

# Rate-Splitting Non-orthogonal Multiple Access: Practical Design and Performance Optimization

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**Abstract.** In this paper we propose a novel rate-splitting non-orthogonal multiple access (RS-NOMA) scheme implemented in power-domain for practical scenarios in 5G mobile communication systems. The proposed scheme is exploited to solve the mismatch between Quality-of-Service (QoS) and channel conditions of users via rate-splitting instead of power reallocation as the conventional NOMA does. The RS-NOMA scheme contributes to both system spectrum efficiency and user flexibility on resource allocation. We investigate the signal-to-noise ratio (SNR) region division for the proposed scheme and then a splitting factor optimizing algorithm to enable the system achieving the maximum throughput. Simulation results show that the RS-NOMA scheme significantly improves the user flexibility with almost no loss to the system spectrum efficiency.

**Keywords:** Rate-splitting · Non-orthogonal multiple access · MCS selection · Spectrum efficiency · User flexibility

## 1 Introduction

With the worldwide spread of 4G wireless technology, mobile internet has been developed rapidly. Explosive growth of traffic volume, massive interconnected devices and new diverse services all challenge the future mobile communications technology [1]. The fifth generation (5G) mobile communication has attracted significant focus and research. Facing with the higher requirements in all aspects, the main demand for 5G systems are still the higher capacity and higher quality of user experience [2, 3].

Due to the urgent demand for high capacity and massive number of connected devices, non-orthogonal multiple access (NOMA) suggests one candidate technique for future radio access owing to its advantages to orthogonal multiple access (OMA) [4]. One of the NOMA schemes proposed in [5] is implemented by multiplexing users in power-domain at the transmitter side and separating signals at the receiver side based on successive interference cancellation (SIC) [5]. Once the power is allocated for each user, the transmit rate is correspondingly determined no matter whether it is appropriate. We would like to refer to the NOMA scheme applied to the uplink in [5] as the Fixed-NOMA scheme.

From a theoretical perspective, the Fixed-NOMA scheme can achieve the vertices of the Gaussian multiple-access channel (GMAC) capacity region characterized in [6]. Furthermore, a rate-splitting approach to the GMAC called rate-splitting multiple accessing (RSMA) was presented in [7] to achieve every point of the boundary of the capacity region using only single-user codes. The main idea behind the RSMA technique is to split some original inputs into more virtual inputs so as to split their rates as well. The RSMA technique enables the users achieving any rate partition while the sum rate of system remains maximum. Conceptually, the RSMA technique can efficiently improve the users flexibility on resource allocation depending on different Quality-of-Service (QoS). Some rate-splitting schemes extended from the RSMA technique in [7] have been discussed (see [8,9]) while the practical design is seldom investigated.

In this paper, we propose a RS-NOMA scheme for 5G mobile communication systems to meet the demand for ultra-high data traffic and ultra-high connection ability [1]. The proposed scheme offers an optimal splitting approach to solve the mismatch between QoS and channel conditions of users. The main idea is to make the users whose QoS are lower while the channel conditions are better to assist the ones whose situations are converse. We would like to refer to the former users as the “strong users” and the latter ones as the “weak users”. Compared with the Fixed-NOMA scheme, the proposed scheme enables the users adjusting their rates flexibly according to their QoS despite the power has already been assigned and constrained. However, different with the Fixed-NOMA scheme, the proposed scheme requires the cooperative communication between the transmitting nodes since the “strong users” demand for the information from the “weak users”. We consider the device-to-device (D2D) communication between the transmitting nodes which has been recognized as an underlay to cellular infrastructures [10]. Simulation results show that the proposed RS-NOMA scheme achieves the maximum throughput during the signal-to-noise ratio (SNR) region where the rates of the “strong users” are higher. It is also verified to be efficient to improve the user flexibility on resource allocation with almost no loss to the system capacity compared with the Fixed-NOMA scheme.

