

Study on Electromagnetic Scattering Characteristics of Bodies of Revolution by Compressive Sensing

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Abstract. Based on discrete wavelet transform (DWT), the discrete wavelet transform (DWT) is pre-processed on the basis of the Bodies of revolution-Method of Moments, and the underdetermined equation is constructed and solved by using the compressive perceptual method. In this method, non-zero lines are extracted from the sparse excitations of the wavelet coefficients, and a small-scale impedance matrix is formed to extract the impedance matrices, which reduce the memory consumption and improve the computational efficiency. This method of adding compression perception can systematically construct the corresponding underdetermined equations to ensure fast acquisition of the signal reconstruction solution.

Keywords: Bodies of revolution-method of moments · Discrete wavelet transform · Compressive sensing

1 Introduction

As a current numerical methods for solving electromagnetic scattering problems, the method of moments [1] is widely used because of its high accuracy. Its core is to describe the interaction between the source and the field by the Green function. The moment method is used to solve the electromagnetic scattering problem, and basic procedure is expand the functional equation by means of the basis function, then the weight function is used to form the matrix equation. Bodies of revolution-method of moments [2–5] to solve the problem of electrical large size problems, need involve the filling and inversion of large scale dense impedance matrix. The calculation process is complex, the computer occupies big memory, so the wavelet analysis tool is introduced. Discrete wavelet transform [6–9] is a sparse preconditioning matrix equation, the rotational symmetric moment method with the addition of wavelet sparse transform. In the premise of maintaining high accuracy, it can effectively reduce the amount of memory. Based on this, the paper introduces compressed sensing [10–12]. The threshold of the excitation vector in the wavelet domain is established and can get no-zero line. The no-zero line is used as a priori to extract the impedance matrix in the

wavelet domain to get the equation. The sparse current solution is recovered by the OMP recovery algorithm. Finally, the real current coefficient vector is obtained by inverse wavelet transform. Results show that the rotational symmetric moment method based on compressive sensing can effectively recover the signal in the field scattering problem. At the same time it reduces the amount of memory and improves the computational efficiency.

2 Theoretical Analysis

2.1 Bodies of Revolution-Method of Moments

The Bodies of Revolution-Method of Moments (BOR-MOM) is mainly used to deal with the electromagnetic scattering from the target of (Bodies), BOR and the target of rotational symmetry. The BOR has the characteristics of rotational symmetry of geometry and constitutive relation. Structural basis functions are constructed by using its structural characteristics. The vector integral equation is transformed into two scalar integral equations, which can reduce the dimension of the vector integral equations.

On the surface of BOR, if the exciting incident magnetic field H^i , then the induction current conductor surface for \vec{J} . According to the rotation symmetry property can be decomposed into:

$$\vec{J} = \vec{J}^t + \vec{J}^\phi \quad (1)$$

Among them, \vec{J}^t is the tangential component of current \vec{J} , \vec{J}^ϕ as a component of current azimuth $\vec{\phi}$. By means of Fourier series expansion and the integral equation of the magnetic field into the surface of the component:

$$\begin{aligned} \vec{n} \times \vec{H}^{inc} = & \left(\sum_{n=-\infty}^{\infty} \sum_{i=1}^N I_{ni}^t \frac{T_i}{\rho} e^{jn\phi} \vec{t} + \sum_{n=-\infty}^{\infty} \sum_{i=1}^N I_{ni}^\phi \frac{T_i}{\rho} e^{jn\phi} \vec{\phi} \right) + \vec{n} \\ & \times \iint_s \left\{ \left(\sum_{n=-\infty}^{\infty} \sum_{i=1}^N I_{ni}^t \frac{T_i}{\rho} e^{jn\phi} \vec{t} + \sum_{n=-\infty}^{\infty} \sum_{i=1}^N I_{ni}^\phi \frac{T_i}{\rho} e^{jn\phi} \vec{\phi} \right) \times \nabla G \right\} dS' \end{aligned} \quad (2)$$

In which, $\nabla G = (\vec{r} - \vec{r}') [1 + jkr] \frac{e^{-jkr}}{4\pi r^3}$, I_{ni}^t , I_{ni}^ϕ said t and ϕ component to calculate the current coefficient, T^i for trigonometric function, ρ is the distance between the two segmentation points of subdivision surface.

This paper use the Galerkin's method, and according to the method of moments, the matrix can be used as follows:

$$ZI = V \quad (3)$$

Z is the impedance matrix, I is unknown current coefficient matrix, V is the excitation matrix.

2.2 Discrete Wavelet Transform Theory

To (3) carry on the wavelet transform, we can get:

$$\tilde{Z}\tilde{I} = \tilde{V} \quad (4)$$

Which, $\tilde{Z} = WZW^H$, $\tilde{I} = WI$, $\tilde{V} = WV$, and W is an orthogonal array constructed by the discrete wavelet transform. $WW^H = U$, U is a unit matrix, W^H is the conjugate transpose matrix of W .

