Interference Alignment in Cognitive Relay Networks Under CSI Mismatch

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Abstract. Interference alignment (IA) is an effective method that can eliminate interferences in wireless networks, and has been applied to spectrum sharing in cognitive radio (CR) networks recently. However, the availability of perfect network channel state information (CSI) is necessary for most existing IA schemes, which is not practical in general due to the realistic communication scenarios and deployment challenges. In this paper, we apply IA to cognitive relay networks under CSI mismatch where the variance of the CSI measurement error depends on signal-to-noise ratio (SNR). An adaptive Max-SINR IA algorithm has been introduced to improve the performance of the secondary network by using the knowledge of CSI error variance. Finally, we analyze the performance of the secondary network in terms of the end-to-end equivalent transmission rate and outage probability. Simulation results indicate that our proposed adaptive Max-SINR IA scheme can greatly improve the performance of the secondary network.

Keywords: Cognitive relay networks · Interference alignment · CSI mismatch · Outage probability

1 Introduction

Recently, due to the ability of alleviating the spectrum shortage problem of wireless communications, spectrum sharing techniques have received substantial interests [1,2]. In spectrum sharing networks, the secondary users (SUs) are allowed to access the licensed spectrum as long as the interference power generated by the secondary communications does not exceed the predefined threshold at the primary user (PU). In parallel, cooperative relay has been demonstrated as an effective way to combat channel fading and improve systems transmission performance. In [3,4], the performance of an underly cognitive relay networks has been studied. Using N-th best relay selection, the outage probability was

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further analyzed for spectrum-sharing relaying networks in [5]. However, to the best of the authors' knowledge, the SU receivers have ignored the interference that caused by primary transmission in above works. In most of practical scenario, both the interference between PU receivers and SU transmitters and the interference between SU receivers and PU transmitters should be considered simultaneously.

Recently, interference alignment (IA) has been proposed as an effective method not only to eliminate the interference to PU receivers but also the interference to SU receivers [6]. However, the application of IA should satisfy some conditions. First of all, the application of IA needs global channel state information (CSI) and it is difficult to maintain. To deal with this issue, the authors in [7] proposed the distributed iterative algorithm to achieve IA, in which the proposed algorithm only need the local CSI and can simplify the implementation of IA. Unfortunately, it will affect the achievable throughput and the total full degrees of freedom (DoF) of the network that only can obtain partial CSI due to the realistic communication scenarios. Thus, IA under CSI mismatch has received great interests. However, due to the complex nature of the issue, different works deal with various CSI uncertainty aspects, such as correlated channels [8], analog channel state feedback [9], and constant variance of the CSI error [10]. The authors of [11] investigate the performance of IA techniques in multiple-input multiple-output (MIMO) interference channels in the presence of CSI mismatch, in which the CSI mismatch model is versatile and treats the channel error variance either as a function of SNR or as independent of it. Moreover, to the best of authors' knowledge, no existing literatures have considered the application of IA in cognitive relay networks under CSI mismatch.

Motivated by these observation, we apply IA to spectrum sharing relay network under the similar CSI mismatch model in [11]. The adaptive Max-SINR algorithm has been introduced to improve the performance of the secondary network by using the knowledge of CSI error variance. Finally, we investigate the performance of the secondary network in terms of the end-to-end equivalent transmission rate and outage probability and derive that IA can increase the sum rate and decrease the outage probability of the secondary network. Simulation results are provided to demonstrate the validity of our analysis and indicate that our proposed adaptive Max-SINR IA scheme can greatly improve the performance of the secondary network.

Notation: $\mathbf{A}^H$ represents the conjugate transpose of matrix $\mathbf{A}$. $rank\left(\mathbf{A}\right)$ is the rank of matrix $\mathbf{A}$. $\mathbf{I}_n$ denotes the $n \times n$ identity matrix.

2 System Model

2.1 Network Description

We consider a spectrum sharing relay network, where the PU network containing a PU source that equipped with M_p antennas and a PU destination that equipped with N_p antennas. Secondary network including three nodes coexists with the PU network. Moreover, the SU network contains a secondary source (SS), a secondary relay (SR) and a secondary destination (SD) that equipped with M_s, M_r and M_d antennas, respectively. Specifically, the PU source transmit information to PU destination directly while the SS transmit information to SD with the help of SR using decode-and-forward (DF) protocol. In the first phase, the received signals at the PU receiver, the SR and the SD are respectively given by

$$\mathbf{y}_{p,1} = \mathbf{H}_{pp}\mathbf{V}_{p,1}\mathbf{x}_p + \mathbf{H}_{ps}\mathbf{V}_{s,1}\mathbf{x}_s + \mathbf{n}_{p,1} \tag{1}$$

$$\mathbf{y}_{r,1} = \mathbf{H}_{rs}\mathbf{V}_{s,1}\mathbf{x}_s + \mathbf{H}_{rp}\mathbf{V}_{p,1}\mathbf{x}_p + \mathbf{n}_{r,1} \tag{2}$$

$$\mathbf{y}_{d,1} = \mathbf{H}_{ds}\mathbf{V}_{s,1}\mathbf{x}_s + \mathbf{H}_{dp}\mathbf{V}_{p,1}\mathbf{x}_s + \mathbf{n}_{d,1} \tag{3}$$