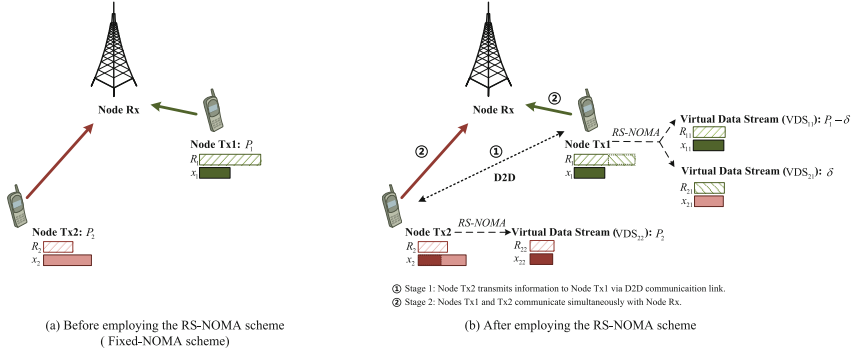
## 2 System Model

We consider the systems illustrated in Fig. 1, where nodes Tx1 and Tx2 denote the transmitting nodes and node Rx denotes the receiving node. We suppose the case of single transmit and receive antenna in this paper.

The initial system is shown in Fig. 1(a). When nodes Tx1 and Tx2 communicate simultaneously with node Rx by transmitting signal  $x_1$  and  $x_2$  respectively, the received signal at node Rx is given by

$$y = \sqrt{P_1}h_1x_1 + \sqrt{P_2}h_2x_2 + n, \quad (1)$$

where  $P_i$  is the transmit power allocated for node Tx*i* and  $h_i$  is the channel coefficient between node Tx*i* and Rx,  $i = 1, 2$ . We assume the additive white Gaussian noise (AWGN)  $n$  with variance  $\sigma_n^2$ .



**Fig. 1.** (a) Communication system with mismatch between QoS and channel conditions. (b) Communication system with RS-NOMA scheme which solving the mismatch between QoS and channel conditions

Assuming  $E[|x_i|^2] = 1$ ,  $i = 1, 2$ , where  $E[\cdot]$  stands for the statistical expectation, the received signal-to-interference and noise ratio (SINR) for  $x_1$  and  $x_2$  can be respectively written as

$$\text{SINR}_1 = \frac{P_1|h_1|^2}{P_2|h_2|^2 + \sigma_n^2}, \text{SINR}_2 = \frac{P_2|h_2|^2}{P_1|h_1|^2 + \sigma_n^2}. \tag{2}$$

In this case, we assume that  $\text{SINR}_1 > \text{SINR}_2$ , therefore  $x_1$  comes first in the decoding order when SIC process is implemented at node Rx [3]. Supposing successful decoding without error propagation, the rate of the data streams from nodes Tx1 and Tx2 can be given by

$$R_1 = \log \left( 1 + \frac{P_1|h_1|^2}{P_2|h_2|^2 + \sigma_n^2} \right), \tag{3}$$

$$R_2 = \log \left( 1 + \frac{P_2|h_2|^2}{\sigma_n^2} \right). \tag{4}$$

Assuming that  $\rho$  represents the SNR at node Rx, we obtain the relation

$$\sigma_n^2 = (P_1 + P_2) \times 10^{\frac{-\rho}{10}}. \tag{5}$$

In Fig. 1, we assume that the channel condition between nodes Tx1 and Rx is good as the short solid arrow indicates while the one between nodes Tx2 and Rx is poor as the long solid arrow represents. Accordingly, the achievable rate of the data stream from node Tx1 is higher than that from Tx2 as the sizes of shadow blocks represent. In the contrary, the QoS of node Tx1 is supposed to be lower than Tx2 as the sizes of filled blocks represent.

Apparently, in Fig. 1(a), the shadow block is longer than the filled one at node Tx1, which means the achievable rate  $R_1$  is surplus for transmitting signal

$x_1$ . However, node Tx2 is in an opposite situation. The shadow block is too short to load the filled one which indicates the achievable rate  $R_2$  is insufficient for transmitting signal  $x_2$ . A mismatch between QoS and channel conditions appears in Fig. 1(a). To compete against the contradiction, we seek for solution where Tx1 helps transmitting Tx2's information using its extra rate, and the RS-NOMA scheme is proposed as shown in Fig. 1(b).

We assume that nodes Tx1 and Tx2 can communicate directly with D2D communication link in Fig. 1(b), and the information from node Tx2 has already been transmitted to node Tx1 in stage 1. In this paper, we emphasize on stage 2 where the proposed scheme is actually adopted.

### 3 Rate-Splitting Non-orthogonal Multiple Access

To solve the mismatch between QoS and channel conditions, we propose a RS-NOMA scheme for practical system, due to its benefit in adjusting the user rates flexibly. The SNR region division for the proposed scheme is investigated and the algorithm to select the optimal splitting factor is designed.

