Research on LMMSE Channel Estimation Algorithm Using SLSM in WPM System

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Abstract. In wavelet packet modulation (WPM) system, the application of the linear minimum mean square error (LMMSE) channel estimation algorithm is limited by the nonlinearity of the fading WPM signal. To solve the problem, a simplified linear signal model (SLSM) matching with the LMMSE algorithm is established in the paper. The establishing of the SLSM is based on the fading channel and the orthogonality of WPM signals. The analysis and simulation results show that, the SLSM is matched with the LMMSE algorithm, and the SLSM based LMMSE algorithm can improve the WPM system performance effectively in frequency-selective fading environment.

Keywords: WPM system · Channel estimation · LMMSE algorithm · Simplified linear signal model

1 Introduction

Multicarrier scheme is a representative technique for high bit rate wireless communications, whose key advantages are multipath immunity, narrowband interference suppression and high band efficiency [1]. WPM system is a novel kind of multicarrier transmission method whose good time-frequency localization motivate a lot of current researches on it [2–5]. In WPM scheme, a packet structure can be divided not only in frequency domain but also in time domain and this unique division brings flexibility to wireless communication.

The wireless channel can be characterized as frequency-selective fading if the signal bandwidth is considerably wider than the coherence bandwidth; it brings a random gain for each scale of WPM signal and leads to selective fading in frequency domain. Generally, linear channel estimation method is used to compensate for this, the studies of channel compensation suitable for WPM system have been done in [6–9]. In [6, 7], least square (LS) algorithm based estimation method was proposed to improve the performance of the system in flat fading channel. In [8], parameters of multi-path Rayleigh fading channel can be obtained by using LS channel estimation and linear interpolation. In [9], Huber channel estimation algorithm was applied to achieve the best performance in non-Gaussian channel. However, LMMSE estimation algorithm,

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the most effective estimation method in frequency selective fading channel was not mentioned in these literatures. This is because in WPM, the inverse wavelet packet transform (IWPT) is employed for signal synthesis, which makes the relationship between the signal and the channel impulse response become more complex and this leads to the mismatch between the WPM signal model and the typical LMMSE method. As a consequence, LMMSE based channel estimation cannot be used in WPM system. Thus the SLSM which cooperates with the typical LMMSE method has been established in this paper. Theory and simulation results show that, the SLSM meets the using requirement of the LMMSE channel estimator, and the proposed estimators based on the SLSM provide significant performance gain for WPM system in frequency-selective channel.

The structure of the paper is organized as follows. After presenting the WPM system in Sect. 2, SLSM is established in Sect. 3. In Sect. 4, LMMSE estimation algorithm based on SLSM are put forward. The robustness of the LMMSE channel estimators based on the SLSM is tested in Sect. 5 and a summary and concluding remark appear in Sect. 6.

2 WPM System

2.1 WPM System Model

WPM is a kind of multicarrier transmission method [10]. The typical WPM system model is displayed in Fig. 1.

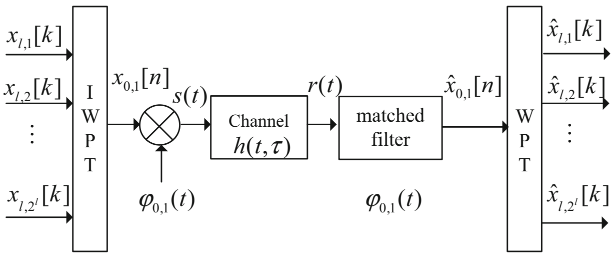

Fig. 1. Equivalent WPM base-band system model.

At the transmitter, the IWPT is applied to the input data $x_{l,m}[k]$ for synthesizing the transmitted symbols $x_{0,1}[n]$, each of which is individually amplitude modulated by scaling function $\varphi_{0,1}(t)$. Then, the synthesis WPM signal can be constructed by

$$\begin{aligned} s(t) &= \sum_{n} x_{0,1}[n]\varphi_{0,1}(t - nT_0) \\ &= \sum_{l \in \Lambda, m \in M_l} \sum_{k} x_{l,m}[k]\varphi_{l,m}(t - kT_l) \end{aligned} \tag{1}$$

where $\varphi_{l,m}(t)$ are orthogonal subcarriers and T_l is the symbol interval. The basic principle of WPM and the performance comparison of OFDM and WPM have been described in the literature [3]. They are very different in form, therefore the effective channel estimation method LMMSE can be used in OFDM but not suitable for WPM.

