A New Approach to the Analysis of Network Observability in Medium and Low Voltage Electrical Grids

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Abstract. Medium and low voltage electrical power grids are typically sparsely instrumented, and thus, not observable in a systems' theory sense. However, this is a requirement to carry out state estimation methods. To this end, many approaches for optimal sensor placement are proposed in the literature. Such methods are typically motivated from a mathematical perspective, not taking the physical properties of the network into account. As a consequence, the dimensionality of the mathematical problem is typically quite large resulting in significant numerical complexity. Therefore, a new approach is proposed here which is based on analyzing the characteristic observable and unobservable nodes by using singular value decomposition (SVD) and the breadth-first search method. The aim of the method is to identify all possibilities for the placement of measuring equipment to achieve observability. The proposed method does render the network observable with a minimal number of sensors. In this way, this reduces the dimensionality for conventional optimal sensor placement algorithms substantially.

Keywords: Electrical grid · State estimation · Singular value decomposition · Breadth-first search · Optimal sensor placement

1 Introduction

Medium and low voltage electrical power grids are typically sparsely instrumented, and thus, such systems are usually not observable in a systems' theory sense [1, 2]. That is, the complete network state cannot be inferred from the available and measured network parameters. There are basically two situations which cause a lack of observability: insufficient measuring equipment and redundant measurements that cannot contribute to the observability of the system. The installation of additional measuring equipment is rather costly, which is why optimal strategies for their placement are of great interest. When additional instrumentation of the network is not feasible, typically pseudo-measurements are used for state estimation instead. Owing to their poor accuracy, it is also important to find the useful placements for such pseudo-measurements in order to achieve good overall estimation quality.

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Classical sensor placement methods consider the whole network and aim at determining a set of measured nodes which is optimal in the sense of minimal state estimation errors [3–5]. Therefore, typically all possible measurements including redundancy are considered one by one. This process is continued until the network is observable, namely the Jacobian matrix of network has full rank. However, there are many possibilities to install new measuring equipment, resulting in a high-dimensional estimation problem. For practical networks the dimensionality and complexity of this mathematical problem is so large that it results in serious numerical and computational issues. To this end, we propose a new method for determining network nodes at which measurements have to be added in order to achieve network observability. The idea is that based on such kind of pre-processing, the computational complexity of the optimization problem can be reduced significantly (Fig. 1).

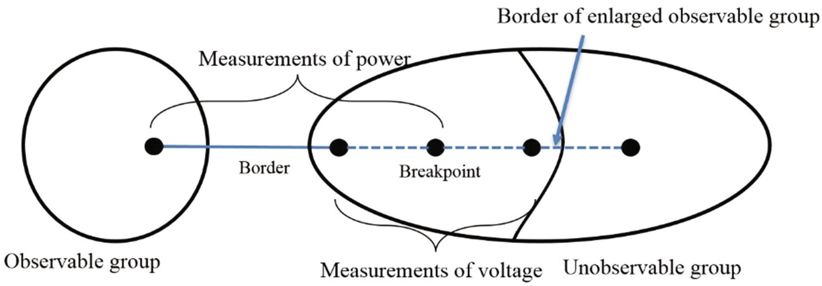

Fig. 1. Possible placements of network based on the proposed observability and voltage flow analysis

The reason for a network being unobservable is that at some nodes the voltage cannot be computed using the forward and backward sweep in the power flow calculation. These nodes are here defined as *breakpoints*. The main idea of the here proposed method is that adding power measurements at the nodes between the border of the observable and the unobservable groups and the breakpoint, adding voltage measurements at the nodes between the border of observable and unobservable groups and border of enlarged observable group or aggregation the nodes on the sides of border will then convert the unobservable group to an observable one. Thus, these nodes are considered as potential points for the placement of extra measuring equipment, pseudo-measurements or aggregation of nodes. In particular for large networks, this pre-determination of appropriate placement positions reduces the dimensionality for conventional optimal sensor placement algorithms substantially.

