# Utility-Based Resource Allocation in OFDMA Relay Systems with Half-Duplex Transmission

Huanglong Teng<sup>1</sup>, Binjie Hu<sup>2</sup>, Hongming Yu<sup>1</sup>, Miao Cui<sup>1,3(⊠)</sup>, and Guangchi Zhang<sup>3</sup>

 <sup>1</sup> China Electronics Technology Group Corporation No. 7 Research Institute, Guangzhou 510310, China single450@163.com
 <sup>2</sup> School of Electronic and Information Engineering, South China University of Technology, Guangzhou 510641, China
 <sup>3</sup> School of Information Engineering, Guangdong University of Technology, Guangzhou 510006, China

**Abstract.** This paper considers resource allocation in an Orthogonal Frequency Division Multiple Access (OFDMA) relay system, which uses either direct transmission or nonregenerative relay transmission strategies in each subchannel. An optimization problem is developed to handle joint dynamic subchannel assignment (DSA), adaptive power allocation (APA), transmission strategy selection and relay selection in the downlink of OFDMA relay system exploiting half-duplex transmission. We aim to obtain the fair usage of the relays with the assumption that one relay's maximum subchannels and the relay power in each subchannel are fixed. A suboptimal greedy algorithm is proposed to optimize all users' overall sum utility, where resources are allocated to the user with the greatest utility increment potential one at a time. Simulation results illustrate that the proposed algorithm significantly outperforms the fixed resource allocation schemes.

Keywords: Resource allocation  $\cdot$  Relay transmission  $\cdot$  OFDMA  $\cdot$  Utility function  $\cdot$  Half-duplex

# 1 Introduction

Wireless relay attracts great interest in wireless communication research [1]. The two main types of relay transmission strategies are the regenerative and the nonregenerative [2]. Orthogonal Frequency Division Multiplexing (OFDM) is robust against frequency selective fading and Orthogonal Frequency Division Multiple Access (OFDMA) is one of the promising multiple-access schemes for future broadband wireless networks, e.g., IEEE 802.16.

The OFDMA systems that employ relays have attracted notable attention [3, 4]. The systems investigated in this paper are used in cellular networks, and are called OFDMA relay systems. Efficient resource allocation schemes can greatly improve system performance, and it is natural to ask the questions: which user transmits at which subchannel, which relay forwards information at which subchannel, how much

power is allocated to each subchannel etc. By means of optimizing the utility function, the balance between efficient and fair resource allocation can be achieved [4, 5].

Using nonregenerative relay transmission, Han et al. [6] proposed an algorithm to handle dynamic subchannel assignment (DSA), adaptive power allocation (APA) and relay selection in an OFDM Time-Division Multiple Access relay system to reduce transmission power. In [3], Li and Liu investigated joint DSA and relay selection with fixed power allocation and transmission strategy in the uplink of an OFDMA relay system. Ng and Yu [4] considered joint DSA, APA, transmission strategy selection and relay selection in an OFDMA relay system with full-duplex transmission. Although the digital full-duplex operation is implemented in wireline systems [7], it cannot be used in wireless systems because the transmitted signal drowns out the received signal [1].

This paper investigates joint DSA, APA, transmission strategy selection and relay selection in the downlink of an OFDMA nonregenerative relay system with half-duplex transmission. A greedy algorithm is proposed to maximize the overall sum utility on the condition that the maximum number of subchannels one relay can assist and the relay power in each subchannel are fixed. These conditions are necessary for ensuring fair usage of the relays. Note that our proposed algorithm is different from the dual decomposition method in [4].

## 2 System Model and Problem Formulation

#### 2.1 System Model

The downlink of the broadband OFDMA relay system consists of one base station and *K* half-duplex users. Denote the user set as  $\mathcal{K} = \{1, 2, ..., K\}$ . In addition to receiving information of their own, the users in a subset of  $\mathcal{K}$  serve as relays. Denote  $\mathcal{V}$  as the relay set with  $\mathcal{V} \subset \mathcal{K}$ ,  $|\mathcal{V}| = L$  ( $1 \le L \le K/2$ ), and  $\overline{\mathcal{V}} = \mathcal{K} - \mathcal{V}$  as the non-relay user set<sup>1</sup>. The slow fading wireless channel is divided into *N* subchannels, and denote  $\mathcal{N} = \{1, 2, ..., N\}$  as the set of all subchannels. The OFDM frames are synchronized [3, 4, 6]. The base station allocates resources according to full channel state information.

