# On Relay Node Selection for Multi-relay Cooperative Communication in Cellular Networks

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**Abstract.** This paper introduces a relaying scheme of combining cellular networks to establish the relay selection mechanism in the heterogeneous cooperative network model, then based on the system capacity analysis, propose an optimal relay selection scheme with the constraint of the specific outage probability and power allocation. And the multi-hop nodes can act as cooperative mobile relays to ensure the normal communication, to save costs and energy consumption caused by deploying a large number of fixed relays. Simulation results show that the proposed relay node selection algorithm can guide how to dispose the fixed number of relays in the cell and how to arrange these relays to improve the performance of the ergodic capacity of the system.

Keywords: Wireless communication  $\cdot$  Cooperative communication  $\cdot$  Channel capacity  $\cdot$  Multi-relay  $\cdot$  Relay node placement

## 1 Introduction

The wireless communication systems are required to provide more high-rate multimedia services and data services than ever before, so the purpose of the cooperative communication is to make full use of the resource [1], and to improve the performance of the node which has the demand of communication with participating into the high-speed and reliable wireless communication [2]. Many contributions have been made by previous researchers on how to choose suitable cooperative relay nodes, the reference [3] mainly introduced the multi-relay cooperative communication into the existing cellular structure and it is considered as one of the most practical improvements under the demand of high rate and high coverage. The work in [4] studied the joint optimization of the relay position and power allocation according to the channel capacity as the performance indicator in the multi-relay cooperative communication. And the reference [5] calculated the exact capacity of multi-relay multiuser cooperative networks based on two-step relay selection scheme. And it advocated the capacity analysis of an amplify-and-forward cooperative communication system model in multi-relay multiuser networks. Determining the number of relay nodes is also a hot issue. Therefore, in this paper, we study the joint optimization of the number and placement of relay nodes according to the ergodic capacity of the multi-relay cooperative system. According to the way of relay nodes processing and forwarding information, the relays can be sorted into Amplify-and-Forward (AF) and Decode-and-Forward (DF) [6].

For the cell in which relay stations are deployed, the number and placement of relay nodes will greatly impact the capacity of the network [7]. On the basis of the former research [8, 9] of the instantaneous capacity, we consider AF in our system model, constitute a cooperative communication model of a multiuser cellular relay network, and investigate the joint optimization problem of the number and placement of relay nodes for multi-relay cooperative communication system, and then we analyze the relationship between the channel ergodic capacity and the number and placement of the relay nodes.

## 2 Relay Model

The earliest model of the cooperative communication is a three-node model [10], followed by the continuous emergence of multi-relay parallel transmission model and multi-hop model. Due to the different locations of users, the communication between the mobile terminal and the base station can be achieved by the assistance of one or more relay stations.

We consider a homogeneous isotropic unitary cell [11] of circular structure as shown in Fig. 1. The system which is fixed to a certain place in the cell choosing a single relay node to communicate with the base station can be treated as a three-node cooperative communication model. Therefore, in order to simplify the notations in the down-link scenario [12, 13], we respectively use the source node *S*, the relay node *R* and the destination node *D* to stand for the base station *BS*, the wireless access point *AP* or the mobile station *MS*. The radius of the cellular circle is *L*, the center of the circle is the location of the base station *S*, and there are *M* relays  $R_m$  (m = 1, 2, ..., M) located in



Fig. 1. A homogeneous isotropic unitary cell

the cell. At the same time, N mobile stations  $D_n$  (n = 1, 2, ...N) are also uniformly distributed in the same area [14]. We consider the symmetry of the circular cell and the uniform distribution of the mobile terminals, and a large number of literature [15, 16] demonstrated by simulation that we can gain the optimal system performance of the cell in which relays are uniformly distributed when the base station is the center of the circular cell.