#### 3.1 Introduction of RS-NOMA Scheme

The main idea of the RS-NOMA scheme is to split original data streams into virtual data streams as depicted in Fig. 2. At the transmitter, original data streams from  $L$  real users are split into at most  $2L - 1$  virtual data streams with splitting rates  $r_i, i = 1, 2, \dots, 2L - 1$ . Transmit power  $p_i, i = 1, 2, \dots, 2L - 1$  with splitting factors are assigned to virtual data streams to adjust the splitting rates. At the receiver, each virtual data stream is regarded as a single user and decoded in an SIC manner.

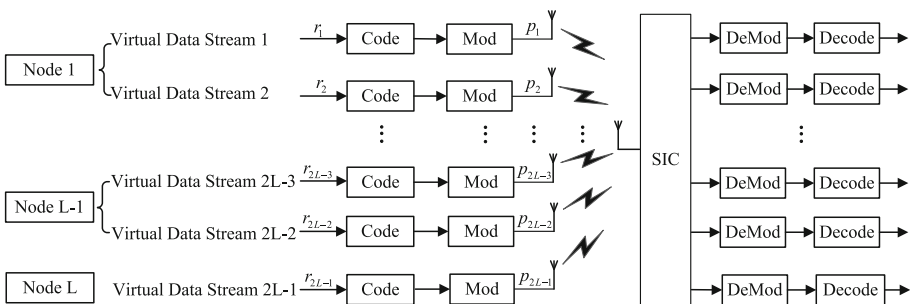


Fig. 2. Block diagram of the RS-NOMA scheme.

Taking Fig. 1(b) as an example, we adopt the RS-NOMA scheme to split the rate of original data stream from node Tx1 in stage 2. The long shadow block

at node Tx1 is divided into two parts, so one of them can be assigned to carry a short filled block from node Tx2 to help node Tx2 transmitting information.

We define  $\delta$  as the splitting factor in power-domain. The received signal at node Rx using the RS-NOMA scheme in system can be written as follows:

$$y = \sqrt{P_1 - \delta} h_1 x_{11} + \sqrt{\delta} h_1 x_{21} + \sqrt{P_2} h_2 x_{22} + n, \quad (6)$$

where  $x_{11}$  is the original signal  $x_1$  of node Tx1,  $x_{21}$  and  $x_{22}$  are split from the original signal  $x_2$  of node Tx2. Apparently, the whole system has transformed from two original data streams of real nodes to three virtual data streams as VDS<sub>11</sub>, VDS<sub>21</sub> and VDS<sub>22</sub>. As shown in Fig. 1(b), VDS<sub>11</sub> and VDS<sub>21</sub> currently carry  $x_{11}$  and  $x_{21}$  respectively at node Tx1 while node Tx2 only transmits  $x_{22}$  which is part of the original signal  $x_2$  via VDS<sub>22</sub>.

In addition, we would like to point out that the modulation and coding scheme (MCS) of the data streams are adaptively chosen via the channel state information as implementation in practical system [11].

### 3.2 SNR Region Division for RS-NOMA Scheme

As introduced earlier, the main idea of the RS-NOMA scheme is to enable the “strong users” assisting the “weak users” by utilizing the extra rates. Therefore, according to the user rates, we can divide the SNR region into general two parts. During the SNR region where the rates of the “strong users” are higher than the “weak users”, we employ the RS-NOMA scheme with the optimal splitting factor and we would like to refer to it as the RS-SNR region. While during the SNR region where the situations are converse, we set the splitting factor to be zero which actually turns into the Fixed-NOMA scheme. The SNR region division offers a reference to use the RS-NOMA scheme flexibly.

In Fig. 1(b), we consider the SNR region where the rates of the original data streams from nodes Tx1 and Tx2 satisfy  $R_1 \geq R_2$ . Substituting (5) back into (3) and (4), the RS-SNR region at receiving node can be written as

$$\rho \leq 10 \log \frac{(P_1 |h_1|^2 - P_2 |h_2|^2) (P_1 + P_2)}{P_2^2 |h_2|^4}. \quad (7)$$

In different scenarios, the RS-SNR region varies and can be calculated by (7) once the RS-NOMA scheme is employed.

### 3.3 Splitting Factor Optimizing Algorithm

The RS-NOMA scheme is capable to achieve every point of the boundary of the GMAC capacity region by choosing different splitting factors similar to the RSMA technique in [7]. During the RS-SNR region obtained by (7), we develop a splitting factor optimizing algorithm consisting of a series of criteria. The proposed algorithm selects the optimal splitting factor for current system to achieve the maximum throughput.