2 Mathematical Formulation of the Simplified Jacobian

The mathematical model relating measured network parameters z, such as nodal voltage amplitudes, active and reactive power, to the nodal voltage magnitude and phase values $\boldsymbol{x}$ is given by [6]

$$z = h(\boldsymbol{x}). \tag{1}$$

In static state estimation, a linearized variant of this relation is considered, leading to the system of linear equations of a power system

$$z = \boldsymbol{H} \cdot \boldsymbol{x}, \tag{2}$$

where z is the vector of measurements, $\boldsymbol{H}$ is the Jacobian of h with respect to $\boldsymbol{x}$, and $\boldsymbol{x}$ is vector of $2n - 1$ states. If for the rank r of the Jacobian $\boldsymbol{H}$ it holds that $r < 2n - 1$, then the power system is not observable in a systems' theory sense. To this end, $2n - 1 - r$ measurements have to be introduced in such a way that the resulting Jacobian $\boldsymbol{H}_{new} = \begin{bmatrix} \boldsymbol{H} \\ \boldsymbol{H}' \end{bmatrix}$ has rank $r = 2n - 1$. This can only be achieved efficiently by a well-structured measurement placement method.

As long as the voltages and admittances are non-zero, their value does not affect the rank of the Jacobian matrix $\boldsymbol{H}$. Thus, the observability analysis can be carried out using the Jacobian matrix with initial value of voltage magnitude being 1 and voltage phase being 0:

$$\boldsymbol{U} = e + \sqrt{-1}f \text{ where } e = 1 \text{ and } f = 0. \tag{3}$$

In the admittance matrix the sum of the elements in each row is zero $\sum_1^n Y_{ij} = 0$. Thus, the partial derivatives of nodal power with respect to nodal voltages that form the Jacobian can be simplified. Hence, the Jacobian matrix for nodal power can be directly written as

$$\boldsymbol{H} = \begin{bmatrix} \frac{\partial P_i}{\partial e} & \frac{\partial P_i}{\partial f} \\ \frac{\partial Q_i}{\partial e} & \frac{\partial Q_i}{\partial f} \end{bmatrix} = \begin{bmatrix} G_i & -B_i \\ -B_i & -G_i \end{bmatrix}. \tag{4}$$

Likewise, the partial derivatives for power from bus i to bus j that form the Jacobian matrix can be simplified. Hence, the part of the Jacobian matrix for power from bus i to bus j is finally given by

$$\begin{aligned} \boldsymbol{H} &= \begin{bmatrix} \frac{\partial P_{ij}}{\partial e} & \frac{\partial P_{ij}}{\partial f} \\ \frac{\partial Q_{ij}}{\partial e} & \frac{\partial Q_{ij}}{\partial f} \end{bmatrix} \\ &= \begin{array}{c} \begin{array}{ccccccccccccccc} & & i & & & j & & & i+n-1 & & & j+n-1 & & & \end{array} \\ \begin{bmatrix} 0 & \cdots & g_{ij} & 0 & \cdots & -g_{ij} & 0 & \cdots & -b_{ij} & 0 & \cdots & b_{ij} & 0 & \cdots \\ 0 & \cdots & -b_{ij} & 0 & \cdots & b_{ij} & 0 & \cdots & -g_{ij} & 0 & \cdots & g_{ij} & 0 & \cdots \end{bmatrix} \end{array}. \end{aligned} \tag{5}$$

where in the row P_{ij} only the element in the columns i is g_{ij}, columns j is $-g_{ij}$, columns $i + n - 1$ is $-b_{ij}$ and columns $j + n - 1$ is b_{ij}, the other elements in this row are equal to zero. In the row Q_{ij} only the element in the columns i is $-b_{ij}$, columns j is b_{ij}, columns $i + n - 1$ is $-g_{ij}$ and columns $j + n - 1$ is g_{ij}, the other elements in this row are equal to zero.

3 Determining the Observable and Unobservable Groups

An unobservable power system can always be partitioned into a set of observable groups and a set of unobservable groups. An observable group is defined as a network region where, without additional measurements, the (complex) nodal voltage for all nodes in this region can be calculated. It is assumed that at least the reference (slack) node is observable.

