

Robust Spectral-Temporal Two-Dimensional Spectrum Prediction

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Abstract. With the development of mobile network, the limited spectrum resources are being running out of. Therefore, there is a harsh need for us to be able to know the current spectrum state as well as predict the future spectrum state. Though a number of studies are about spectrum prediction, some fundamental issues still remain unresolved: (i) the existing studies do not account for anomaly data, which causes serious performance degradation, (ii) they do not account for missing data, which may not hold in reality. To address these issues, in this paper, we develop a robust spectral-temporal spectrum prediction (R-STSP) framework from corrupted and incomplete observations. Firstly, we present data analytic of real-world spectrum measurements to analyze the impact of anomalies on the rank distribution of spectrum matrices. Then, from a spectral-temporal spectrum perspective, we formulate the R-STSP as a matrix recovery problem and develop an optimization method to efficiently solve it. We apply the formulated R-STSP to real-world VHF spectrum data and the results show that R-STSP outperforms state-of-the-art schemes.

Keywords: Spectrum prediction · Anomaly data · Missing data · Matrix completion and recovery

1 Introduction

The rapid development of mobile network is running out of the limited spectrum resource, which is a signal that we need more probable spectrum usage to adapt to this trend [1, 2]. To achieve this goal, we need to know the current spectrum state as well as predict the future spectrum state. Spectrum sensing helps us determines the current spectrum state using various signal detection methods [3–5], while spectrum prediction gives us the future spectrum data. Spectrum prediction’s applications in wireless networks has many merits, for example,

it increases system's throughput of spectrum access and reduces the time delay in spectrum sensing and so on.

As is mentioned above, spectrum sensing is usually used to obtain spectrum state. However, due to the limitations of hardware processing speed, the cost of equipment and network deployment cost in real world, we can just only get sparse spectrum data on time, frequency or space through spectrum sensing. On the other hand, the measured spectrum data analysis both at home and abroad [6, 7] indicates that any spectrum data does not exist in isolation. They have a close correlation in time, frequency and space dimensions. The sparsity of spectrum sensing data sample caused by the limitations of hardware processing speed, the cost of equipment and network deployment cost can be overcome by fully modeling, analyzing, mining and using the correlations in all dimensions and then predicting spectrum state. The current domestic and international researches on spectrum prediction have made staggered results. The early researches on spectrum prediction mainly focused on the time domain. With the deepening of the data analysis based on real-world spectrum measurements, spectrum correlation phenomenon (the relationship between different channels of spectrum) gradually attracts researchers' attention [8, 9] and spectrum prediction algorithm based on spectrum's frequency domain correlation also constantly emerges. The related researches can be found in [6, 10, 11].

There are mainly three challenges in spectrum prediction, namely anomalies, measurement errors and missing data. Anomalies and the wrong data are common, for the error in the process of electromagnetic wave transmission is unavoidable [12]. Data missing is also inevitable for three reasons. Firstly, the data missing in the transmission process is normal [13]. Secondly, the limitation of measuring equipment brings the fact that it is unrealistic to measure all the spectrum bands [14]. Thirdly, the existing measurement algorithm is not perfect [15].

In this paper, we consider the spectrum state matrix. The columns correspond to time slots and the rows correspond to spectrum bands. Next, we analyze real-world spectrum data to excavate the correlation structure between time slots and spectrum bands. Then from a two-dimensional perspective, we regard spectrum prediction as a matrix recovery optimization problem from incomplete and corrupted historical data. We develop an alternating direction optimization method to solve it. Finally, we apply the algorithm to real-world VHF spectrum data and the results show that it outperforms state-of-the-art schemes.