The downlink transmission slot includes two subslots of equal length. In the first subslot, the base station transmits in all subchannels, and all relay users listen; in the second subslot, the relay users forward the amplified signals in relay transmission subchannels, the non-relay users listen. For simplicity, assume that a relay user transmits the information at the same subchannel where it receives information. It is assumed that one relay user assists one subchannel at most [3, 4].

For  $i \in \mathcal{K}$ , denote  $\mathcal{A}_i$  as the subchannel set where user *i* selects direct transmission, and  $\mathcal{B}_{ii}$  as the subchannel set where user *i* selects relay transmission and  $\mathcal{S}_i = \mathcal{A}_i \cup \mathcal{B}_i$ as subchannel set of user *i*.  $A_i \cap B_i = \emptyset$ . As the relay users transmit in the second subslot, they cannot receive at the same time due to the half-duplex constraint. Therefore they only select direct transmission in their subchannels, e.g.,  $\mathcal{B}_j = \emptyset$  for  $j \in \mathcal{V}$ , while non-relay users may select both direct transmission and relay transmission in their different subchannels.

<sup>&</sup>lt;sup>1</sup> The procedure of determining  $\mathcal{V}$  is out of the scope of this paper.

With direct transmission, user *i* receives from the base station in the first subslot in the subchannels of  $A_i$ . The direct transmission rate in subchannel *n* is given by

$$R_{i,n}^{direct} = \frac{W}{2} \log_2 \left( 1 + \frac{p_{0,n} |H_{0i,n}|^2}{N_0 W} \right), \quad n \in \mathcal{A}_i,$$
(1)

where *W* is the subchannel bandwidth,  $p_{i,n}$  is the power that user *i* allocates to subchannel *n*,  $H_{ji,n}$  is the channel gain of subchannel *n* from user *j* to user *i*, and  $N_0$  is the power spectral density of the additive white Gaussian noise. User 0 denotes the base station.

With relay transmission, user *i* receives the amplified signal from its relays in the second subslot in the subchannels of  $\mathcal{B}_i$ . The relay transmission rate in subchannel *n* is given by [2]

$$R_{i,n}^{relay} = \frac{W}{2} \log_2 \left( 1 + \frac{p_{0,n} p_{j,n} |H_{0j,n}|^2 |H_{ji,n}|^2}{N_0 W(p_{0,n} |H_{0j,n}|^2 + p_{j,n} |H_{ji,n}|^2 + N_0 W)} \right), \quad n \in \mathcal{B}_i, \quad (2)$$

where user j is the relay in the subchannel. The total rate of user i is given by

$$R_i = \sum_{n \in A_i} R_{i,n}^{direct} + \sum_{n \in B_i} R_{i,n}^{relay}.$$
(3)

#### 2.2 Problem Formulation

To optimize all users' sum utility, the resource allocation scheme determines the subchannel set of each user, e.g.,  $S_i$  for  $i \in \mathcal{K}$ , the subchannel set that each relay assists, denoted as  $Q_i$  for  $i \in \mathcal{V}$ , and power allocated at each subchannel. Note that  $\mathcal{B}_i = S_i \cap \left(\bigcup_{j \in V} Q_j\right)$ . The problem is formulated as

$$\max_{p_{j,n},S_i,Q_i} \qquad \sum_{i=1}^K U_i(R_i) \tag{4}$$

subject to 
$$\sum_{n=1}^{N} p_{j,n} \le P_j^{tot}, \, \forall j \in \{0\} \cup \mathcal{V}$$
(5)

$$p_{j,n} \ge 0, \, \forall j \in \{0\} \cup \mathcal{V}, \,\, \forall n \in \mathcal{N}$$
(6)

$$\bigcup_{i\in\mathcal{K}}\mathcal{S}_i\subseteq\mathcal{N}\tag{7}$$

$$\bigcup_{j\in\mathcal{V}}\mathcal{Q}_{j}\subseteq\mathcal{N}-\bigcup_{i'\in\mathcal{V}}S_{i'}$$
(8)

$$\mathcal{S}_i \cap \mathcal{S}_{i'} = \emptyset, \,\forall i, \, i' \in \mathcal{K} \tag{9}$$

$$\mathcal{Q}_{j} \cap \mathcal{Q}_{j'} \varnothing = \forall j, j' \in \mathcal{V} \tag{10}$$