Using the singular value decomposition (SVD) [7], the $m \times 2n - 1$ dimensional Jacobian matrix $\boldsymbol{H}$ can be decomposed into the matrix product

$$\boldsymbol{H} = \boldsymbol{U}\boldsymbol{S}\boldsymbol{V}^T. \tag{6}$$

The columns of the $m \times m$ unitary matrix $\boldsymbol{U}$ are the eigenvectors of $\boldsymbol{H}\boldsymbol{H}^T$; the non-zero elements of the $m \times 2n - 1$ matrix $\boldsymbol{S}$, which only has non-negative real numbers along the main diagonal, are the square roots of the non-zero eigenvalues of $\boldsymbol{H}^T\boldsymbol{H}$; the columns of the $2n - 1 \times 2n - 1$ unitary matrix $\boldsymbol{V}$ are the eigenvectors of $\boldsymbol{H}^T\boldsymbol{H}$. Assuming the elements $\sigma_{11}, \sigma_{22} \ldots \sigma_{mm}$ of the matrix $\boldsymbol{S}$ are non-zero, the columns $v_1, v_2 \ldots v_m$ of $\boldsymbol{V}$ are the eigenvectors corresponding to the non-zero eigenvalues of $\boldsymbol{H}^T\boldsymbol{H}$, and the columns $v_{m+1}, v_{m+2} \ldots v_{2n-1}$ corresponding to the vanishing eigenvalues. Thus, the columns $v_{m+1}, v_{m+2} \ldots v_{2n-1}$ form an orthonormal basis for the solutions to the homogeneous equation $\boldsymbol{H}x = 0$. Rows for which all these columns of $\boldsymbol{V}$ have zeros, belong to the observable network nodes. Thus, the remaining nodes are unobservable.

4 Enlarging the Observable Groups

The breadth-first search algorithm [8] can be used to assign connected observable und unobservable nodes to groups. Firstly, for each unobservable group the node which is connected with an observable node is chosen as “top point” in the network topology. From there, all neighbouring nodes up to the first branching point in the network are identified. Say there are n connections $(L_1, L_2 \ldots L_n)$ between the observable und unobservable groups as shown in Fig. 2.

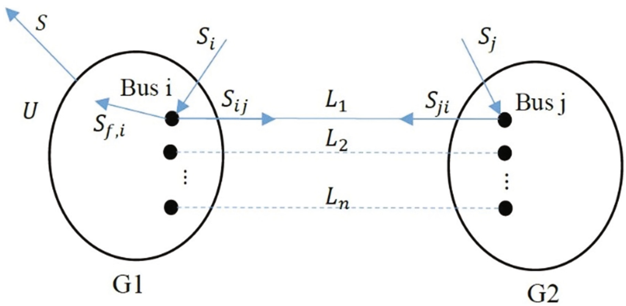

Fig. 2. Observable and unobservable group connected by n lines

In order to identify the points where instrumentation has to be added, the parts of the unobservable groups have to be added to the observable groups by *assumed* or *virtual* measurements.

All values in the observable group G1 are known, but in the group G2 the voltages cannot be calculated due to missing measurements. The bus i from G1 is connected to bus j from G2. For observability, a new virtual measurement, which can be either the complex power S_{ij} from bus i to bus j, the complex power S_{ji} from bus j to bus i, nodal power S_i of bus i or S_j of bus j, has to be added to enable the calculation of the nodal voltage at bus j, so that the observable group can be enlarged.

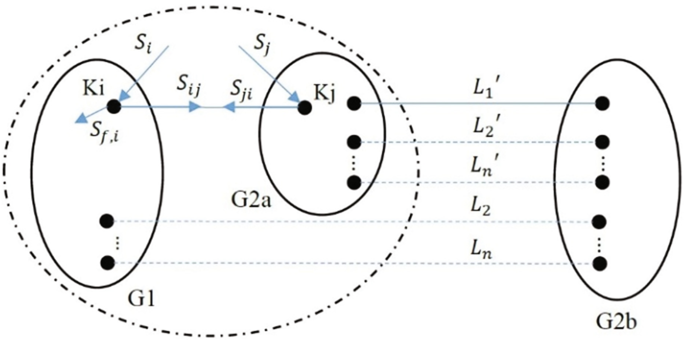

Fig. 3. Enlarged observable group

By adding the virtual measurements to the unobservable group, the observable group will be enlarged and the unobservable group will be decreased. The overall structure of the network can then be illustrated as shown in Fig. 3.