2 System Model

As stated before, we consider a spectrum matrix $\mathbf{X} \in \mathcal{R}^{F \times T}$. The rows correspond to frequency bands and the columns correspond to time slots. Each element $x_{f,t}$, $f \in \{1, \dots, F\}$, $t \in \{1, \dots, T\}$ represents the spectrum state in the t -th time slot on the f -th frequency band. Each row $\mathbf{x}_{f,\cdot} := [x_{f,1}, x_{f,2}, \dots, x_{f,T}]$, $f \in \{1, \dots, F\}$ represents the state evolution of T successive time slots over the f -th frequency band. Each column $\mathbf{x}_{\cdot,t} := [x_{1,t}, x_{2,t}, \dots, x_{F,t}]'$, $t \in \{1, \dots, T\}$ represents

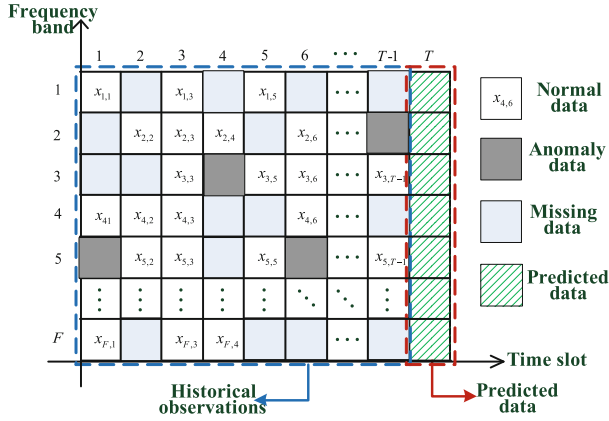


Fig. 1. System model.

the state distribution of F frequency bands in the t -th time slot. As is shown in Fig. 1, the abscissa axis represents time slot, the vertical axis represents frequency band. The data from column 1 to column $T - 1$ is historical data, among which there exist anomaly data and missing data. What we do is to predict the data in column T from a spectral-temporal 2D perspective by exploiting the relationships among historical data and predicted data. To achieve this objective, there are two critical issues:

- There are many factors contributing to practical spectrum data matrices, including signals, anomalies and noise.
- Unlike the conventional matrix completion or interpolation that elements are missing uniformly and randomly, an entire column of the matrix is known in the case of spectrum prediction.

As for the first issue, we consider the original dataset as a mixture of all these effects and then decompose the original spectrum matrix into a low-rank component, a sparse component and a dense noise component, which capture the major effects of signals, anomalies and noise, respectively. As for the second issue, we utilize some essential properties of spectrum matrices and add the time series forecasting into the matrix interpolation.

3 Analysis of Datasets

3.1 Real-World Spectrum Measurement Dataset

As shown in Fig. 2, in this paper we use a software defined radio NI USRP N2920 to perform real-world spectrum measurement in the basement of a 10-floor building. The frequency band spans from 50 MHz to 75 MHz with a frequency resolution 25 kHz. In total 1000 bands are measured and each band is measured 100 times. Therefore, the data size is 1000×100 . The spectrum measurement in terms of power spectral density (dbm/25 kHz) is shown in Fig. 3.

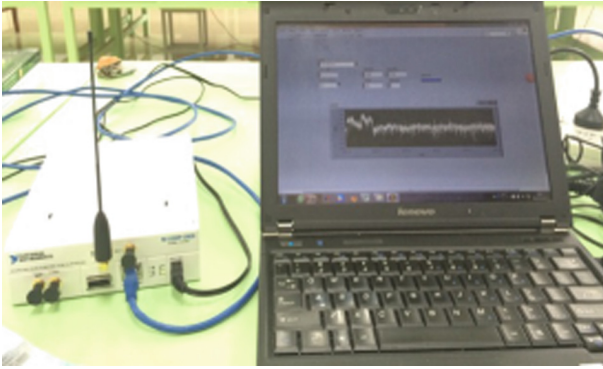


Fig. 2. A real-world spectrum measurement platform.

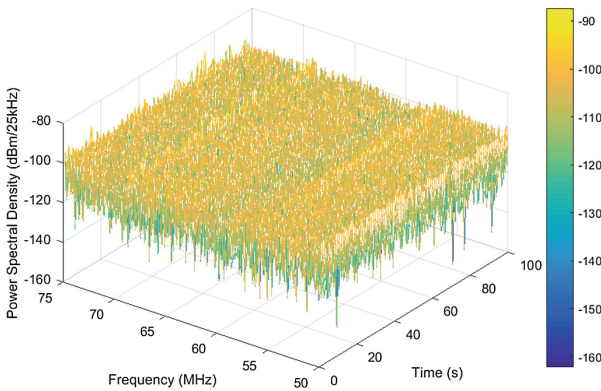


Fig. 3. The spectrum measurement data across various frequency bands and time slots.

