Distributed Compressive Sensing for Correlated Information Sources

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Abstract. The abstract should summarize the contents of the paper and should Distributed Compressive Sensing (DCS) improves the signal recovery performance of multi signal ensembles by exploiting both intra- and inter-signal correlation and sparsity structure. In this paper, we propose a novel algorithm, which improves detection performance even without a priori-knowledge on the correlation structure for arbitrarily correlated sparse signal. Numerical results verify that the propose algorithm reduces the required number of measurements for correlated sparse signal detection compared to the existing DCS algorithm.

Keywords: Compressive sensing · Distributed source coding · Sparsity · Random projection · Sensor networks

1 Introduction

Baron et al. [1] introduced Distributed Compressive Sensing (DCS), which exploits not just intra-, but also inter- correlation of signals to improve detection performance. In [1], they assumed Wireless Sensor Network (WSN) consisting of arbitrary number of sensors and one sink node, where each sensor carries out compression without cooperation of the other sensors and transmits the compressed signal to the sink node. At the sink node, it jointly reconstructs the original signals from the received signals. Here, a key of DCS is a concept of joint sparsity, defined as the sparsity of the entire signal ensemble. Three models have been considered as a joint sparse signal model in [1]. In the first model, each signal is individually sparse, and also there are common

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components shared by every signal vector, called common information. In the second model, all signals share supports, which means the locations of the nonzero coefficient. In the third model, although no signal is sparse itself, they share the large amount of common information, which makes it possible to compress and recover the signals using CS. The third model can be considered as a modified version of the first model. The second joint sparsity model (JSM-2) has been actively explored in many existing literatures [2–7]. A joint orthogonal matching pursuit (JOMP) for DCS was proposed to improve the target detection performance of MIMO radar [4]. A precognition matching pursuit (PMP) which used the knowledge of common support from Fr´echet mean was proposed to reduce the number of required measurements in WSNs [5]. DCS was shown to be feasible for a realistic wireless sensor WSNs by implementing on real commercial off-the-shelf (COTS) hardware with providing good trade-off between performance and energy consumption [6]. Exploiting common information across the multiple EGS signals, simultaneous orthogonal matching pursuit (SOMP) for DCS with learned dictionary was shown to provide accurate reconstruction with the reduced number of measurements [7]. However, to the best of authors' knowledge, the first model (JSM-1) has been studied relatively little. In addition, a limited ensemble of signals that have single common information is considered in most cases.

In this paper, we propose a generalized (GDCS). While the key idea of DCS [1] is that we can exploit common information during joint reconstruction process to achieve performance improvement, the key of the GDCS framework is that we can exploit not only conventional common information, but also partial common information newly defined in this paper. The proposed GDCS algorithm, therefore, can provide better performance than the DCS algorithm in [1] in a generalized, and practical signal environment.

The remainder of this paper is organized as follows. We summarize the background of CS briefly in Sect. 2. A novel detection algorithm is proposed to capitalize on the GDCS in a practical environment in Sect. 3. In Sect. 4, numerical simulations are provided. Conclusions are made in Sect. 5.

Before going further, some terminologies are clarified as follows.

- Full common information: the set of signal components that are measured by every sensor in a system.
- Partial common information: the set of signal components that are measured by a set of sensor set Π, where its cardinality is $1 < |\Pi| < J$. J is the number of sensors in a system.
- Innovation information: the set of signal components that are measured by a single sensor.
- DCS algorithm: Algorithm presented in [1] to exploit signal structure in the presence of full common information.

2 Compressive Sensing

In many cases, we can represent a real value signal $\mathbf{x} \in R^N$ as sparse coefficients with a particular basis $\Psi = [\psi_1, \ldots, \psi_N]$. We can write

$$\mathbf{x} = \sum_{n=1}^{N} \psi_n \varpi(n) \tag{1}$$

where $\varpi(n)$ is the n th component of sparse coefficients ϖ. Let assume $\|\varpi\|_0 = K$, where $\|\varpi\|_0$ is the number of nonzero elements in vector ϖ. In a matrix multiplication form, it can be represented as

$$x = \Psi\varpi \tag{2}$$

Including the widely used Fourier and wavelet basis, various expansions, e.g., Gabor bases [8] and bases obtained by Principal Component Analysis (PCA) [9], can be used as a sparse basis. For convenience, we use the identity matrix I for a sparse basis $\Psi\mathbb{Z}\mathbb{R}$. Without loss of generality, an arbitrary sparse basis can be easily incorporated.