**Criterion I: Appropriate Detecting Order**

According to the RSMA technique proposed in [7], all virtual data streams are detected based on SIC at receiver, and the detecting order is decided by SINR. Besides, the detecting order of the un-split users must be placed among the split ones to adjust the user rates by altering the splitting factors (see [7]). The SINR of the virtual data streams in Fig. 1(b) are given by

$$\text{SINR}_{11} = \frac{(P_1 - \delta) |h_1|^2}{P_2 |h_2|^2 + \delta |h_1|^2 + \sigma_n^2}, \tag{8}$$

$$\text{SINR}_{21} = \frac{\delta |h_1|^2}{P_2 |h_2|^2 + (P_1 - \delta) |h_1|^2 + \sigma_n^2}, \tag{9}$$

$$\text{SINR}_{22} = \frac{P_2 |h_2|^2}{(P_1 - \delta) |h_1|^2 + \delta |h_1|^2 + \sigma_n^2}. \tag{10}$$

When  $\delta < P_1/2$ , we can obviously obtain from (8) and (9) that  $\text{SINR}_{21} < \text{SINR}_{11}$ . Then the SINR of the three virtual data streams which determines the detecting order should satisfy the follows:

$$\text{SINR}_{21} < \text{SINR}_{22} < \text{SINR}_{11}. \tag{11}$$

Submitting (8)–(10) to (11), we can obtain constraint of splitting factors as

$$\delta < \frac{P_2^2 |h_2|^4 + P_1 P_2 |h_1|^2 |h_2|^2 + P_2 |h_2|^2 \sigma_n^2}{P_1 |h_1|^4 + |h_1|^2 \sigma_n^2 + P_2 |h_1|^2 |h_2|^2} \tag{12}$$

when  $P_1 |h_1|^2 > 2P_2 |h_2|^2$ , or

$$\delta < \frac{P_1^2 |h_1|^4 - P_2^2 |h_2|^4 + (P_1 |h_1|^2 - P_2 |h_2|^2) \sigma_n^2}{P_1 |h_1|^4 + |h_1|^2 \sigma_n^2 + P_2 |h_1|^2 |h_2|^2} \tag{13}$$

when  $P_1 |h_1|^2 < 2P_2 |h_2|^2$ .

Assuming decoding successfully without error propagation, the rates of the three virtual data streams are given by

$$R_{21} = \log \left( 1 + \frac{\delta |h_1|^2}{\sigma_n^2} \right), \tag{14}$$

$$R_{22} = \log \left( 1 + \frac{P_2 |h_2|^2}{\delta |h_1|^2 + \sigma_n^2} \right), \tag{15}$$

$$R_{11} = \log \left( 1 + \frac{(P_1 - \delta) |h_1|^2}{\delta |h_1|^2 + P_2 |h_2|^2 + \sigma_n^2} \right), \tag{16}$$

which can be altered by varying  $\delta$ . When  $\delta > P_1/2$ , the constraint of splitting factors can be calculated in the same way as described above.

### Criterion II: Appropriate SINR for Successful Decoding

Using the RS-NOMA scheme, we would like to ensure each virtual data stream being decoded successfully. We consider the stream whose detecting order is the last to be on reliable communication [12].

When  $\delta < P_1/2$ ,  $VDS_{21}$  is the last one to be detected by comparing (8)–(10). To ensure  $VDS_{21}$  on reliable communication, the splitting factors are subject to follows:

$$\begin{aligned} \frac{E_b}{\sigma_n^2} &> \frac{2^{R_{AMC}/W} - 1}{R_{AMC}/W} \Rightarrow \frac{\delta}{R_{AMC} \times \sigma_n^2} > \frac{2^{R_{AMC}} - 1}{R_{AMC}/W} \\ &\Rightarrow \delta > \sigma_n^2 \times (2^{R_{AMC}} - 1) \times W, \end{aligned} \quad (17)$$

where  $E_b$  is the energy per bit,  $R_{AMC}$  is the practical rate depending on adaptive MCS level, and  $W$  is the system bandwidth. Equation (17) gives another constraint of splitting factors.

When  $\delta > P_1/2$ ,  $VDS_{11}$  becomes the last one to be detected and the constraint of splitting factors can be calculated as follows:

$$\delta < P_1 - \sigma_n^2 \times (2^{R_{AMC}} - 1) \times W. \quad (18)$$

### Criterion III: Appropriate MCS Level

In practical system, the MCS level of signals are adaptive to the channel quality information (CQI). Therefore, the criteria for optimal splitting factor should be jointly designed with the MCS level.