3.2 Rank Analysis

From the knowledge of linear algebra we know that a higher correlation of a matrix generally means low rank. For each data matrix, we first make a processing by subtracting from each row its mean value. Then we apply singular value decomposition (SVD) to analyze the rank distribution of all mean-centered spectrum data matrices. In Fig. 4, we plot the normalized singular values in a descending order for VHF bands and for both the cases with and without anomalies. For comparison, we also analyze an i.i.d Gaussian random signal dataset.

From Fig. 4, it is suggested in the case of spectrum data matrices without anomaly, the energy is always contributed by the top several singular values in measured practical data matrices, which reflects the fact that practical spectrum data matrices show approximate low-rank structure, and this is quite different from the Gaussian random signal dataset. In the case with anomaly, we use the

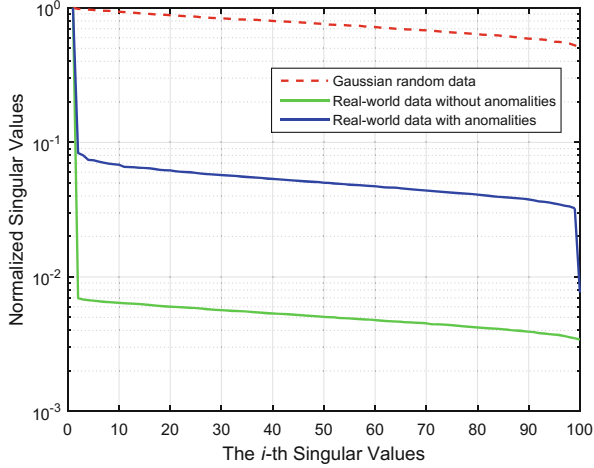


Fig. 4. Normalized singular values of spectrum datasets.

standard anomaly injection method [16] and inject anomalies to a portion of the entries in the original matrices. As a result, the presence of anomalies has a destructive effect on the approximate low-rank structure.

4 Problem Formulation and Algorithm Design

4.1 Problem Formulation

We use $x_{f,t}$ to express the measured spectrum data in the f -th frequency band over the t -th time slot, then we have

$$x_{f,t} = z_{f,t} + a_{f,t} + v_{f,t}, f = 1, \dots, F, t = 1, \dots, T, \quad (1)$$

where $z_{f,t}$ denotes the signal of interest, $a_{f,t}$ denotes the anomaly component and $v_{f,t}$ denotes the additive noise component. As for $z_{f,t}$, because the signal of interest is not always present, so we have

$$z_{f,t} = h_{f,t} \cdot p_{f,t} \quad (2)$$

where $h_{f,t}$ is a function indicating the presence or absence of the signal and $p_{f,t}$ is the signal strength in the t -th time slot and the f -th frequency band. If the signal is present, then $h_{f,t} = 1$. If the signal is absent, then $h_{f,t} = 0$.

Introduce the matrix $\mathbf{X}_T := [x_{f,t}]$, $\mathbf{Z}_T := [z_{f,t}]$, $\mathbf{A}_T := [a_{f,t}]$, $\mathbf{V}_T := [v_{f,t}] \in \mathbb{R}^{F \times T}$, then Eq. (1) can be further rewritten as follows:

$$\mathbf{X}_T = \mathbf{Z}_T + \mathbf{A}_T + \mathbf{V}_T \quad (3)$$

where \mathbf{Z}_T represents a low-rank signal component, \mathbf{A}_T a sparse anomaly component and \mathbf{V}_T a dense noise component. The low-rank property of the signal has

been observed from Fig. 4. The introduction of dense noise component makes the low-rank structure of $\mathbf{Z}_T + \mathbf{V}_T$ approximate and the injection of sparse anomaly component \mathbf{A}_T makes the approximate low-rank structure does not hold at more.