In the wavelet domain, σ make the threshold. If the value of the matrix element is smaller than the threshold value is 0,

$$\sigma = \tau \|\tilde{Z}\|_1 / N = \tau \times \max_m \sum_n |\tilde{Z}(m, n)| / N \quad (5)$$

Among them, N is the dimension of the matrix, τ is the control variable to control the impedance matrix is sparse.

2.3 Compressive Sensing (CS)

Known (4) type \tilde{V} is sparse in a wavelet domain excitation vector, \tilde{V} and \tilde{Z} also selected in the important position of the M line. The formation of small scale matrix $\tilde{V}_{M \times 1}^{CS}$ and $\tilde{Z}_{M \times N}^{CS}$ respectively into underdetermined equations:

$$\tilde{Z}_{M \times N}^{CS} \tilde{I}_{N \times 1} = \tilde{V}_{M \times 1}^{CS} \quad (M \ll N) \quad (6)$$

Which, $\tilde{I}_{N \times 1}$ is a sparse vector in wavelet domain, according to the theory of compressive sensing, $\tilde{V}_{M \times N}^{CS}$ can think of is known to be measured for sparse vector $\tilde{I}_{N \times 1}$ value, And \tilde{V} in the position where the zero vector elements can be regarded as a priori knowledge, it's used to extract the specific location of the line in \tilde{Z} , the measurement matrix is constructed so as to form a fixed $\tilde{Z}_{M \times N}^{CS}$, underdetermined equations.

$\tilde{I}_{N \times 1}$ use OMP technology to reconstruct the recovery. Then the formula (7) inverse wavelet transform format to obtain the true coefficient of current vector $I_{N \times 1}$

$$I = W^H \tilde{I} \quad (7)$$

The computational complexity of the iterative solution of the traditional moment method is $O(PN^2)$, Where P is the number of iterations. The computational complexity of (6) is $O(SMN)$, which is $S \ll P$, $M \ll N$, and $\tilde{Z}_{M \times N}^{CS}$ build a sparse measurement matrix calculation will further reduce the complexity of OMP. Therefore, the method proposed in this paper can reduce the CPU memory loss and increase the speed of computation compared with the traditional method.

3 Instance Verification

Cases 1. The magnetic field integral equation calculation of radius R as the ideal conductor sphere 1 m, the current solution of the incident wave frequency is 300 MHz, the bus section sphere was divided into 257 segments. This example uses the Db8 wavelet to carry on the sparse processing, under the condition of selecting the excitation threshold 0.01, can get sparse excitation vector $\tilde{V}_{512 \times 1}$, the non-zero element number is $K = 64$, As shown in Fig. 1. According to the diagram (a) for no-zero position of the impedance matrix $\tilde{Z}_{512 \times 512}$. As a Fig. 2 extraction of $M = K = 64$ lines, the construction of a 64×512 small scale impedance matrix, Fig. 3. From the Figs. 4 and 5, we can see that the use of the traditional method of moments, wavelet sparse method and the CS method of the current solution is basically consistent, and the reconstruction error of CS is less than 1%. Using the method proposed in this paper, the current solution is obtained at 0° and 90° respectively.

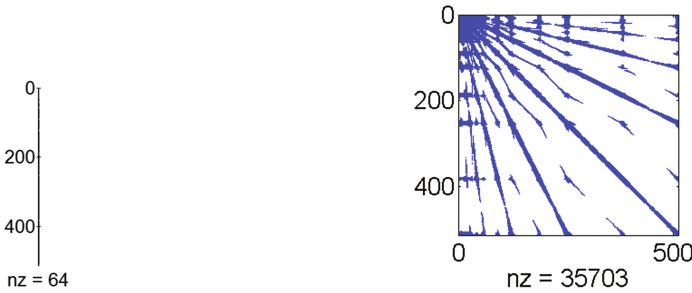


Fig. 1. The non-zero element line

Fig. 2. Non-zero elements position (shadows)

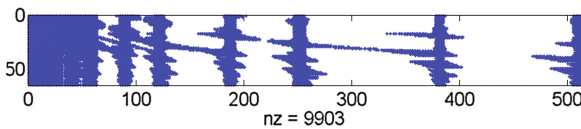


Fig. 3. Non-zero elements in the row of extraction

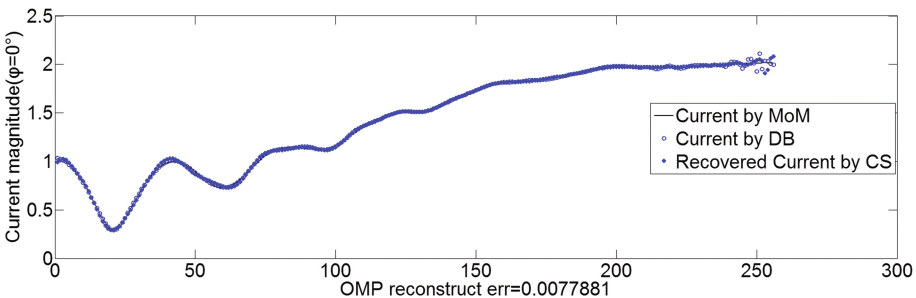


Fig. 4. When the incident angle is 0° , the current solution to the tangential direction of the plane