### 3 The Selection Algorithm of Relay Nodes

#### 3.1 The Relay Model

The corresponding instantaneous  $SNR_S$  of the links of S to  $D_n$ , S to  $R_m$  and  $R_m$  to  $D_n$  are denoted by  $\gamma_{S,D_n}$ ,  $\gamma_{S,R_m}$  and  $\gamma_{R_m,D_n}$ . Then  $\gamma_{S,D_n} = \frac{P_S}{N_O} |h_{S,D_n}|^2$ ,  $\gamma_{S,R_m} = \frac{P_S}{N_O} |h_{S,R_m}|^2$  and  $\gamma_{R_m,D_n} = \frac{P_R}{N_O} |h_{R_m,D_n}|^2$ .  $\overline{\gamma}_{S,R_m} = \frac{P_S}{N_O} E(|h_{S,R_m}|^2)$  and  $\overline{\gamma}_{R_m,D_n} = \frac{P_R}{N_O} E(|h_{R_m,D_n}|^2)$  are the means of  $\gamma_{S,R_m}$  and  $\gamma_{R_m,D_n}$  [17].

When the source node S send information to the destination node  $D_n$  directly, the direct channel capacity [18]  $C_{D_n}$  is

$$C_{D_n} = \log_2(1 + \gamma_{S,D_n}). \tag{1}$$

The multi-node cooperative transmission instantaneous capacity using AF protocol  $C_{AF}$  can be written as

$$C_{AF} = \frac{1}{2} \log_2(1 + \gamma_{S,D_n} + \gamma_{S,R_m,D_n}).$$
 (2)

Where  $\gamma_{S,R_m,D_n}$  is the instantaneous equivalent *SNR* of the forwarding path [5], which can be written as

$$\gamma_{S,R_m,D_n} = \frac{\gamma_{S,R_m} \gamma_{R_m,D_n}}{\gamma_{S,R_m} + \gamma_{R_m,D_n} + 1}.$$
(3)

Therefore, the AF cooperative state ergodic capacity is

$$\overline{C_{AF}} = E_h[C_{AF}] = E_{\gamma_{S,D_n},\gamma_{S,R_m},\gamma_{R_m,D_n}} [\frac{1}{2} \log_2(1 + \gamma_{S,D_n} + \gamma_{S,R_m,D_m})].$$
(4)

 $\gamma_{S,R_m}$  and  $\gamma_{R_m,D_n}$  are two random variables satisfying the exponential distribution, so we assume the parameters are  $\lambda_{S,R_m}$  and  $\lambda_{R_m,D_n}$ . When the *SNR* is high,  $\gamma_{S,R_m,D_n}$  is still an approximate exponential random variable [19], so we set the parameter is  $\lambda_{S,R_m,D_n}$ .

$$\lambda_{S,R_m,D_n} = \lambda_{S,R_m} + \lambda_{R_m,D_n} \tag{5}$$

Let  $\overline{\gamma}_{S,R_m,D_n} = \frac{1}{\lambda_{S,R_m,D_n}} = \frac{1}{d_{S,R_m}^2 \overline{P_S} + d_{R_m,D_n}^{R_m} \overline{P_S}}$ . According to the formula (4) and Jensen inequality, we can obtain the upper bound for the AF cooperative state ergodic capacity which is:

$$\overline{C}_{AF} \le \frac{1}{2} \log_2(1 + \overline{\gamma}_{S,D} + \overline{\gamma}_{S,R_m,D_n}) = \frac{1}{2} \log_2(1 + \frac{1}{d_{S,D_n}^{\alpha} \frac{N_O}{P_S}} + \frac{1}{d_{S,R_m}^{\alpha} \frac{N_O}{P_S}} + d_{R_m,D_n}^{\alpha} \frac{N_O}{P_R}).$$
(6)