Compared (15) with (4), we can obtain that the rate of un-split node Tx2 is reduced which leads to a drop of MCS level. In addition, we would like to point out that the rate of un-split user in the RS-NOMA scheme always decreases as well as the MCS level. To avoid that the rate of un-split user is too low, we propose a constraint of modulation scheme for the un-split user. Supposing that  $m = \log_2 M$  denotes the modulation level, where  $M$  is the modulation order, the modulation scheme of the un-split node Tx2 should satisfy:

$$|m_{Tx2} - m_{VDS_{22}}| \leq 2, \quad (19)$$

where  $m_{Tx2}$  and  $m_{VDS_{22}}$  are adaptively determined by CQI.

The above criteria are organized by their importance. When all splitting factors have been traversed according to the criteria, we can obtain a set of splitting factors  $\mathcal{S} = \{\delta_1, \delta_2, \dots, \delta_k\}$ . The optimal splitting factor could be selected based on the practical QoS. We suppose that the system in Fig. 1(b) demands for higher transmission rate for  $VDS_{21}$ , which means that  $VDS_{21}$  is expected to carry more information for node Tx2. Taking (14), (17) and (18) into account, the optimal splitting factor  $\delta_{opt}$  should be finally decided by being maximized when ensuring all virtual data streams on reliable communication. As a summary, the proposed algorithm to select the optimal splitting factor during the RS-SNR region is described in Algorithm 1.

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**Algorithm 1.** Splitting factor optimizing algorithm

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**Input:** Power assigned to each transmitting node  $P_i$ , each user's channel coefficient  $h_i$ , noise variance  $\sigma_n^2$  and step of splitting factor  $\Delta\delta$ ;  
**Output:** Optimal splitting factor  $\delta_{opt}$ ;

- 1: Initialize a set  $\mathcal{S} = \emptyset$ ,  $\delta = 0$ ;
- 2: Calculate the RS-SNR region by (8) for the RS-NOMA scheme;
- 3: **for**  $\delta = \delta + \Delta\delta \rightarrow P_i(\text{split node})$  **do**
- 4:   **if**  $\delta$  is in the intervals calculated by criterion I **then**
- 5:     **if**  $\delta$  satisfies the constraint in criterion II **then**
- 6:       **if** the MCS of the virtual data streams determined by  $\delta$  satisfies criterion III **then**
- 7:          Go to step 17;
- 8:       **else**
- 9:          Go to step 3;
- 10:      **end if**
- 11:    **else**
- 12:      Go to step 3;
- 13:    **end if**
- 14:  **else**
- 15:    Go to step 3;
- 16:  **end if**
- 17:  Add current  $\delta$  to  $\mathcal{S}$ ;
- 18: **end for**
- 19: Select the optimal splitting factor  $\delta_{opt}$  by practical QoS from  $\mathcal{S} = \{\delta_1, \delta_2, \dots, \delta_k\}$ .

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## 4 Performance Analyses

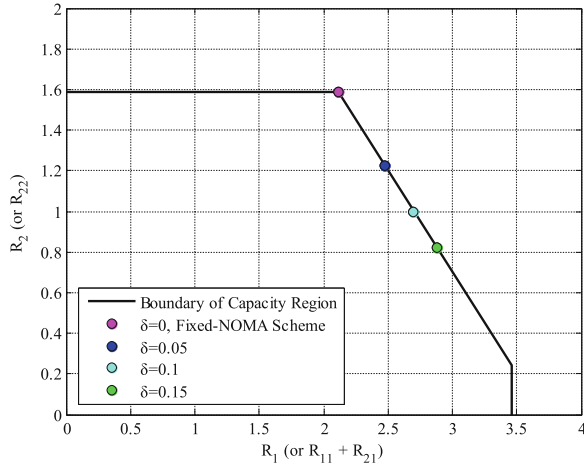
In this section, we study the performance of the proposed RS-NOMA scheme through numerical and simulation results. We consider the communication system as illustrated in Fig. 1(b) and we only analyze stage 2. For simplicity, we normalize the transmit power as  $P_1 = P_2 = 1$ , and we assume that  $|h_1| = \sqrt{2}$ ,  $|h_2| = \sqrt{0.4}$ .

