be formulated as:

$$\begin{aligned} \max_{(\lambda, \eta)} \quad & \eta \lambda \pi R^2 \bar{R}^{\text{D2D}} + (1 - \eta) \lambda \pi R^2 \bar{R}^{\text{CU}} \\ \text{s.t.} \quad & P_{\text{cov}}^{\text{CU}} > 1 - \gamma_c \\ & P_{\text{cov}}^{\text{D2D}} > 1 - \gamma_d, \end{aligned} \quad (20)$$

where γ_c and γ_d denote the outage probability for cellular link and D2D link, respectively.

5 Numerical Analysis

In this section, we assume that there are totally N user (i.e. equals $\lambda\pi R^2$), which contains M (i.e. equals $(1 - \eta)\lambda\pi R^2$) CUs and K (i.e. equals $\eta\lambda\pi R^2$) D2D pairs. Now, we can conclude that the scale factor η is equal to K/N or λ_d/λ . The BS is located at the center of the cell, and the simulation parameters are set as follows. The cell radius is set to be 500 m, the power of cellular and that of DT are set to be 100 mw and 1 mw, respectively (i.e.: $P_c = 100$ mw, $P_d = 1$ mw). We assume the interval between DT and corresponding DR is established at 50 m. And the $\mu = 1$; $\alpha = 4$.

In Fig. 2, the interrelationship between the total data rate R^{TOTAL} and the density of DUs (i.e. λ_d) with varying CUs (i.e. $M = 2, 6, 10$) is depicted. In spite of the number of CUs/DUs, the interference will always be applied along with the increase of system capacity. For a given number of CUs, an best total rate does exist and shows with the increase of the density of DUs. Note that when the density of DUs is set to be a certain constant, the total data rate of the system will fall off as the raising of CUs. We may then draw a conclusion that the total rate can be improved by employing D2D mode, but it has a maximum limit. And when a single sub-channel is allocated only to an individual CU, the system performance is inclined to ameliorate. Therefore, by properly adjusting the density of DUs we can optimize the sum system rate effectively.

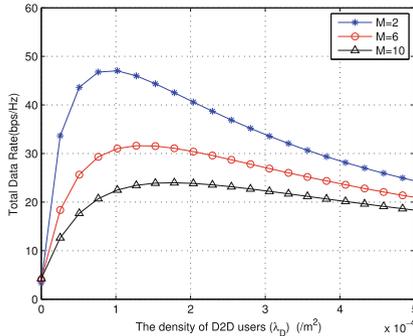


Fig. 2. Curves of total data rate of the system as functions of the density of DUs (λ_d) when the number of CUs (M) equals 2, 6 and 10, respectively.

In Fig. 3, the interrelationship between the total rate and the total number of users (i.e. N) is presented, provided that $\eta = 0.4, 0.8$. As is shown in the figure, for an established η , an best total rate does exist with the increase of users' density (a higher η implying a higher total rate). Furthermore, when the condition $\bar{R}^{D2D} > \bar{R}^{CU}$ is satisfied, the total rate will then be elevated. Otherwise, if $\bar{R}^{D2D} < \bar{R}^{CU}$, the employment of D2D mode may not benefit the system, but, on the contrary, will degrade the total rate. The point of intersection A denotes the best user density for total data rate, implying that more mobile

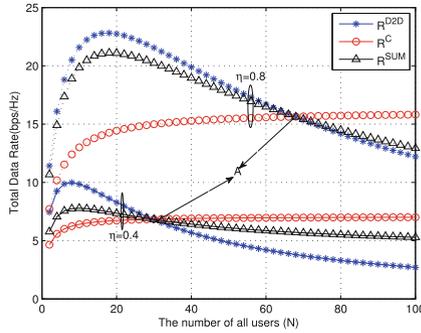


Fig. 3. Curves of total data rate as functions of the total number of users (N) when the scale factor η equals 0.4 and 0.8, respectively.

users can be managed and served for a higher η , the reason is that the interference applied by the D2D links is much lower than that applied by the CUs. Note that when $\eta = 0.8$, the best number of users is 1.5 times that of $\eta = 0.4$. Consequently, we can ameliorate the system performance in terms of total rate can be obtained by increasing η , and the total rate can be optimized by fitly fine-tuning η (i.e. the proportion of D2D users out of total users) in the presence of a certain given number of total users.

6 Conclusions

In this paper, we proposed a new single cell system model where all M CUs and N D2D pairs are co-sharing the same licensed uplink spectrum resources. We analyzed the system performance in terms of coverage probability and the total system rate (when there was no noise), and then gave out the approximate expressions for each link, respectively. Furthermore, the optimization strategy for hybrid network was proposed based on the total users and the scale factor of D2D pairs. Numerical results show that by introducing D2D communication underlying cellular communication and controlling the D2D proportion and the total number of users at the same time, the system performance can be greatly improved.

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