In Eq. (4),  $y = U_i(R_i)$  is the utility function of user *i*, which is concave increasing in this paper [4, 5]. In Eq. (5),  $P_j^{tot}$  is the total transmission power. Equation (8) guarantees that the relay users cannot select relay transmission; Eqs. (9) and (10) guarantee that each subchannel is assigned to one user and assisted by at most one relay [3, 4].

## **3** Greedy Algorithm for Resource Allocation

The above problem is combinatorial, and the optimal solution is hard to find. To prevent that some relays are used in excess [3], our algorithm guarantees that each relay assists at most  $\Omega$  subchannels, and that the relay power at each relay transmission subchannel is  $p_{j,n} = p_r, j \in \mathcal{V}$ , so the maximum transmission power of each relay user is  $P_j^{tot} = \Omega p_r, j \in \mathcal{V}$ .

The greedy algorithm allocates resources to the user with the greatest utility increment potential under the available resources one at a time. Similar to [8], the user rate is updated with water-filling power allocation given in the Appendix. The algorithm continues until all the resources have been allocated. The proposed algorithm is given below, where  $P_i$  is the total power allocated to user *i* from the base station, e.g.,  $P_i = \sum_{n \in S_i} p_{0,n}$ ,  $[R_i, \{p_{0,n}\}_i] = WF(P_i, A_i, B_i)$  denotes water-filling power allocation for user *i* based on subchannels and transmission strategy of it, and  $\{p_{0,n}\}_i$  denotes the allocated power by the base station to the subchannels of user *i*.

Greedy algorithm for resource allocation:

- 1. Initialization.  $C = \mathcal{N}, \mathcal{S}_i = \emptyset, \mathcal{Q}_i = \emptyset, \mathcal{A}_i = \emptyset, \mathcal{B}_i = \emptyset, P_i = 0$  for  $i \in \mathcal{K}$ .
- 2. Calculate the maximum possible rate  $R_i^{pos}$  for each user.
  - (a) Find subchannel *n* with the greatest channel gain,  $n \in C$ ;

$$[R_{i,1}^{pos}, \{p_{0,n}\}_i] = WF(P_i + P_0^{tot}/N, \mathcal{A}_i \cup \{n\}, \mathcal{B}_i).$$

(b) If  $i \in \mathcal{V}$ ,  $R_i^{pos} = R_{i,1}^{pos}$ . Else, find (n, j) where the maximum rate is achieved with relay *j* at subchannel *n*,  $n \in \mathcal{C}$  and  $j \in \{k | k \in \mathcal{V}, |\mathcal{Q}_k| < \Omega\}$ ;

$$[R_{i,2}^{pos}, \{p_{0,n}\}_i] = WF(P_i + P_0^{tot}/N, \mathcal{A}_i, \mathcal{B}_i \cup \{n\}); R_i^{pos} = \max(R_{i,1}^{pos}, R_{i,2}^{pos}).$$

- 3. Resource allocation on the user with maximum utility increment potential.
  - (a) Find  $i^*$  that  $U_{i^*}(R_{i^*}^{pos}) U_{i^*}(R_{i^*}) \ge U_k(R_k^{pos}) U_k(R_k), \forall i^*, k \in \mathcal{K};$

$$P_{i^*} = P_{i^*} + P_0^{tot} / N; R_{i^*} = R_{i^*}^{pos}$$

(b) Subchannel  $n^*$  is the best,  $S_{i^*} = S_{i^*} \cup \{n^*\}$ ,  $C = C - \{n^*\}$ . If  $i^* \notin \mathcal{V}$ ,  $R_{i^*,1}^{pos} < R_{i^*,2}^{pos}$  and the user  $j^*$  is the best relay,  $\mathcal{Q}_{j^*} = \mathcal{Q}_{j^*} \cup \{n^*\}$ ,

$$\mathcal{B}_{i^*} = \mathcal{B}_{i^*} \cup \{n^*\}; \text{ Else } \mathcal{A}_{i^*} = \mathcal{A}_{i^*} \cup \{n^*\}.$$

If C is not empty, go to step 2; else water-filling power allocation for each user and stop.