To further model missing data, introduce the operator $P_{\Omega_T}(\cdot)$, which sets the entries of its matrix argument not in Ω_T to zero, and keeps the rest unchanged, then the spectrum state data can be further given as

$$P_{\Omega_T}(\mathbf{X}_T) = P_{\Omega_T}(\mathbf{Z}_T + \mathbf{A}_T + \mathbf{V}_T) \quad (4)$$

As is stated before, the objective in this paper is to predict the data in column (*i.e.*, $\mathbf{x}_{\cdot,T}$) from a two-dimensional perspective, based on the incomplete and historical data $P_{\Omega_{T-1}}(\mathbf{X}_{T-1})$. Now this objective falls into the field of joint (low-rank) Matrix Completion and (sparse) Matrix Recovery (MCMR).

4.2 Algorithm Design

Consider the fact that the spectrum data in the T -th time slots are completely unknown, conventional MCMR methods cannot function well, so we first forecast a few frequency bands of large evolution regularity $f \in S_{LER}$. Specifically, for any band $f \in S_{LER}$, the forecast spectrum state is given as follows:

$$\bar{x}_{f,T} = \begin{cases} TSF(P_{\Omega_{T-1}}(\mathbf{X}_{T-1})) & f \in S_{LER} \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

where TSF stands for various time series forecasting functions. After studying the evolution trajectories of TV and ISM spectrum, we find that there are always several bands in each service that their spectrum evolution trajectories are highly predictable.

Based on $\bar{x}_{f,t}$, the spectrum matrix for further processing is as follows:

$$P_{\Omega_T}(\bar{\mathbf{X}}_T) = [P_{\Omega_{T-1}}(\mathbf{X}_{T-1}), \bar{\mathbf{x}}_{\cdot,T}] \quad (6)$$

In addition, a natural estimator leveraging the low-rank property of \mathbf{Z}_T and the sparsity property of \mathbf{A}_T attempts to fit the incomplete data $P_{\Omega_T}(\bar{\mathbf{X}}_T)$ to $\mathbf{Z}_T + \mathbf{A}_T$ in the least-squares error sense. Meanwhile, the estimator minimize the rank of \mathbf{Z}_T measured by its nuclear norm $\|\mathbf{Z}_T\|_*$ and the number of nonzero entries of \mathbf{A}_T measured by its l_1 norm $\|\mathbf{A}_T\|_1$. Therefore, we have

$$\min_{\mathbf{Z}, \mathbf{A}} \frac{1}{2} \|P_{\Omega_T}(\bar{\mathbf{X}}_T - \mathbf{Z} - \mathbf{A})\|_F^2 + \lambda_T^* \|\mathbf{Z}\|_* + \lambda_T^1 \|\mathbf{A}\|_1, \quad (7)$$

where rank-controlling parameter $\lambda_T^* \geq 0$ and sparsity-controlling parameter $\lambda_T^1 \geq 0$. In order to provide a effective resolution to the above problem, we face the following challenges: (i) This is a non-smooth optimization problem due to the fact that the nuclear and l_1 norms are not differentiable from the very beginning; (ii) The scale of the problem can easily become very large since the quantity of optimization variables $2 * F * T$ grows with time.

To address the above challenges, first we introduce a constraint that $\text{rank}(\hat{\mathbf{Z}}) \leq r$, where $\hat{\mathbf{Z}}$ is the estimate obtained in Eq. (6) and r is the upper bound rank of the signal part in Eq. (2). Next we factorize the matrix as $\mathbf{Z} = \mathbf{P}\mathbf{Q}'$ through a bilinear decomposition. \mathbf{P} and \mathbf{Q} are $F \times r$ and $T \times r$ matrices, respectively. Furthermore, consider the following alternative property of the nuclear norm [17, 18].