In the second phase, if the SR can decode the message from the SS successfully, the SR will forward the signal to the SD. The received signals of PU destination and SD are respectively expressed as

$$\mathbf{y}_{p,2} = \mathbf{H}_{pp}\mathbf{V}_{p,2}\mathbf{x}_p + \mathbf{H}_{pr}\mathbf{V}_{r,2}\mathbf{x}_r + \mathbf{n}_{p,2} \tag{4}$$

$$\mathbf{y}_{d,2} = \mathbf{H}_{dr}\mathbf{V}_{r,2}\mathbf{x}_r + \mathbf{H}_{dp}\mathbf{V}_{p,2}\mathbf{x}_r + \mathbf{n}_{d,2} \tag{5}$$

In the formulas above, $\mathbf{H}_{ab} \in C^{N_a \times M_b}$ denotes the channel fading matrix from node b to node a, $\mathbf{x}_b \in C^{d_b \times 1}$ and $\mathbf{V}_{b,t} \in C^{M_b \times d_b}$ are the transmitting data and the precoding matrix at node a in the tth phase, and $\mathbf{n}_{a,t} \sim N_C\left(\mathbf{0}, \sigma^2 \mathbf{I}_{N_a}\right)$ is the noise vector at node a in the tth phase, where $b \in \{p, s, r\}$, $a \in \{p, r, d\}$ and $t \in \{1, 2\}$.

Using the interference suppression matrix to null the interference at receiver, the recovered signal vector can be expressed as

$$\mathbf{r}_{d,t} = \mathbf{U}_{d,t}^H \mathbf{y}_{d,t} \tag{6}$$

where $\mathbf{U}_{d,t} \in C^{N_d \times d_d}$ is the truncated unitary interference suppression matric. And the SD will use maximum ratio combination (MRC) protocol to combine the recovered signals of the two phases to recover the message.

For notational convenience, we assume that the transmit power of all nodes is P, and $\rho = P/\sigma^2$ is the nominal transmitting SNR.

2.2 CSI Mismatch Model

According to the assumption in [11], we assume that all precoding matrixs are defined with the knowledge of unified CSI mismatch. And the CSI mismatch model can be expressed as

$$\hat{\mathbf{H}}_{ab} = \mathbf{H}_{ab} + \mathbf{E}_{ab} \tag{7}$$

where the channel measurement error $\mathbf{E}_{ab}$ is thought to be independent of actual channel matrix $\mathbf{H}_{ab}$. We consider $\mathbf{E}_{ab}$ as a Gaussian matrix, which consists of i.i.d. elements with mean zero and variance τ, i.e.,

$$\begin{aligned} vec\left(\mathbf{E}_{ab}\right) &\sim \mathrm{N}_C\left(\mathbf{0}, \tau\mathbf{I}\right) \\ with\, \tau &\triangleq \beta\rho^{-\alpha}, \beta > 0, \alpha \geq 0 \end{aligned} \tag{8}$$

where $\alpha \neq 0$ denotes the error variance depends on SNR and $\alpha = 0$ denotes the error variance is independent of SNR.

3 Iterative Algorithm for Interference Alignment in Cognitive Relay Networks

In this section, we give the iterative IA algorithm in cognitive relay networks under CSI mismatch model.

For the CSI mismatch case, we assume that all precoding matrices are derived based on imperfect CSI $\hat{\mathbf{H}}_{ab}$. According to the perfect CSI case that in [7], in the first phase, the precoding matrices should satisfy

$$\hat{\mathbf{U}}_{p,1}^H \hat{\mathbf{H}}_{ps} \hat{\mathbf{V}}_{s,1} = 0, \quad rank\left(\hat{\mathbf{U}}_{p,1}^H \hat{\mathbf{H}}_{pp} \hat{\mathbf{V}}_{p,1}\right) = d_p \tag{9}$$

$$\hat{\mathbf{U}}_{r,1}^H \hat{\mathbf{H}}_{rp} \hat{\mathbf{V}}_{p,1} = 0, \quad rank\left(\hat{\mathbf{U}}_{r,1}^H \hat{\mathbf{H}}_{rs} \hat{\mathbf{V}}_{s,1}\right) = d_s \tag{10}$$

$$\hat{\mathbf{U}}_{d,1}^H \hat{\mathbf{H}}_{dp} \hat{\mathbf{V}}_{p,1} = 0, \quad rank\left(\hat{\mathbf{U}}_{d,1}^H \hat{\mathbf{H}}_{ds} \hat{\mathbf{V}}_{s,1}\right) = d_s \tag{11}$$