### 4.1 Theoretical Results

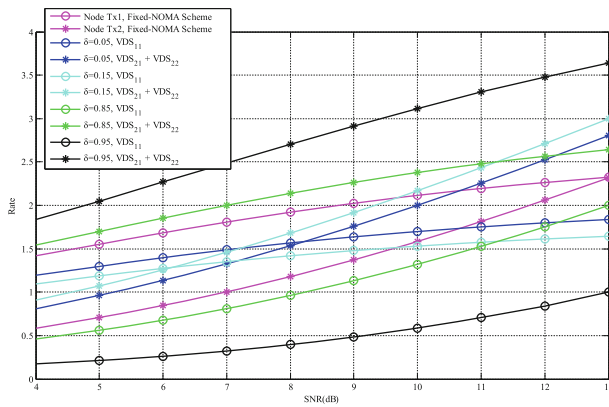
Figure 3 shows the theoretical system capacity region. The legend  $\delta = 0$  denotes the achievable sum rate of the Fixed-NOMA scheme based on (3) and (4) which is a vertex of the capacity region. Only by successful SIC process at node Rx can the system in Fig. 1(a) achieve it. The RS-NOMA scheme achieves different points of the boundary of the capacity region when splitting factors vary. The achievable sum rate can be calculated by (14)–(16). The figure demonstrates that the RS-NOMA scheme enables the system flexibly achieving the maximum sum rate with alternative splitting factors from a theoretical perspective.

Figure 4 depicts the theoretical separate rate of original data stream (Fixed-NOMA scheme) and virtual data stream with various splitting factors. To compare the performance between the Fixed-NOMA scheme and the RS-NOMA





**Fig. 3.** Theoretical achievable system capacity of the Fixed-NOMA scheme and the RS-NOMA scheme with various  $\delta$ .



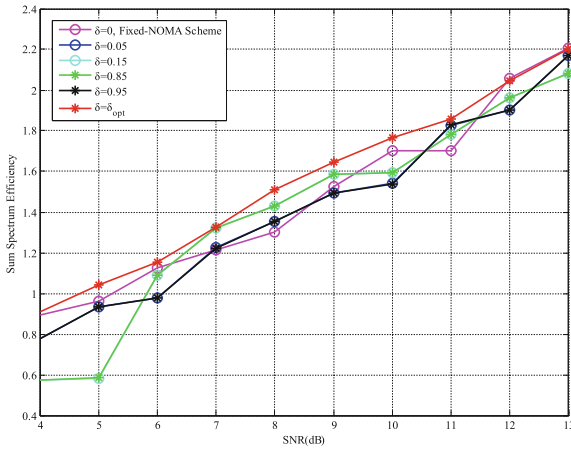
**Fig. 4.** Theoretical separate rate of original data stream (Fixed-NOMA scheme) and virtual data stream with various  $\delta$ .

scheme, we equally regard the  $VDS_{11}$  as one stream from a node and combine the  $VDS_{21}$  and  $VDS_{22}$  as one stream from another node. From Fig. 4, we observe that the RS-SNR region for the RS-NOMA scheme is  $\rho \leq 13$  dB which proves to be consistent with the results calculated by (7) using current parameters. Different splitting factors lead to different rate of each VDS and the proposed RS-NOMA scheme is exploited to select the optimal splitting factor from all these available factors.

## 4.2 Simulation Results

The above analyses are based on Gaussian signal model. However, in practical implementation, the modulation and coding scheme will bring some deviations to theoretical results. We use the schemes in 3GPP LTE standard [13] as a reference.

Figure 5 exhibits the system sum spectrum efficiency comparison between the Fixed-NOMA scheme and the RS-NOMA scheme with different splitting factors including the optimal one during the RS-SNR region. The simulation results illustrate that the proposed algorithm selects the optimal splitting factor to achieve the maximum throughput. In addition, it verifies that the proposed RS-NOMA scheme contributes to current system on both spectrum efficiency and user flexibility. We have to point out that the adaptive selection of MCS level leads to the jitters of curves.



**Fig. 5.** System sum spectrum efficiency comparison between the Fixed-NOMA scheme and the RS-NOMA scheme with various  $\delta$  in simulation.

## 5 Conclusion

We propose a practical design and performance optimization method for RS-NOMA scheme. The proposed scheme contributes to a better match between QoS and channel conditions of users by rate-splitting. We investigated the SNR region division for the RS-NOMA scheme and the splitting factor optimizing algorithm. Simulation results show that by appropriately exploiting the optimal splitting factor, the proposed scheme significantly improves the user flexibility with almost no loss to the system spectrum efficiency.

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