$$\|\mathbf{Z}\|_* := \min_{\mathbf{P}, \mathbf{Q}} \frac{1}{2} \{ \|\mathbf{P}\|_F^2 + \|\mathbf{Q}\|_F^2 \}, \text{ s.t. } \mathbf{Z} = \mathbf{P}\mathbf{Q}' \quad (8)$$

Apply Eq. (7) to Eq. (6) and we have

$$\arg \min_{\mathbf{P}, \mathbf{Q}, \mathbf{A}} \frac{1}{2} \|\Gamma_{\Omega_T}(\bar{\mathbf{X}}_T - \mathbf{P}\mathbf{Q}' - \mathbf{A})\|_F^2 + \frac{\lambda_T^*}{2} \{ \|\mathbf{P}\|_F^2 + \|\mathbf{Q}\|_F^2 \} + \lambda_T^1 \|\mathbf{A}\|_1, \quad (9)$$

Obviously, on condition that $\text{rank}(\hat{\mathbf{Z}}) \leq r$, the separable Frobenius-norm regularization in Eq. (8) does not damage the optimality relative to Eq. (6). So far, the optimization in Eq. (8) can be solved by the standard method introduced in [19].

5 Experiment Results

In this section, spectrum measurements are used to validate the effectiveness of the proposed robust spectral-temporal two-dimensional spectrum prediction (R-STSP) scheme over the joint (two-dimension) spectral-temporal spectrum prediction (J-STSP) scheme [20].

We quantify the spectrum prediction performance in terms of prediction error. Root mean square error (RMSE) in dB is used to quantify the prediction error, which is defined as:

$$\text{RMSE}(T) = 10 \log_{10} \left(\frac{\|\mathcal{P}_{\bar{\omega}_T}(\hat{\mathbf{z}}_T - \mathbf{z}_T)\|_2^2}{\|\mathcal{P}_{\bar{\omega}_T}(\mathbf{z}_T)\|_2^2} \right), \quad (10)$$

where \mathbf{z}_T and $\hat{\mathbf{z}}_T$ are the ground-truth and predicted spectrum data in the T -th time slot, respectively. $\bar{\omega}_T$, the complementary set of ω_T , contains the indices of missing/incomplete observations, while the corresponding sampling operator $\mathcal{P}_{\bar{\omega}_t}(\cdot)$ sets the entries not in $\bar{\omega}_t$ to zero, and keep the rest unchanged.

Figure 5 shows the cumulative distribution functions (CDFs) of RMSE in dB for the two schemes. It shows that: (i) the prediction performance of both J-STSP and R-STSP decrease with an increasing percentage of anomaly data; (ii) R-STSP always outperforms the J-STSP under different configurations; (iii) the prediction performance of the proposed R-STSP is improved with a decreasing percentage of anomaly data.

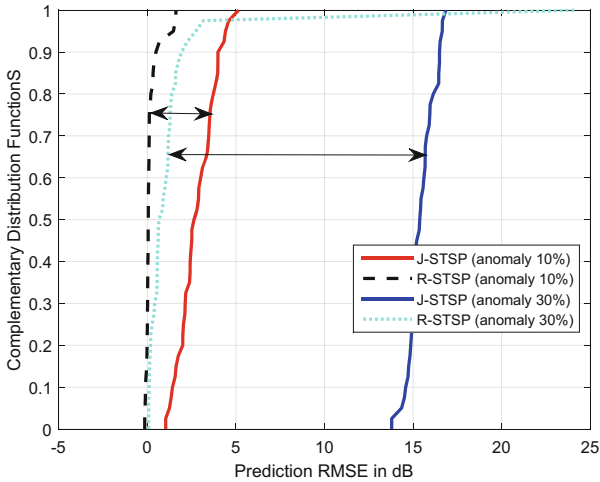


Fig. 5. Illustration of RSTSP with anomaly data.

6 Conclusion

This paper considered spectrum prediction as a matrix recovery optimization problem from incomplete and false historical data. We developed an optimization method to solve it. Finally, we apply the algorithm to real-word VHF spectrum data and the results show that R-STSP outperforms state-of-the-art schemes. One future work is to further develop online algorithms to perform real-time prediction and reduce the delay.

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