Similarly, in the second phase, the precoding matrices should satisfy

$$\hat{\mathbf{U}}_{p,2}^H \hat{\mathbf{H}}_{pr} \hat{\mathbf{V}}_{r,2} = 0, \quad rank\left(\hat{\mathbf{U}}_{p,2}^H \hat{\mathbf{H}}_{pp} \hat{\mathbf{V}}_{p,2}\right) = d_p \tag{12}$$

$$\hat{\mathbf{U}}_{d,2}^H \hat{\mathbf{H}}_{dp} \hat{\mathbf{V}}_{p,2} = 0, \quad rank\left(\hat{\mathbf{U}}_{d,2}^H \hat{\mathbf{H}}_{dr} \hat{\mathbf{V}}_{r,2}\right) = d_r \tag{13}$$

To derive the precoding matrices, we firstly derive the normalized interference plus noise covariance matrix the following Lemma.

Lemma 1. The normalized interference plus noise covariance matrix associated with the lth stream of node k is derived as

$$\begin{aligned} \hat{\mathbf{Q}}_k^l = &\sum_{d=1}^{d_j} \hat{\mathbf{H}}_{kj} \hat{\mathbf{V}}_j^{[*d]} \hat{\mathbf{V}}_j^{[*d]H} \hat{\mathbf{H}}_{kj}^H + \sum_{d=1, d\neq l}^{d_k} \hat{\mathbf{H}}_{kk} \hat{\mathbf{V}}_k^{[*d]} \hat{\mathbf{V}}_k^{[*d]H} \hat{\mathbf{H}}_{kk}^H \\ &+ \underbrace{\left(\left(d_s + d_p - 1\right)\tau\left(1+\tau\right) + \rho^{-1}(1+\tau)^2\right)}_{\mu_1} \mathbf{I}_{N_k}, \end{aligned} \tag{14}$$

Proof. The interference plus noise covariance matrix associated with the lth stream of user k can be shown as that in (15).

$$\begin{aligned}
&\mathbf{Q}_k^l = \sum_{d=1}^{d_j} P\mathbf{H}_{kj}\hat{\mathbf{V}}_j^{[*d]}\hat{\mathbf{V}}_j^{[*d]H}\mathbf{H}_{kj}^H + \sum_{d=1,d\neq l}^{d_k} P\mathbf{H}_{kk}\hat{\mathbf{V}}_k^{[*d]}\hat{\mathbf{V}}_k^{[*d]H}\mathbf{H}_{kk}^H + \sigma^2\mathbf{I}_{N_k}\\
&= \sum_{d=1}^{d_j} P\left(\tfrac{1}{1+\tau}\hat{\mathbf{H}}_{kj} + \breve{\mathbf{H}}_{kj}\right)\hat{\mathbf{V}}_j^{[*d]}\hat{\mathbf{V}}_j^{[*d]H}\left(\tfrac{1}{1+\tau}\hat{\mathbf{H}}_{kj} + \breve{\mathbf{H}}_{kj}\right)^H\\
&+ \sum_{d=1,d\neq l}^{d_k} P\left(\tfrac{1}{1+\tau}\hat{\mathbf{H}}_{kk} + \breve{\mathbf{H}}_{kk}\right)\hat{\mathbf{V}}_k^{[*d]}\hat{\mathbf{V}}_k^{[*d]H}\left(\tfrac{1}{1+\tau}\hat{\mathbf{H}}_{kk} + \breve{\mathbf{H}}_{kk}\right)^H + \sigma^2\mathbf{I}_{N_k}\\
&= \sum_{d=1}^{d_j} \tfrac{P}{(1+\tau)^2}\hat{\mathbf{H}}_{kj}\hat{\mathbf{V}}_j^{[*d]}\hat{\mathbf{V}}_j^{[*d]H}\hat{\mathbf{H}}_{kj}^H + \sum_{d=1,d\neq l}^{d_k} \tfrac{P}{(1+\tau)^2}\hat{\mathbf{H}}_{kk}\hat{\mathbf{V}}_k^{[*d]}\hat{\mathbf{V}}_k^{[*d]H}\hat{\mathbf{H}}_{kk}^H\\
&+ \underbrace{\sum_{d=1}^{d_j}\frac{P}{1+\tau}\left(\hat{\mathbf{H}}_{kj}\hat{\mathbf{V}}_j^{[*d]}\hat{\mathbf{V}}_j^{[*d]H}\breve{\mathbf{H}}_{kj}^H + \breve{\mathbf{H}}_{kj}\hat{\mathbf{V}}_j^{[*d]}\hat{\mathbf{V}}_j^{[*d]H}\hat{\mathbf{H}}_{kj}^H\right)}_{\mathbf{J}_1}\\
&+ \underbrace{\sum_{d=1,d\neq l}^{d_k}\frac{P}{1+\tau}\left(\hat{\mathbf{H}}_{kk}\hat{\mathbf{V}}_k^{[*d]}\hat{\mathbf{V}}_k^{[*d]H}\breve{\mathbf{H}}_{kk}^H + \breve{\mathbf{H}}_{kk}\hat{\mathbf{V}}_k^{[*d]}\hat{\mathbf{V}}_k^{[*d]H}\hat{\mathbf{H}}_{kk}^H\right)}_{\mathbf{J}_2}\\
&+ \underbrace{\sum_{d=1}^{d_j} P\breve{\mathbf{H}}_{kj}\hat{\mathbf{V}}_j^{[*d]}\hat{\mathbf{V}}_j^{[*d]H}\breve{\mathbf{H}}_{kj}^H + \sum_{d=1,d\neq l}^{d_k} P\breve{\mathbf{H}}_{kk}\hat{\mathbf{V}}_k^{[*d]}\hat{\mathbf{V}}_k^{[*d]H}\breve{\mathbf{H}}_{kk}^H}_{\mathbf{J}_3} + \sigma^2\mathbf{I}_{N_k}
\end{aligned} \tag{15}$$