So the under link state ergodic capacity of multi-user [19] can be expressed as

$$\overline{C} = \begin{cases} \frac{1}{2} \log_2 \left( 1 + \frac{1}{d_{s,D_n}^{\alpha} \frac{N_0}{P_s}} + \frac{1}{d_{s,R_m}^{\alpha} \frac{N_0}{P_s} + d_{R_m,D_n}^{\alpha} \frac{N_0}{P_R}} \right), \ (l,\theta) \in S_m (m = 1, \dots M); \\ \log_2 \left( 1 + \frac{1}{d_{s,D_n}^{\alpha} \frac{N_0}{P_s}} \right), \ (l,\theta) \in S_0 \end{cases}$$
(7)

Then we give the specific analysis of the scene of the circular cell. According to Fig. 1, we can obtain

$$\Omega_{S,D_n} = l^{-\alpha}, \Omega_{R_m,D_n} = d^{-\alpha}$$
  

$$\Omega_{S,R_n} = (l^2 + d^2 - 2ld \cos(\theta - 2\pi m/M))^{-\alpha/2}$$
(8)

By substituting formula (8) into formula (7), we can obtain the state erode capacity  $\overline{C}$  as

$$\overline{C} = \begin{cases} \frac{1}{2} \log_2 \left( 1 + P_S^{\prime l - \alpha} + \frac{1}{(l^2 + d^2 - 2ld \cos \theta)^{\alpha/2} \frac{N_0}{P_S} + d^{\alpha} \frac{N_0}{P_R}} \right), d_{th} < l < L; \\ \log_2 \left( 1 + \frac{P_S^{l - \alpha}}{N_o} \right), 0 \le l \le d_{th} \end{cases}$$
(9)

Because of the good symmetry of the circular cell and the hypothesis of the uniform distribution of all the users and relays, if we intend to calculate the expectation of the state ergodic capacity of all user's position in the cell, we only need to research on the sector coverage of one-single relay, then we can obtain the ergodic capacity of the whole cell by multiplying the number of relays *M*. The capacity of users in the interval is  $\overline{C}_{S/S_{AF}}(M, d, l, \theta), \theta \in [-\pi/M, \pi/M]$ . Based on the above formula (9), we can calculate the ergodic capacity of the system as

$$\widehat{C}(M,d) = E_{l,\theta}[\widehat{C}(M,d,l,\theta)] 
= M \begin{pmatrix} \int_{-\frac{\pi}{M}}^{\frac{\pi}{M}} \int_{0}^{d_{th}} \psi(l,\theta)\overline{C_{D}}(M,d,l,\theta) dld\theta \\ + \int_{-\frac{\pi}{M}}^{\frac{\pi}{M}} \int_{d_{th}}^{L} \psi(l,\theta)\overline{C_{AF}}(M,d,l,\theta) dld\theta \end{pmatrix} = \frac{2M}{\pi L^{2}} \begin{pmatrix} \int_{0}^{\frac{\pi}{M}} \int_{0}^{d_{th}} l(M,d,l,\theta) dld\theta \\ + \int_{0}^{\frac{\pi}{M}} \int_{d_{th}}^{L} l\overline{C_{AF}}(M,d,l,\theta) dld\theta \end{pmatrix}.$$
(10)

To sum up, the objective function of the joint optimization in the multi-relay amplify and forward cooperative communication is

$$\max_{M,d} \hat{C}(M,d). \tag{11}$$

The constraint conditions are 
$$\begin{cases} M \ge 3 \text{ and } M \in \mathbb{Z}^+ \\ 0 \le d \le L \end{cases}$$

#### 3.2 Performance Analysis of the Relayed System

We study on the impact of the relay number *M* and the relay radius *d* on the objective function  $\widehat{C}(M, d)$ , respectively.

(1) The monotonicity relationship between  $\widehat{C}(M, d)$  and M.