All enlarged observable groups can be determined by repeatedly adding virtual measurements. For radial networks, such as some medium and low voltage network, the lines $L_2, L_3, \cdots L_n$ *and* $L_2', L_3', \cdots L_n'$ in Figs. 2 and 3 should not be considered.

In order to determine the exact placement positions, we analyse the local Jacobian matrices for the unobservable groups as follows.

5 Determining Breakpoints Using the Jacobian Matrix

In the unobservable subgroup G2a, which can be added to the observable group exactly one point exists at which the voltage flow calculation breaks down due to missing measurements. We call this point a *breakpoint*. This breakpoint can be identified by analysing the simplified Jacobian of the local topology as follows.

The Jacobian matrix $\boldsymbol{H}_p$ of G2a, which includes n_p, is extracted from the whole system. Because exactly one measurement is missing to render this subgroup observable, the rank of $\boldsymbol{H}_p$ is equal to $2n_p - 2$. The matrix $\boldsymbol{H}_p$ can be written as

$$\boldsymbol{H}_p = [\boldsymbol{H}_{pRe} \quad \boldsymbol{H}_{pIm}], \tag{7}$$

where the block-matrix H_{pRe} consists of the partial derivatives of powers $S = (P, Q)^T$ with respect to the real parts of nodal voltage, and H_{pIm} consists of the partial derivatives with respect to the imaginary parts of nodal voltage. The matrices H_{pRe} and H_{pIm}, which have the same rank $n_p - 1$, can be transformed by Gaussian elimination to the form

$$H_{pRe}, H_{pIm} = \begin{bmatrix} 1 & & & & & & a_{1,n_p} \\ & 1 & & & & & a_{2,n_p} \\ & & \ddots & & & & \vdots \\ & & & 1 & & & a_{i,n_p} \\ & & & & \ddots & & \vdots \\ & & & & & 1 & a_{n_p-1,n_p} \end{bmatrix}. \tag{8}$$

The last column is $a_1, a_2 \ldots a_i \neq -1$ and $a_{i+1}, a_{i+2} \ldots a_{n_p-1} = -1$ for some index i. Assume that the ordering of the Jacobian matrix follows the ordering of nodes in the considered network group. For the successful calculation of the voltage and power flow in the network, the local Jacobian then has to be of the following structure

$$J = \begin{bmatrix} g & -g \\ b & -b \end{bmatrix}. \tag{9}$$

The elements in the last column after Gaussian elimination, which are equal to -1 indicate that at these nodes this structure is satisfied. Hence, the element $\mathrm{a_{i+1}}$ corresponds to the breakpoint for that group. Consequently, a power measurement has to be added between the previously identified border and the breakpoint of the subgroup.

If complex voltage (PMU) can be measured in the subgroup G2a, it means, a new Jacobian matrix H_v with only one row, which has one nonzero element, will be inserted in the Jacobian matrix H_{pRe} and H_{pIm}

$$H'_{pRe} = \begin{bmatrix} H_{pRe} \\ H_v \end{bmatrix}, H'_{plm} = \begin{bmatrix} H_{pRe} \\ H_v \end{bmatrix}. \tag{10}$$

No matter which element is nonzero, the new matrices H'_{pRe} and H'_{plm} have full rank. Hence a complex voltage measurement can be placed at any node of the subgroup to ensure the group becomes observable.

The method of summarising voltages is a special case of complex voltage measurement. The according node s have the same voltage, namely the loss at the line is ignored then. Thus, only the node s at the vicinity of the border should be summarized.

The above steps of identifying and grouping of the remaining unobservable nodes and the subsequent identification of breakpoints have to be repeated until the whole network is observable.

6 Example

For illustrating the application of the proposed method, a 16 buses 15 branches network model is employed. The network is shown in the figure below, where positions of power measurements are marked with blue arrows (Fig. 4).