All precoders and combiners are constructed upon channel $\hat{\mathbf{H}}_{kj}$ and are independent of $\breve{\mathbf{H}}_{kj}$, therefore $\mathrm{E}_{\hat{\mathbf{H}}_{kj},\breve{\mathbf{H}}_{kj}}\mathbf{J}_1 = 0$, $\mathrm{E}_{\hat{\mathbf{H}}_{kj},\breve{\mathbf{H}}_{kj}}\mathbf{J}_2 = 0$, $\mathrm{E}_{\hat{\mathbf{H}}_{kj},\breve{\mathbf{H}}_{kj}}\mathbf{J}_3 = P\left(d_j + d_k - 1\right)\frac{\tau}{1+\tau}\mathbf{I}$ based on that in [7]. This way, we can normalized $\mathbf{Q}_k^l$ by divided $\frac{P}{(1+\tau)^2}$ in (14) and approximate it with a simpler from, i.e., $\hat{\mathbf{Q}}_k^l$, as follows that in (14).

Given randomly initialized precoders and with respect to the fact that only imperfect channel estimates are available, in the cognitive relay network considered in this paper in the first phase, the normalized interference plus noise covariance matrices for stream l at the PU receiver, the SR and SD can be evaluated as

$$\begin{aligned}
&\hat{\mathbf{Q}}_{p,1}^l = \sum_{d=1}^{d_s}\hat{\mathbf{H}}_{ps}\hat{\mathbf{V}}_{s,1}^{[*d]}\hat{\mathbf{V}}_{s,1}^{[*d]H}\hat{\mathbf{H}}_{ps}^H + \sum_{d=1,d\neq l}^{d_p}\hat{\mathbf{H}}_{pp}\hat{\mathbf{V}}_{p,1}^{[*d]}\hat{\mathbf{V}}_{p,1}^{[*d]H}\hat{\mathbf{H}}_{pp}^H\\
&+ \underbrace{\left(\left(d_s + d_p - 1\right)\tau\left(1+\tau\right) + \rho^{-1}(1+\tau)^2\right)}_{\mu_1}\mathbf{I}_{N_p},
\end{aligned} \tag{16}$$

$$\begin{aligned}\hat{\mathbf{Q}}_{r,1}^{l} = \sum_{d=1}^{d_p} \hat{\mathbf{H}}_{rp}\hat{\mathbf{V}}_{p,1}^{[*d]}\hat{\mathbf{V}}_{p,1}^{[*d]H}\hat{\mathbf{H}}_{rp}^{H} + \sum_{d=1,d\neq l}^{d_s} \hat{\mathbf{H}}_{rs}\hat{\mathbf{V}}_{s,1}^{[*d]}\hat{\mathbf{V}}_{s,1}^{[*d]H}\hat{\mathbf{H}}_{rs}^{H} \\ + \underbrace{\left(\left(d_s + d_p - 1\right)\tau\left(1+\tau\right) + \rho^{-1}(1+\tau)^2\right)}_{\mu_1}\mathbf{I}_{N_r}\end{aligned} \tag{17}$$