Candes, Romberg, and Tao [10] showed that a reduced set of linear projections can contain enough information to recover a sparse signal, naming this framework as Compressive Sensing (CS). In CS, a compression is simply projecting a signal onto measurement matrix $\Phi \in R^{M \times N}$ where $M << N$ as follows.

$$\mathbf{y} = \Phi\mathbf{x} \ \mathit{where} \ \mathbf{y} \in R^M \tag{3}$$

This system is ill-posed, however, it can be reconstructed if the restricted isometry property (RIP) of Φ [10] is satisfied with an appropriate constant. According to [10], the original signal $\mathbf{x}\mathbb{R}\mathbb{Z}\mathbb{R}$ can be reconstructed by

$$\varpi_e = \arg\min\|\varpi\|_0 \ \ s.t. \ \ \mathbf{y} = \Phi\Psi\varpi \tag{4}$$

However, because of NP-hardness of l_0 minimization, we use l_1 minimization, paying more measurements [10] as a cost of a tractable algorithm.

$$\varpi_e = \arg\min\|\varpi\|_1 \ \ s.t. \ \ \mathbf{y} = \Phi\Psi\varpi \tag{5}$$

This approach is called Basis Pursuit. Contrary to l_0 minimization, we can solve l_1 minimization with bearable complexities, which is polynomial in N. In addition to Basis Pursuit, an iterative greedy algorithm can be used for finding the original signal. Orthogonal Matching Pursuit (OMP) [11] is the most typical algorithm.

3 Iterative Signal Detection with Sequential Correlation Search

In this section, we discuss a method that can exploit signal structure without any a priori-knowledge to improve the performance of signal recovery. This is a main obstacle of exploiting partial common information in practical implementation. To compare the requirement of a priori-knowledge of the DCS and the proposed GDCS, the problem formulation of the DCS model [1] is described adopting the same notations.

$$\mathbf{X} = [\mathbf{x}_1^T \ \mathbf{x}_2^T \ \ldots \mathbf{x}_J^T]^T \in R^{NJ} \tag{6}$$

$$\mathbf{Z} := [\mathbf{z}_C^T \, \mathbf{z}_1^T \ldots \ \mathbf{z}_J^T]^T \in R^{N(J+1)} \tag{7}$$

$$\mathbf{x}_j = \mathbf{z}_C + \mathbf{z}_j \ \ where\ j \in \Lambda \tag{8}$$

$$\bar{\Phi} := \begin{bmatrix} \Phi_1 & \Phi_1 & 0 & . & 0 \\ \Phi_2 & 0 & \Phi_2 & . & 0 \\ . & . & . & . & . \\ \Phi_J & 0 & 0 & . & \Phi_J \end{bmatrix} \in R^{JM \times (J+1)N} \tag{9}$$

$$\mathbf{Y} = \bar{\Phi}\mathbf{Z} \tag{10}$$

$$\mathbf{Z}_e = \arg\min \|\mathbf{W}_C \mathbf{z}'_C\|_1 + \|\mathbf{W}_1 \mathbf{z}'_1\|_1 + \ldots + \|\mathbf{W}_J \mathbf{z}'_J\|_1 \quad s.t.\ \ \mathbf{Y} = \bar{\Phi}\mathbf{Z}' \tag{11}$$

where $\mathbf{W}_C$ and $\mathbf{W}_j, j \in \Lambda$ are weight matrices, which could be obtained by [12]. Thanks to a joint recovery, improved recovery performance can be obtained compared to disjoint recovery.

Similarly, we can consider a case of the proposed GDCS model, in which a single partial common information is measured by a set of sensors $\Lambda \backslash \{1, 2, 3\}$. This case can be formulated as the following problem by using the proposed GDCS model.

$$\mathbf{X} = [\mathbf{x}_1^T \ \mathbf{x}_2^T \ \ldots \mathbf{x}_J^T]^T \in R^{NJ} \tag{12}$$

$$\mathbf{Z} := [\mathbf{z}_C^T \, \mathbf{z}_1^T \ \ldots \mathbf{z}_J^T]^T \in R^{N(J+1)},\ where\ \Pi = \Lambda \backslash \{1, 2, 3\} \tag{13}$$

$$\mathbf{x}_j = \begin{cases} \mathbf{z}_{i_j}, & if\ j \notin \Pi \\ \mathbf{z}_{C_\Pi} + \mathbf{z}_{i_j}, & else \end{cases} \tag{14}$$

$$\bar{\Phi} = \begin{bmatrix} 0 & \Phi_1 & 0 & 0 & 0 & . & 0 \\ 0 & 0 & \Phi_2 & 0 & 0 & . & 0 \\ 0 & 0 & 0 & \Phi_3 & 0 & . & 0 \\ \Phi_4 & 0 & 0 & 0 & \Phi_4 & . & 0 \\ . & . & . & . & . & . & 0 \\ \Phi_J & 0 & 0 & 0 & 0 & 0 & \Phi_J \end{bmatrix} \in R^{JM \times (J+1)N} \tag{15}$$