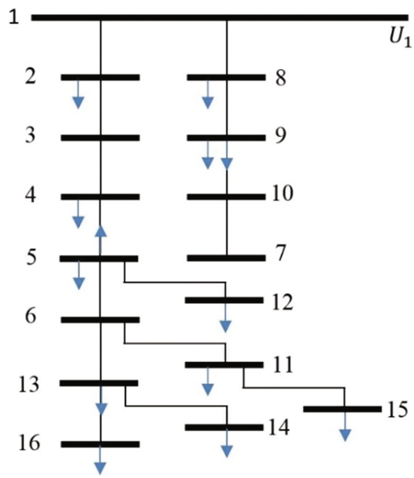

Fig. 4. Topology and measurements of the example network (Color figure online)

The possible placements of power measuring equipment are $\{S_1, S_{1,2}, S_{2,1}\ldots\}$, which are in total 33 possibilities.

The simplified Jacobian matrix is calculated by Eqs. (4) and (5). The observable group and the unobservable group of the network can be determined using the singular value decomposition (SVD) method by Eq. (6). The matrix S is a diagonal 27×31 matrix with number of the non-zero elements σ_1, $\sigma_2 \ldots \sigma_{27}$, corresponding to the rank of the Jacobian matrix, that 3 measuring equipment are required to let the network observable.

From the 31×31 dimensional matrix $\boldsymbol{V}$, the non-zero entries in columns $v_{28}, v_{29}, v_{30}, v_{31}$ indicate the elements of the observable group. In the first iteration of the algorithm for this example, the rows {1, 8, 9, 10, 23, 24, 25} of these columns of $\boldsymbol{V}$ are zero. Consequently, the resulting observable group include the nodes {1, 8, 9, 10} and unobservable group {2, 3, 4, 5, 6, 7, 11, 12, 13, 14, 15, 16}.

In the considered network topology, the nodes of the observable group are connected to each other. In contrast, the nodes of the unobservable group are partitioned by the breadth-first search algorithm into two groups {2, 3, 4, 5, 6, 11, 12, 13, 14, 15, 16} and {7}, which are separated by the observable group.

The nodes at the border are found to be 1, 2 and 10, 7, for the different groups respectively. Consequently, additional virtual measurements of powers $S_{1,2}$ and $S_{10,7}$ are inserted respectively such that the observable group is enlarged as described in the

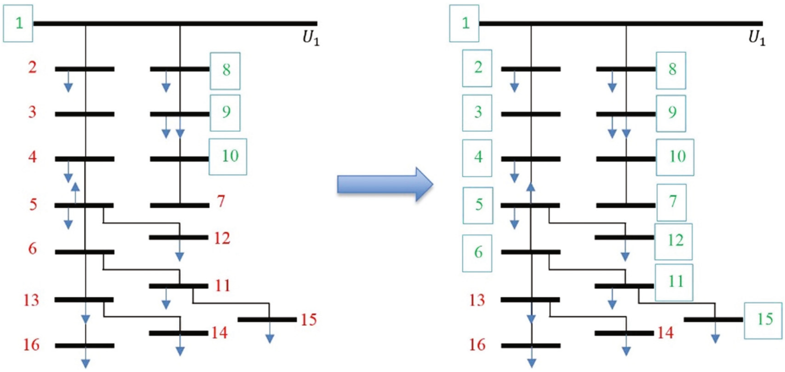

Fig. 5. Observable group before and after first iteration

Sect. 4. The additional nodes in the enlarged groups are then {2, 3, 4, 5, 6, 11, 12, 15}, which before belonged to the unobservable group {2, 3, 4, 5, 6, 11, 12, 13, 14, 15, 16} and node {7}, which before belonged to the unobservable group {7}., see Fig. 5.

Exactly one breakpoint exists in each enlarged group, and it can be found by the Jacobian matrix Eq. (8) as described in the Sect. 5. In our example, the Jacobian matrix of the two enlarged groups {2, 3, 4, 5, 6, 11, 12, 15} and {7} are calculated with the original measurements, which are $\left\{S_2, S_4, S_5, S_{11}, S_{12}, S_{15}, S_{5,4}\right\}$ for the first group and no measurement for the second group. For the first group, the Jacobian matrix with respect to the real part with dimension 7×8 of nodal voltage can be simplified by Gaussian elimination to

$$H_{pRe} = \begin{bmatrix} 1 & 0 & \cdots & 0 & -0.5 \\ 0 & 1 & \cdots & 0 & -1 \\ \vdots & \ddots & \ddots & \vdots & \vdots \\ 0 & \cdots & 0 & 1 & -1 \end{bmatrix} \tag{11}$$

From the last column, the breakpoint in this group is identified to be bus 3. In the second group there is no measurement, and hence, the identified breakpoint is bus 7.