$$\begin{aligned}\hat{\mathbf{Q}}_{d,1}^{l} = \sum_{d=1}^{d_p} \hat{\mathbf{H}}_{dp}\hat{\mathbf{V}}_{p,1}^{[*d]}\hat{\mathbf{V}}_{p,1}^{[*d]H}\hat{\mathbf{H}}_{dp}^{H} + \sum_{d=1,d\neq l}^{d_s} \hat{\mathbf{H}}_{ds}\hat{\mathbf{V}}_{s,1}^{[*d]}\hat{\mathbf{V}}_{s,1}^{[*d]H}\hat{\mathbf{H}}_{ds}^{H} \\ + \underbrace{\left(\left(d_s + d_p - 1\right)\tau\left(1+\tau\right) + \rho^{-1}(1+\tau)^2\right)}_{\mu_1}\mathbf{I}_{N_d},\end{aligned} \tag{18}$$

respectively. In the reciprocal network, the normalized interference plus noise covariance matrices for stream l for the second phase at the PU transmitter and the SS are given by

$$\begin{aligned}\tilde{\hat{\mathbf{Q}}}_{p,1}^{l} = \sum_{d=1}^{d_s} \tilde{\hat{\mathbf{H}}}_{pr}\tilde{\hat{\mathbf{V}}}_{r,1}^{[*d]}\tilde{\hat{\mathbf{V}}}_{r,1}^{[*d]H}\tilde{\hat{\mathbf{H}}}_{pr}^{H} + \sum_{\mathrm{d}=1}^{d_d} \tilde{\hat{\mathbf{H}}}_{pd}\tilde{\hat{\mathbf{V}}}_{d,1}^{[*d]}\tilde{\hat{\mathbf{V}}}_{d,1}^{[*d]H}\tilde{\hat{\mathbf{H}}}_{pd}^{H} \\ + \sum_{d=1,d\neq l}^{d_p} \tilde{\hat{\mathbf{H}}}_{pp}^{H}\tilde{\hat{\mathbf{V}}}_{p,1}^{[*d]}\tilde{\hat{\mathbf{V}}}_{p,1}^{[*d]H}\tilde{\hat{\mathbf{H}}}_{pp} + \underbrace{\left(\left(d_s + d_d + d_p - 1\right)\tau\left(1+\tau\right) + \rho^{-1}(1+\tau)^2\right)}_{\tilde{\mu}_1}\mathbf{I}_{N_p}\end{aligned} \tag{19}$$

$$\begin{aligned}\tilde{\hat{\mathbf{Q}}}_{s,1}^{l} = \sum_{d=1}^{d_p} \tilde{\hat{\mathbf{H}}}_{sp}\tilde{\hat{\mathbf{V}}}_{p,1}^{[*d]}\tilde{\hat{\mathbf{V}}}_{p,1}^{[*d]H}\tilde{\hat{\mathbf{H}}}_{sp}^{H} + \sum_{d=1,d\neq l}^{d_r} \tilde{\hat{\mathbf{H}}}_{sr}\tilde{\hat{\mathbf{V}}}_{r,1}^{[*d]}\tilde{\hat{\mathbf{V}}}_{r,1}^{[*d]H}\tilde{\hat{\mathbf{H}}}_{sr}^{H} \\ + \sum_{d=1,d\neq l}^{d_d} \tilde{\hat{\mathbf{H}}}_{sd}\tilde{\hat{\mathbf{V}}}_{d,1}^{[*d]}\tilde{\hat{\mathbf{V}}}_{d,1}^{[*d]H}\tilde{\hat{\mathbf{H}}}_{sd}^{H} + \underbrace{\left(\left(d_s + d_d + d_p - 1\right)\tau\left(1+\tau\right) + \rho^{-1}(1+\tau)^2\right)}_{\tilde{\mu}_1}\mathbf{I}_{N_s}\end{aligned} \tag{20}$$

As mentioned earlier, due to the coupled nature of the problem, finding precoders and combiners requests an iterative algorithm in general. With respect to the fact that only imperfect channel estimates are available, and with knowledge of error variance τ in advance, the proposed adaptive Max-SINR algorithm in the first phase can be concisely presented as follows:

Adaptive Max-SINR in the first phase

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1. Set the adaptive factors $\mu_1 = \left(d_s + d_p - 1\right)\tau\left(1+\tau\right) + \rho^{-1}(1+\tau)^2$, $\tilde{\mu}_1 = \left(d_s + d_d + d_p - 1\right)\tau\left(1+\tau\right) + \rho^{-1}(1+\tau)^2$.
2. The PU source and SS respectively set arbitrary precoding matrices $\hat{\mathbf{V}}_{p,1}$ and $\hat{\mathbf{V}}_{s,1}$ under CSI mismatch, which satisfies $\hat{\mathbf{V}}_{p,1}^{H}\hat{\mathbf{V}}_{p,1} = \mathbf{I}_{d_p}$, $\hat{\mathbf{V}}_{s,1}^{H}\hat{\mathbf{V}}_{s,1} = \mathbf{I}_{d_s}$.
3. According to (16)–(18), the PU destination, the SR and the SD compute their interference plus noise covariance matrices $\hat{\mathbf{Q}}_{p,1}$, $\hat{\mathbf{Q}}_{r,1}$ and $\hat{\mathbf{Q}}_{d,1}$, respectively.

4. The PU destination, the SR and the SD calculate their interference suppression vectors $\hat{\mathbf{U}}_{p,1}^{[*d]}$, $\hat{\mathbf{U}}_{r,1}^{[*d]}$ and $\hat{\mathbf{U}}_{d,1}^{[*d]}$ under CSI mismatch by

$$\hat{\mathbf{U}}_{i,1}^{[*d]} = \frac{\left(\hat{\mathbf{Q}}_{i,1}^{l}\right)^{-1}\hat{\mathbf{H}}_{i,j}\hat{\mathbf{V}}_{i,1}^{[*d]}}{\left\|\left(\hat{\mathbf{Q}}_{i,1}^{l}\right)^{-1}\hat{\mathbf{H}}_{i,j}\hat{\mathbf{V}}_{i,1}^{[*d]}\right\|} \tag{21}$$

   where $i \in \{p, r, d\}$ and $j \in \{p, s\}$.
5. Setting $\tilde{\hat{\mathbf{V}}}_{i,1} = \hat{\mathbf{U}}_{i,1}$ $(i \in \{p, r, d\})$. Then, according to (19) and (20), we compute the interference plus noise covariance matrices at the PU source and SS, i.e. $\tilde{\hat{\mathbf{Q}}}_{p,1}$, $\tilde{\hat{\mathbf{Q}}}_{s,1}$, respectively.
6. The PU source and SS compute their interference suppression matrices $\tilde{\hat{\mathbf{U}}}_{p,1}^{[*d]}$, $\tilde{\hat{\mathbf{U}}}_{s,1}^{[*d]}$ under CSI mismatch, respectively, satisfying

$$\tilde{\hat{\mathbf{U}}}_{a,1}^{[*d]} = \frac{\left(\tilde{\hat{\mathbf{Q}}}_{a,1}^{l}\right)^{-1}\tilde{\hat{\mathbf{H}}}_{a,b}\tilde{\hat{\mathbf{V}}}_{a,1}^{[*d]}}{\left\|\left(\tilde{\hat{\mathbf{Q}}}_{a,1}^{l}\right)^{-1}\tilde{\hat{\mathbf{H}}}_{a,b}\tilde{\hat{\mathbf{V}}}_{a,1}^{[*d]}\right\|} \tag{22}$$

   where $a \in \{p, s\}$ and $b \in \{p, r, d\}$.
7. Reverse the communication direction again and set $\hat{\mathbf{V}}_{a,1} = \tilde{\hat{\mathbf{U}}}_{a,1}$ $(a \in \{p, s\})$.
8. Repeat the procedure from 2 to 7 till convergence.

In the second phase, the adaptive Max-SINR algorithm can be proposed as that in the first phase. And the detailed analysis is omitted due to space constraints.

4 Performance Analysis

According to [12], we evaluate the achievable rate for each user in this section. Specifically, the achievable rates for the transmission from SS to SR and SS to SD in the first phase can be derived as

$$\begin{aligned} R_{rs} &= \tfrac{1}{2}\log_2 \det\left(\mathbf{I}_{d_r} + \left(\mathbf{I}_{d_r} + \hat{\mathit{\Phi}}_{r,1}\right)^{-1}\hat{\mathit{\Psi}}_{r,1}\right) \\ &= \tfrac{1}{2}\log_2 \det\left(\mathbf{I}_{d_r} + \hat{\mathit{\Theta}}_{r,1}\right) \end{aligned} \tag{23}$$

where $\mathit{\Phi}_{r,1} = P\hat{\mathbf{U}}_{r,1}^{H}\mathbf{H}_{rp}\hat{\mathbf{V}}_{p,1}\hat{\mathbf{V}}_{p,1}^{H}\mathbf{H}_{rp}^{H}\hat{\mathbf{U}}_{r,1}$, $\mathit{\Psi}_{r,1} = P\hat{\mathbf{U}}_{r,1}^{H}\mathbf{H}_{rs}\hat{\mathbf{V}}_{s,1}\hat{\mathbf{V}}_{s,1}^{H}\mathbf{H}_{rs}^{H}\hat{\mathbf{U}}_{r,1}$.