$$\mathbf{Y} = \bar{\Phi}\mathbf{Z} \tag{16}$$

$$\mathbf{Z}_e = \arg\min \|\mathbf{W}_{C_\Pi} \mathbf{z}'_{C_\Pi}\|_1 + \|\mathbf{W}_{i_1} \mathbf{z}'_{i_1}\|_1 + \ldots + \|\mathbf{W}_{i_J} \mathbf{z}'_{i_J}\|_1 \quad s.t.\ \ \mathbf{Y} = \bar{\Phi}\mathbf{Z}' \tag{17}$$

where $\mathbf{W}_{C_\Pi}$ and $\mathbf{W}_{i_j}, j \in \Lambda$ are weight matrices, which could be obtained by [12]. As shown above, to exploit partial common information, we have to find the sensor set for partial common information Π, in this case $\Lambda \backslash \{1, 2, 3\}$. Unfortunately, it is not straightforward to find this set. Since each sensor compresses an acquired signal without cooperation of other sensors, there is nothing we can do to determine the correlation structure in a compression process. In a recovery process, although we can find the correlation structure by an exhaustive search, it demands approximately 2^J number of searches, which is not practical. We, therefore, need a moderately complex algorithm that finds the correlation structure.

In this notion, a novel algorithm is proposed for finding a correlation structure, which means a sensor set measuring partial common information. The algorithm iteratively selects the least correlated signal so that we can approximate the sensor set for partial common information Π. For simplicity, we assume a joint sparse signal ensemble $\mathbf{X}$ with partial common information $\mathbf{z}_{C_\Pi}$, where $\Pi = \Lambda \backslash \{1, 2, 3\}$ as in (13). However, since we have no knowledge on the correlation structure, we cannot formulate the measurement matrix as in (15). Instead of that, we refine the correlation structure. Let's assume that the given signal ensemble $\mathbf{X}$ has partial common information, and the correlation structure is not known. We first consider that the given signal ensemble $\mathbf{X}$ has full common information only. Then, the recovery algorithm forcefully makes full common information, while this artificially made full common information is compensated in the innovation information part. For this reason, sensors that do not have partial common information would have more innovation information than the sensors that have partial common information. By using this intuition, we compare l_1 norm of the recovered innovation information part, and remove the sensor whose l_1 norm is maximum from the correlation structure. Repeating this, we can obtain the exact correlation structure.

Although this phenomenon is difficult to understand at the first glance, it is quite straightforward. We should note that the forcefully found full common information may have some relation with the real partial common information. Actually, the forcefully found full common information is likely to be similar to the partial common information to minimize l_1 norm of the solution vector. Then, if the sensor j is one of the sensors that measure the partial common information, a joint recovery process successfully divides the energy of the signal into a joint recovery part (the first column of $\bar{\Phi}$ in (9)) and a disjoint recovery part (the rest of the columns of $\bar{\Phi}$ in (9)). However, if the sensor j, is one of the sensors that do not have the partial common information, the innovation information of the existing DCS for the sensor $j \notin \Pi$ must be made to compensate the forcefully found full common information, causing increase in l_1 norm of the innovation information part.

Thus, it can be exploited only if forcefully found full common information is made to be similar to partial common information. If only a small number of sensors can measure partial common information, i.e., $|\Pi|$ is small, the forcefully found full common information is likely to be different from the partial common information. In this case, we cannot expect to find the sensor set Π based on the above observation. Therefore, in this paper, we assume that any partial common information can be measured by a sufficient number of sensors. This assumption can be justified by the fact that significant performance gain of the proposed GDCS framework can be achieved when a sufficient number of sensors measure partial common information.

Exploiting the above intuition, an iterative signal detection with a sequential correlation search algorithm which we call "GDCS algorithm" throughout this paper is proposed. It is assumed for simplicity that the number of measurements at each sensor is M. The concatenated received signal is denoted by $\mathbf{Y} \in R^{MJ}$. The GDCS algorithm consists of two phases, inner and outer phases respectively. In the inner phase, correlation structure is identified for a given common information by excluding a sensor index one by one from the candidate sensor set. In the outer phase, it determines whether it is going to stop searching a new common information or continue. The details of the proposed algorithm will be elaborated in the paper in preparation for journal publication.