Taking the relay radius *d* as a constant,  $\widehat{C}(M, d)$  is a function whose variable is the relay number *M*. According to formula (11), we can obtain

$$\widehat{C}(M) = C_1 + C_2 M \int_0^{\pi/M} \int_{d_{th}}^L l \log_2(C_3 + [C_4(l^2 + d^2 - 2ld \cos \theta)^{\alpha/2} + C_5]^{-1}) dl d\theta.$$
(12)

Where  $C_i(i = 1, ..., 5)$  can be regarded as constant values which have nothing to do with M, and  $C_i > 0$ .

Combining with formula (12) and the integral mean value theorem, we can obtain

$$\frac{\partial \widehat{C}(M)}{\partial M} = C_2 \int_0^{\frac{\pi}{M}} \int_{d_{th}}^{L} l \log_2 \left( C_3 + \left[ C_4 (l^2 + d^2 - 2ld \cos \theta)^{\alpha/2} + C_5 \right]^{-1} \right) dl d\theta$$
  

$$- C_2 \frac{\pi}{M} \int_{d_{th}}^{L} l \log_2 \left( C_2 + \left[ C_4 (l^2 + d^2 - 2ld \cos \frac{\pi}{M})^{\alpha/2} + C_5 \right]^{-1} \right) dl$$
  

$$= C_2 \frac{\pi}{M} \int_{d_{th}}^{L} l \log_2 \left( C_3 + \left[ C_4 (l^2 + d^2 - 2ld \cos \theta)^{\alpha/2} + C_5 \right]^{-1} \right) dl$$
  

$$- C_2 \frac{\pi}{M} \int_{d_{th}}^{L} l \log_2 \left( C_2 + \left[ C_4 (l^2 + d^2 - 2ld \cos \frac{\pi}{M})^{\alpha/2} + C_5 \right]^{-1} \right) dl$$
  
(13)

Where  $\xi \in (0, \pi/M)$ , because  $M \ge 2$ , there is  $0 < \xi < \frac{\pi}{M} \le \frac{\pi}{2}$ . So  $\frac{\partial C(M)}{\partial M} > 0$ , namely, the ergodic capacity of the system is the monotonically increasing function about M.

(2) The monotonicity relationship between  $\widehat{C}(M,d)$  and d.

Taking the relay number M as a constant,  $\widehat{C}(M, d)$  is a function whose variable is the relay radius *d*. Here in order to simplify the derivation, we set  $\phi(d) = d$ ,  $P'_S = P'_R = P$ ,  $\alpha = 4$ . And according to formula (10), we can obtain

$$\widehat{C}(d) = \frac{2}{L^2} \int_0^d l \log_2(1+Pl^{-4}) dl + \frac{1}{L^2} \int_d^L l \int_0^{\frac{\pi}{M}} \log_2\left(1+Pl^{-4} + \frac{P}{(l^2+d^2-2ld\cos\theta)^2+d^4}\right) d\theta dl$$
(14)

Because the derivation process of  $\widehat{C}(d)$  about d is very complicated, here the conclusion after the derivation; is given directly as

$$\frac{\partial^2 \widehat{C}(d)}{\partial d^2} < 0. \tag{15}$$

According to formula (15), we can know that the figure of  $\widehat{C}(d)$  is convex arc which increases first and then decreases. When  $\frac{\partial \widehat{C}(d)}{\partial d} = 0$ , we can obtain the optimal position (namely the maximum ergodic capacity) of the relay node  $d^*(M)$  based on the fixed relay numbers.

Here we give the following conclusions:  $\widehat{C}(M, d)$  is a monotonically increasing function about *M*. For any *M*, there is only one optimal position  $d^*(M)$  corresponding to the maximum ergodic capacity  $C^*(M)$ , namely

$$\frac{\partial \widehat{C}(M,d)}{\partial d} = 0 \Rightarrow d * (M) \Rightarrow \widehat{C}(M) = \widehat{C}(M,d * (M)).$$
(16)

(3) The determination of the optimal relay number M and relay radius d.