$$\begin{aligned} R_{ds} &= \tfrac{1}{2}\log_2 \det\left(\mathbf{I}_{d_d} + \left(\mathbf{I}_{d_d} + \hat{\mathit{\Phi}}_{d,1}\right)^{-1}\hat{\mathit{\Psi}}_{d,1}\right) \\ &= \tfrac{1}{2}\log_2 \det\left(\mathbf{I}_{d_d} + \hat{\mathit{\Theta}}_{d,1}\right) \end{aligned} \tag{24}$$

where $\mathit{\Phi}_{d,1} = P\hat{\mathbf{U}}_{d,1}^H\mathbf{H}_{dp}\hat{\mathbf{V}}_{p,1}\hat{\mathbf{V}}_{p,1}^H\mathbf{H}_{dp}^H\hat{\mathbf{U}}_{d,1}$, $\mathit{\Psi}_{d,1} = P\hat{\mathbf{U}}_{d,1}^H\mathbf{H}_{ds}\hat{\mathbf{V}}_{s,1}\hat{\mathbf{V}}_{s,1}^H\mathbf{H}_{ds}^H\hat{\mathbf{U}}_{r,1}$.

In the second phase, the achievable rate for the transmission from SR to SD can be written as

$$\begin{aligned} R_{dr} &= \tfrac{1}{2}\log_2\det\left(\mathbf{I}_{d_d} + \left(\mathbf{I}_{d_d} + \hat{\mathit{\Phi}}_{d,2}\right)^{-1}\hat{\mathit{\Psi}}_{d,2}\right) \\ &= \tfrac{1}{2}\log_2\det\left(\mathbf{I}_{d_d} + \hat{\mathit{\Theta}}_{d,2}\right) \end{aligned} \tag{25}$$

where $\mathit{\Phi}_{d,2} = P\hat{\mathbf{U}}_{d,2}^H\mathbf{H}_{dp}\hat{\mathbf{V}}_{p,2}\hat{\mathbf{V}}_{p,2}^H\mathbf{H}_{dp}^H\hat{\mathbf{U}}_{d,2}$, $\mathit{\Psi}_{d,2} = P\hat{\mathbf{U}}_{d,2}^H\mathbf{H}_{dr}\hat{\mathbf{V}}_{r,2}\hat{\mathbf{V}}_{r,2}^H\mathbf{H}_{dr}^H\hat{\mathbf{U}}_{d,2}$.

Hence, the transmission rate and the outage probability of the spectrum sharing relay network are derived as

$$R = \min\left\{\frac{1}{2}\log_2\det\left(\mathbf{I}_{d_r} + \hat{\mathit{\Theta}}_{r,1}\right), \frac{1}{2}\log_2\det\left(\mathbf{I}_{d_d} + \hat{\mathit{\Theta}}_{d,1} + \hat{\mathit{\Theta}}_{d,2}\right)\right\} \tag{26}$$

and

$$P_{out} = \Pr\left\{R < R_{th}\right\} \tag{27}$$

respectively, where R_{th} represents the target transmission rate of the spectrum sharing relay network.

5 Numerical Results

In this section, Monte Carlo simulation results are performed to evaluate the performance of the proposed IA algorithm in the spectrum sharing relay network. For notational convenience, we assume that each user has equal power. We focus on two representative cases: $\alpha = 0$ (the error variance is independent of SNR), and $\alpha = 0.5$ (the error variance is depend on SNR).

Figure 1 shows the transmission rate of the spectrum sharing relay network when adopting different algorithms. In this figure, we observe that the transmission rate of the adaptive Max-SINR algorithm is better than that of the original Max-SINR algorithm described in [7]. We also analyze the transmission rate of Alt-Min algorithm given in [7] and observe that Max-SINR algorithm outperforms Alt-Min algorithm when the CSI is perfect. However, due to CSI mismatch, the adaptive Max-SINR achieves notably better performance than both Max-SINR and Alt-Min algorithms. And it demonstrates the validity of our proposed adaptive Max-SINR scheme in the cognitive relay system with CSI mismatch.