### 4 Simulation Results

We have conducted a simulation to evaluate the proposed algorithm. In the simulation, our algorithm was compared with two other resource allocation schemes: the fixed resource allocation (FRA) scheme and the fixed subchannel assignment with adaptive power allocation (FSA-APA) scheme. In the FRA scheme, all resource allocations are fixed. The FSA-APA scheme is the same with the FRA except adaptive water-filling power allocation.

The OFDM relay system has N = 128 subchannels with 5 MHz bandwidth. Each channel has a six-tap multipath, where each tap component is simulated by Clarke's model [9]. The path loss exponent is 3, and the energy of the *t*th tap is

$$E[|h_t|^2] = (\frac{d_0}{d})^3 e^{-(t-1)}, \ t = 1, 2, \dots, 6,$$
(11)

where *d* is the distance with  $d_0 = 10$  m as the reference. The utility function of each user is  $U_i(R_i) = \ln R_i$ , so the resource allocations are proportionally fair [10].  $P_0^{tot} = 1$  W,  $P_i^{tot} = 0.25$  W,  $\Omega = 32$ ,  $p_{j,n} = p_r = 7.8125$  mW ( $j \in V$ ) and  $N_0 = -80$  dBW/Hz.

In the two-user case, all users locate in the same line. User 1, the relay, locates between the base station and user 2. The distance from base station to the user 1 is fixed at  $D_{01} = 100$  m. Figure 1 shows the average rate of each user versus the distance from base station to user 2  $D_{02}$ . The rates of both users with our algorithm are much higher than the other two schemes. Figure 2 shows that our algorithm achieves greater sum utility.



Fig. 1. Average rate of each user in the two-user case.



Fig. 2. Average sum utility in the two-user case.

In the four-user case, users 0 to 4 are located at (0, 0), (60, 80), (48, 36), (102, 136) and (160, 120), and the unit of measurement is meter. User 1 and 2 are relays. Figure 3 shows that with our algorithm, although user 2 suffers a slight rate loss, the other three users have significant rate gain. The average sum utility of our algorithm is 28.521, greater than the ones of the FSA-APA and FRA which are 26.857 and 26.698 respectively.



Fig. 3. Average rate of each user in the four-user case.

# 5 Conclusions

In this paper, by fixing the maximum number of subchannels that a relay assists and relay power at each subchannel, a utility-based resource allocation algorithm is proposed for joint DSA, APA, transmission strategy selection and relay selection in an OFDMA relay system. Simulation results show that the greedy and suboptimal algorithm significantly outperforms the fixed resource allocation schemes.

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# Appendix

The water-filling power allocation problem for user i is given by

$$\max_{\substack{p_{0,n} \\ \text{s. t.} \sum_{n \in S_i} p_{0,n}} R_i = P_i, p_{0,n} \ge 0.$$
 (A.1)

Using the Karush-Kuhn-Tucker (KKT) conditions [11], after some algebraic manipulations, we get

$$p_{0,n} = \left(\lambda - \frac{N_0 W}{|H_{0i,n}|^2}\right)^+, n \in A_i,$$
 (A.2)

$$p_{0,n} = \frac{N_0 W}{|H_{0r,n}|^2} \left[ \frac{p_{r,n} |H_{ri,0}|^2}{2N_0 W} \left( \sqrt{1 + \frac{4|H_{0r,n}|^2}{\lambda p_{r,n} |H_{ri,n}|^2}} - 1 \right) - 1 \right]^+, n \in B_i, \quad (A.3)$$

where *r* denotes the relay in each subchannel, and constant  $\lambda$  is chosen to satisfy  $\sum_{n \in S_i} p_{0,n} = P_i$ .

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