This process is to be repeated until the group of observable nodes contains all nodes in the network, see Fig. 6.

Overall, in this example three additional measurements are required in order to achieve observability. The algorithm deduced that power measuring equipment can be installed respectively between bus 1 and 3 for first group, between bus 6 and 13 for the second group and between bus 10 and 7 for the third group. Or measuring equipment of power can be installed in any bus of the three enlarged groups in Fig. 6.

The quantity of possible placements of power measuring equipment is thus reduced by a factor of three from 33 to 11. A optimal placement method can use this information to determine the optimal placement of measuring equipment with respect to the chosen optimality criterion.

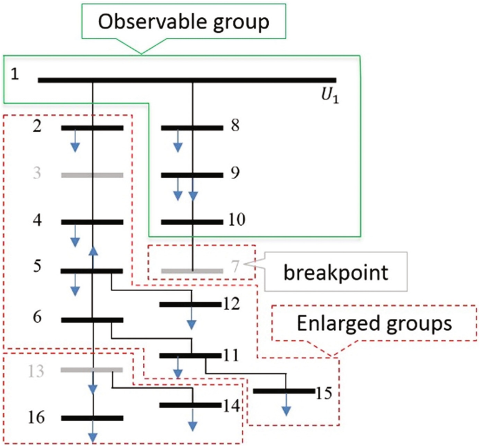

Fig. 6. The result of algorithm

7 Conclusion

In this paper a new algorithm based on the determination of the border between groups of observable and unobservable nodes and so-called *breakpoint* has been proposed to determine placements of measurements by taken into account the physicalities of the network. The observable and unobservable nodes are divided utilizing the Singular Value Decomposition (SVD) and the breadth-first search method. It was shown how adding virtual measurements can enlarge the group of observable nodes. By analysing the Jacobian matrix of the enlarged group, the breakpoint of computable power flow can be identified. The proposed method was illustrated using a 16-bus test system, where the algorithm found the correct minimum number of sensors to be placed and all possible placements of measurements. Using the proposed algorithm, the dimensionality of the optimal placement problem could be reduced by a factor of three.

Classical optimal placement methods are based on solving a complex mathematical discontinuous optimization and problem. The here proposed method utilizes the network topology and technical arguments to determine possible placement settings. The outcome of this algorithm could then be used to overcome the computational complexity of classical placement methods. The clear separation into observable and unobservable nodes and the subsequent algorithmic enlargement of the observable groups can help to make the network observable with fewer measurements, because nodes which do not contribute to the observability can be ignored.

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References

1. Heins, W., Ell, N., Beck, H.-P., Bohn, C.: State observation in medium-voltage grids with incomplete measurement infrastructure through online correction of power forecasts. In: Proceeding of European Control Conference (ECC) (2014)
2. Wu, J., He, Y., Jenkins, N.: A robust state estimator for medium voltage distribution networks. IEEE Trans. Power Syst. **28**(2), 1008–1016 (2013)
3. Korres, G., Xygkis, T., Manousakis, N.: Optimal location of measurement devices in distribution grids via Boolean convex optimization, Lyon, 15–18 June 2015
4. Jiang, C., Soh, Y.C., Li, H.: Sensor placement by maximal projection on minimum eigenspace for linear inverse problems. IEEE, June 2015
5. Ketabi, A., Hosseini, S.A.: A new method for optimal harmonic meter placement. ISSN 1546-9239
6. Abur, A., Expósito, A.G.: Power System State Estimation: Theory and Implementation. Marcel Dekker, New York (2004)
7. Wall, M.E., Rechtsteiner, A., Rocha, L.M.: Singular value decomposition and principal component analysis, pp. 99–109. LANL LA-UR-02-4001
8. Leiserson, C.E., Schardl, T.B.: A work-efficient parallel Breadth-First Search Algorithm (or how to cope with the nondeterminism of reducers) (PDF). In: ACM Symposium on Parallelism in Algorithms and Architectures (2010)