Figure 2 shows the outage probability of the spectrum sharing relay network when $R_{th} = 1bit/s/Hz$. As observed, the Max-SINR algorithm achieves better outage performance than the Alt-Min algorithm when the CSI is perfect. Moreover, when the CSI is imperfect, the achieved outage performance improvement of Max-SINR algorithm can be negligible compared to Alt-Min algorithm especially at high SNRs, while the adaptive Max-SINR algorithm achieves greatly better outage performance. Therefore, similar to that in Fig. 1, the adaptive Max-SINR algorithm is high necessary when the CSI is imperfect.

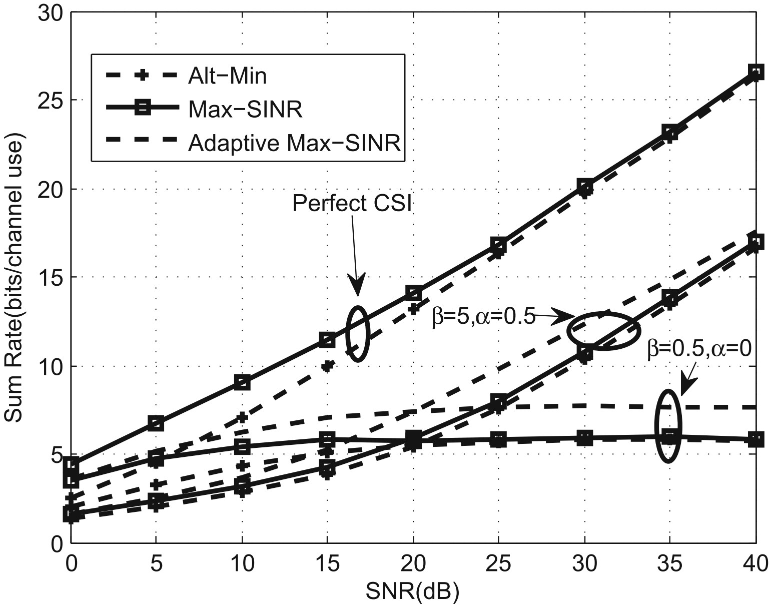

Fig. 1. Average sum rate for $d = 2$, $M = 4$ and for the cases $\beta = 0.5, \alpha = 0$ and $\beta = 5, \alpha = 0.5$.

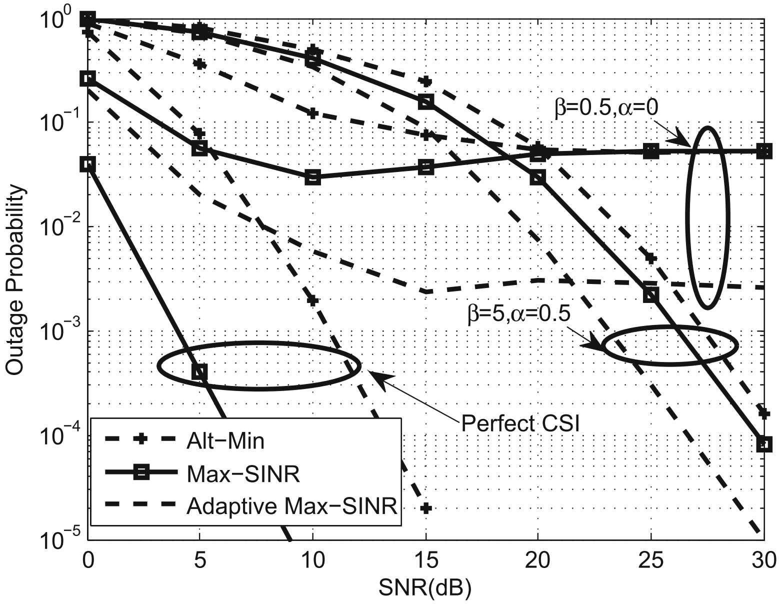

Fig. 2. Outage probability for $d = 2$, $M = 4$ and for the cases $\beta = 0.5, \alpha = 0$ and $\beta = 5, \alpha = 0.5$.

6 Conclusions

In this paper, we apply IA to relay network to improve the performance of the secondary system. The results show that interference between the PU network and the SU network can be nulled using the iterative IA algorithm. We further quantify the performance of IA under CSI mismatch where the variance of the CSI measurement error depends on the SNR. Then, we propose an adaptive Max-SINR to improve the performance of the spectrum sharing relay network under CSI mismatch. Finally, we investigate the performance of the spectrum sharing relay network and conclude that IA can improve the sum rate and outage performance of the considered network, which demonstrates that IA is a promising solution to interference avoidance in the spectrum sharing relay network.

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