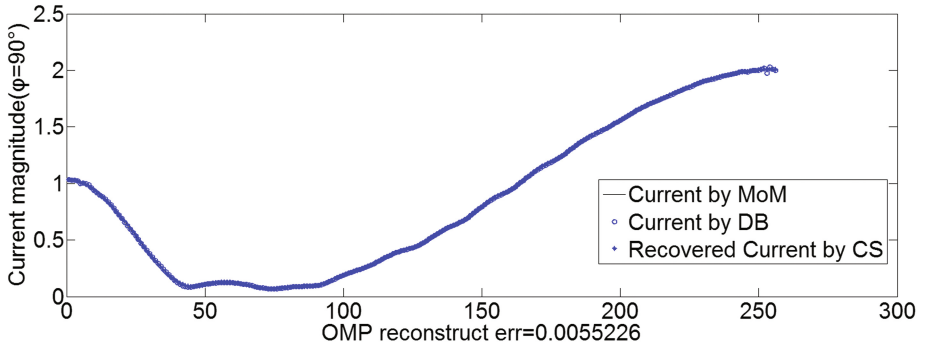


Fig. 5. When the incident angle is 90° , the current solution of the direction of the azimuthal direction of the plane.

Case 2. At a radius of 5 m, the cone angle is 60° on the ideal cone sphere of incident 300 MHz plane wave. The cone ball bus is divided into 257 sections, including angle bus into $N_1 = 128$ segments, and spherical segment bus into $N_2 = 129$ segments. This example also uses the DB8 wavelet sparse. After sparse extraction to get non zero line $K = 245$, $M = K = 245$ line extraction, it can construct a small scale impedance matrix of 245×512 . Using the method proposed in this paper, the current solution is obtained at 0° and 90° respectively. The reconstruction error of the visible CS is less than 1%, as shown in Figs. 6 and 7:

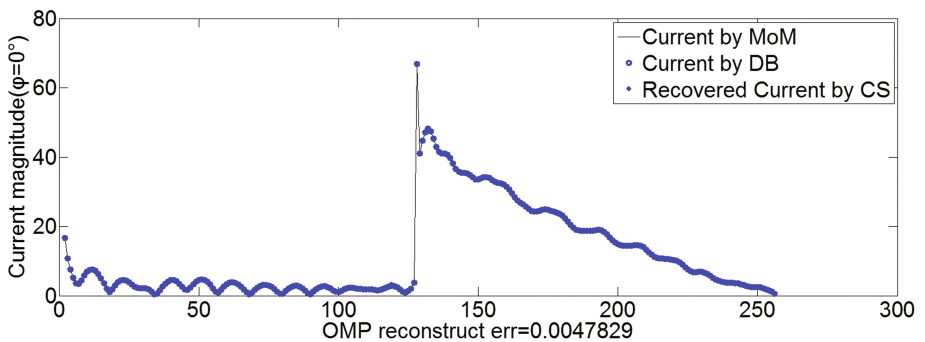


Fig. 6. When the incident angle is 0° , the current solution to the tangential direction of the plane

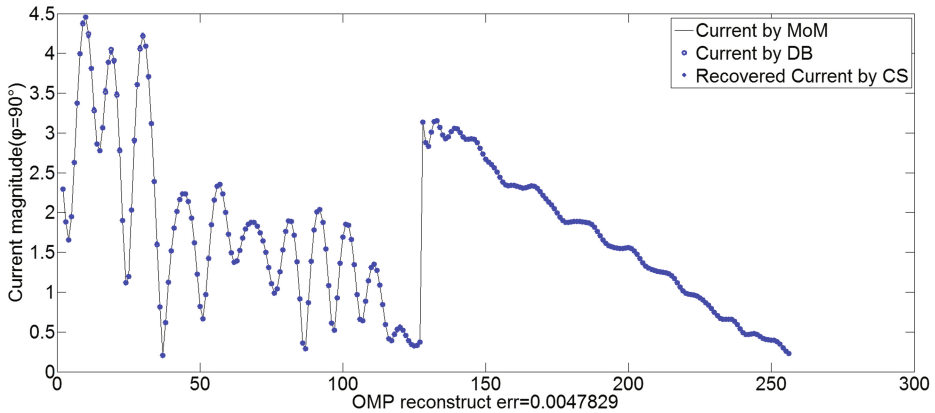


Fig. 7. When the incident angle is 90° , the current solution of the direction of the azimuthal direction of the plane.

4 Conclusion

In this paper, a rotational symmetry method based on compressive perception is presented. By analyzing the electromagnetic scattering properties of the two-dimensional half-targets, it is verified that the new method has more memory and higher precision than the traditional method, high probability reconstruction signal and other characteristics. The method is also applicable to the analysis of electromagnetic scattering properties of other large-scale rotationally symmetric targets.

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