The following inspection is the enhancement of the percentage of the capacity based on the relay number M, namely, to fix the relay radius d, and we compare the corresponding ergodic capacity  $\widehat{C}(M, d)$  when the relay number is M with the corresponding ergodic capacity  $\widehat{C}(M - 1, d)$  when the relay number is M - 1. We calculate the increasing percentage taking the cost consideration K(M) into account when we increase the relay number.

$$K(M) = \frac{\widehat{C}(M,d) - \widehat{C}(M-1,d)}{\widehat{C}(M-1,d)}$$
(17)

Taking the cost consideration into account when we increase the relay number, and combining with the experience in the project application, we determine the threshold  $\varepsilon$  of the increasing percentage of the ergodic capacity as the relay number *M* increases. Then according to the following formulas, we determine the optimal M\* and d\* as

$$M* = \arg\left\{\min_{M} \left(K^{M}(M)\right) \le \varepsilon\right\}$$
(18)

$$\frac{\partial \widehat{C}(M^*, d)}{\partial d} = 0 \Rightarrow d * (M^*)$$
(19)

#### **4** Numerical Results and Analysis

In order to have an intuitive knowledge of the relationship between the given exact value of the ergodic capacity and the upper bound, we use  $P_T$  to denote the summation of  $P_S$  and  $P_R$ . Figures 3 and 4 respectively give the comparison between the upper bound of ergodic capacity and numerical integration for  $P_S = P_R = P_T/2$  and  $P_S = P_R = 2P_T/3$ .

As shown in Figs. 2 and 3, we can see that the numerical curves and the simulation curves of the state ergodic capacity are in good agreement and very close to the upper bound under any channel variance and power allocation. Therefore, according to the upper bound of formula (5), we can obtain relatively accurate state ergodic capacity of multiuser.

In Fig. 4 we give the curves of the increasing percentage of the ergodic capacity  $\hat{C}(M, R_r)$  about the relay number M when different relay nodes place at different radius  $R_r$ . The radius of the cell L = 1000 m,  $N_O = 100$  dBm, the path loss exponent  $\alpha = 4$ , and we set  $P_S$  and  $P_R$  as a same fixed value, namely,  $P_S = P_R = 30$  dBm. As for the relay radius, we respectively choose three typical value of  $R_r = 100$  m near the station,  $R_r = 100$  m near the edge of the cell and  $R_r = 320$  m near the optimal position.



Fig. 2. Comparison between the upper bound of ergodic capacity and numerical integration for  $P_S = P_R$ 

As can be seen from the Fig. 4, regardless of the position of the relay node, the increasing percentage of the ergodic capacity decreases when M increases. When M is small, the ergodic capacity increases largely, but when M increases to a certain extent, and then add M, the increasing percentage of the ergodic capacity approaches to zero. In addition, when the relay number M is determined, the relay radius also plays a significant role.



Fig. 3. Comparison between the upper bound of ergodic capacity and numerical integration for  $2P_S = P_R$ 



Fig. 4. The promoting capacity ratio versus M with various  $R_{rs}$  for  $S/S_{AF}$ 

## 5 Conclusion

This paper investigates the joint optimization problem of the number and placement of relay nodes based on the ergodic capacity of multiuser downlink, when multiuser use AF relay mode for downlink communication in a circular relay cell. We give the numerical expression of the multiuser downlink ergodic capacity, based on which we analyze the relationship between the ergodic capacity and the number and placement of relay nodes, the conclusions are as follows: (1) When the number of relay nodes is small, increasing the quantity of relay nodes can greatly increase the ergodic capacity, while the number of relay nodes is large, the performance improvement of the cell is slight for the further increasing of the quantity of relay nodes. (2) When the relay number is given, we can determine the optimal placement of the relay node by the algorithm and the simulation. Finally, we propose a method to determine the optimal number and placement of relay nodes, and provide an analysis and a theoretical basis for how to dispose the fixed number of relays in the cell and how to arrange these relays to improve the performance of the ergodic capacity of the system.

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