3 The SLSM for WPM in Frequency Selective Fading Channel

3.1 Wireless Channel Model

Assume a multipath fading channel consisting of N resolvable paths; the channel impulse response $h(t,\tau)$ can be closely approximated as

$$h(t,\tau) = \sum_{i=0}^{N-1} \alpha_i(t)\delta(\tau - \tau_i) \tag{2}$$

In (2), N is the total number of propagation path and $\delta(\cdot)$ denotes the impulse function. For the ith path, $\alpha_i(t)$ is the complex impulse response and τ_i is the path delay. In the case, the received signal at the receiver can be expressed as

$$\begin{aligned} r(t) &= s(t) * h(t,\tau) + w(t) \\ &= \int_0^{\tau_{\max}} s(t-\tau)h(t,\tau)d\tau + w(t) \end{aligned} \tag{3}$$

where $w(t)$ is an additive white Gaussian noise with zero mean value and variance σ_w^2, and we can safely assume the noise is uncorrelated with the channel impulse response.

3.2 The SLSM

For analysis, a special case of two-path frequency selective fading channel is taken into account. In the condition, the equivalent low-pass received signal is modeled as

$$r(t) = h_1(t)s(t) + h_2(t)s(t-\tau) + v(t) \tag{4}$$

where $s(t)$ is the multiplexed WPM signal defined in (1), $h_1(t)$ and $h_2(t)$ are channel gain which are mutually independent complex Gaussian processes. At the receiver, $r(t)$ is passed through the matched filter $\varphi_{0,1}(t)$ for sampling and matching firstly. Since $h_1(t)$ and $h_2(t)$ are slow processes and can be regarded as a constant in several bit duration, the output of the match filter is

$$\begin{aligned} r_s[n] &= \int r(t)\varphi_{0,1}(t-nT_0)dt \\ &= h_1[n]x_{0,1}[n] + h_2[n]\sum_n x_{0,1}[k]R_\varphi(nT_0 - kT_0 - \tau) + v[n] \end{aligned} \tag{5}$$

where $h_1[n]$ and $h_2[n]$ are the sampled values of $h_1(t)$ and $h_2(t)$ at instant nT_0, and $R_\phi(\cdot)$ denotes the autocorrelation function of scale function $\varphi_{0,1}(t)$.

Re-arrangement of the right hand of Eq. (5) results in

$$\begin{aligned} r_s[n] &= \big(h_1[n] + h_2[n]R_\varphi(-\tau)\big)x_{0,1}[n] \\ &\quad + \left\{ h_2[n]\sum_{k'\neq 0} x_{0,1}[n-k']R_\varphi(k'T_0-\tau) + v[n] \right\} \\ &= h_s[n]x_{0,1}[n] + v_s[n] \end{aligned} \tag{6}$$

where $h_s[n]$ and $v_s[n]$ can be expressed as

$$\begin{cases} h_s[n] = h_1[n] + h_2[n]R_\varphi(-\tau) \\ v_s[n] = h_2[n]\sum\limits_{k'\neq 0} x_{0,1}[n-k']R_\varphi(k'T_0-\tau) + v[n] \end{cases} \tag{7}$$

In (7), $v_s[n]$ represents the total noise including the Gaussian noise and the multi-path interference.

It is well known that LMMSE is more stable and accurate method. The applicable condition of the LMMSE is that the estimation value is the linear function of the observe value which can be modeled as

$$A = \boldsymbol{B}\theta + \boldsymbol{c} \tag{8}$$

where $\boldsymbol{c}$ is the noise with zero mean value and is irrelevant to θ. Obviously, the form of formula (6) does not meet the requirements, so it is not suitable for the algorithm LMMSE. Therefore, we need to simplify the formula (6), so that it can be used for LMMSE algorithm.

In frequency selective fading case, $\tau > T_0$. It is well known that $R_\varphi(\tau)$ has a good correlation ship. When $\tau > T_0$, $R_\varphi(-\tau)$ approaches to zero and $R_\varphi(0)$ is equal to one. Therefore, Eq. (5) can be transformed into the following form

$$r_s[n] = h_1[n]x_{0,1}[n] + h_2[n]x_{0,1}\left[n - \frac{\tau}{T_0}\right] + v[n] \tag{9}$$