4 Simulation Results

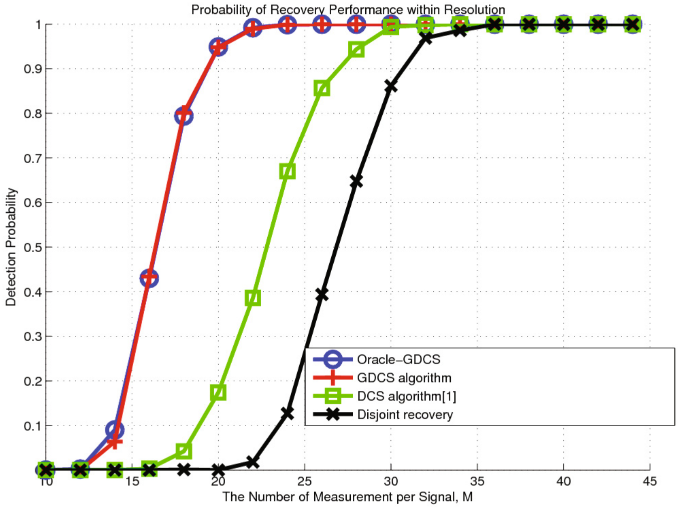
In this section, we demonstrate the GDCS through numerical experiments. Assuming various inter-signal correlations, we compare the detection performance of GDCS algorithm with Oracle-GDCS, which means GDCS with a priori-knowledge of correlation structure the DCS algorithm, and disjoint recovery.

The simulation environment is as follows. Each signal element is generated by an i.i.d. standard Gaussian distribution, and the supports are chosen randomly. The signal size N and the number of sensors J are fixed to 50 and 9, respectively. As aforementioned, the identity matrix is used as a sparse basis without loss of generality.

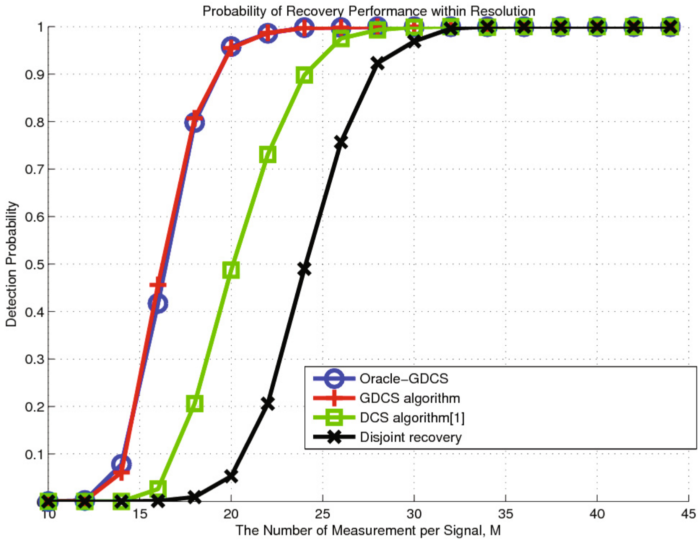
The measurement matrix is composed of i.i.d. Gaussian entries with a variance $1/M$. We assume a noiseless condition in all simulations. The types of common information and the sparsity of the information are determined as simulation parameters, and the corresponding sensors involved in the correlation are chosen randomly.

We use MATLAB as a simulation tool, and YALL1 solver is used for solving the weighted l_1 minimization. We use an iterative weighted l_1 minimization method introduced in [12] to obtain adequate weight matrices within a reasonable time. The probability of estimation error within the resolution is used as a performance measure where error is calculated by $\|\mathbf{X} - \mathbf{X}_e\|_2 / \|\mathbf{X}\|_2$ and the resolution is set to 0.1.

In Fig. 1(a) and (b), GDCS algorithm outperforms the DCS algorithm when there exists single partial common information while performing as well as oracle-GDCS. It reduces the required number of measurements by 23% and 18%. It is also noted that different performance is due to difference in sparsity of partial common information. We compare the consumed CPU time for GDSS with SCS and the DCS algorithm when the individual number of measurements are 25, 30, and 35. We average the CPU time over 100 different realizations with the simulation setting associated with Fig. 1(a) and (b). The CPU time is measured in seconds. In the (a) environment, the CPU time of the DCS algorithm are 1.42, 1.33, 1.32, respectively, while those of GDCS with SCS are 4.07, 3.92, 3.89, respectively. In the (b) environment, the CPU time of the DCS algorithm are 1.10, 1.08, 1.12, respectively, while those of the GDCS algorithm are 3.52, 3.30, 3.36, respectively. We can observe that the GDCS improves the performance with marginal increase in CPU time.



(a)



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(b)

Fig. 1. Performance comparison of Oracle GDCS, GDCS algorithm, DCS algorithm and disjoint recovery when (a) $K_{i_j} = 4$, $K_{C_\Pi} = 6$, $|\Pi| = 6$, (b) $K_{i_j} = 4$, $K_{C_\Pi} = 4$, $|\Pi| = 6$

5 Conclusions

In this paper, we proposed a new framework, a generalized version of the conventional one [1] so that it can be applicable to a more realistic environment. The proposed GDCS model refines the existing model so that it can exploit signal structure associated with partial common information in the joint recovery process. In this notion, we proposed GDCS algorithm to exploit this information in joint signal recovery without a priori-knowledge. Numerical simulation verifies that the proposed algorithm can reduce the required number of measurements compared to the DCS algorithm.

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