Similarly, if the total number of propagation paths is L, and for the ith path, $\tau_i = iT_0$. At the same time assume that the channel is slow fading case, the received signal can be derived as follow

$$\boldsymbol{r}_f = \boldsymbol{X}\boldsymbol{h} + \boldsymbol{v} \tag{10}$$

where $\boldsymbol{v}$ is the AWGN with zero mean value and variance σ_v^2, and matrix $\boldsymbol{X}$ and vector $\boldsymbol{h}$ can be described as

$$\boldsymbol{h} = [h_0, h_1, \cdots, h_{L-1}]^T \tag{11}$$

$$\boldsymbol{X} = \begin{bmatrix} x_{0,1}[1] & x_{0,1}[N] & \cdots & x_{0,1}[N-L+2] \\ x_{0,1}[2] & x_{0,1}[1] & \cdots & x_{0,1}[N-L+3] \\ \vdots & \vdots & \ddots & \vdots \\ x_{0,1}[N] & x_{0,1}[N-1] & \cdots & x_{0,1}[N-L+1] \end{bmatrix} \tag{12}$$

The received signal model described in (10) is SLSM. From (11) and (12) we can find that, the relationship between the estimation value $\hat{\boldsymbol{h}}$ and the observe value $\boldsymbol{r}_f$ satisfies the condition described in (8), which denotes that the established SLSM meets the application condition of LMMSE algorithm.

4 LMMSE Channel Estimation Adopt SLSM

LMMSE uses the statistic properties of the channel coefficients and the additive noise to reduce the mean square error, and its estimation value comes from the minimum mean square error described as follow

$$\frac{\partial J}{\partial \hat{\boldsymbol{h}}} = \frac{\partial E\left[\left(\boldsymbol{h} - \hat{\boldsymbol{h}}\right)^T \left(\boldsymbol{h} - \hat{\boldsymbol{h}}\right)\right]}{\partial \hat{\boldsymbol{h}}} = 0 \tag{13}$$

Based on (13), the estimation value $\boldsymbol{h}$ can be described as follow

$$\begin{aligned} \hat{\boldsymbol{h}}_{LMMSE} &= \boldsymbol{\mu} + \boldsymbol{P}\boldsymbol{X}^T \left[\boldsymbol{X}\boldsymbol{P}\boldsymbol{X}^T + R\right]^{-1} \left[\boldsymbol{r}_f - \boldsymbol{X}\boldsymbol{\mu}\right] \\ &= \left[\boldsymbol{P}^{-1} + \boldsymbol{X}^T R^{-1} \boldsymbol{X}\right]^{-1} \left[\boldsymbol{P}^{-1}\boldsymbol{\mu} + \boldsymbol{X}^T R^{-1} \boldsymbol{r}_f\right] \end{aligned} \tag{14}$$

According to the SLSM established in (10), the $\boldsymbol{h}$, $\boldsymbol{r}_f$,$\boldsymbol{v}$ and $\boldsymbol{X}$ can be denoted as (11) and (12). In (14), $\boldsymbol{\mu}_{(L\times 1)}$ and $P_{(L\times L)}$ are the mean value and variance of $\boldsymbol{h}$, and the superscripts $[\cdot]^{-1}$ and $[\cdot]^H$ denote matrix inversion and Hermitian transpose respectively. $R = E\{\boldsymbol{v}\boldsymbol{v}^H\}$ is the auto-correlation matrix of $\boldsymbol{v}$. The result described in (14) has proved the matching between the proposed model and the LMMSE algorithm in theory.

5 Simulation and Results

Perfect synchronization has been done since the aim is to observe the channel compensation performance. The received signal model was shown as (10), and the simulation parameters were shown in Tables 1 and 2. In the experiment, Daubechies 12 wavelet was used to synthesize and decompose the WPM symbols, the channel was modeled as 6-ray model, and the corresponding amplitude and phase are defined in Table 2. As the channel was frequency selective fading, the block-type pilot was used [11].

Table 1. Simulation parameters

Wavelet pattern	Subcarriers number	Modulation scheme	Path number
Daubechies 12	M = 512	QPSK	L = 6

Table 2. The amplitude and phase of h

Delay time τ_i	Amplitute h_i	Phase ϕ_i
$\tau_1 = 0$	$\lvert h_0 \rvert = 1$	1.2567
$\tau_1 = T_0$	$\lvert h_1 \rvert = 0.6065$	0.6283
$\tau_1 = 2T_0$	$\lvert h_2 \rvert = 0.3679$	−1.2567
$\tau_1 = 3T_0$	$\lvert h_3 \rvert = 0.2231$	−1.2567
$\tau_1 = 4T_0$	$\lvert h_4 \rvert = 0.1353$	0.6283
$\tau_1 = 5T_0$	$\lvert h_5 \rvert = 0.0821$	0.6283

As shown in Figs. 2 and 3, the SLSM based estimator can improve the system performance significantly, although the performance of the LMMSE estimator is much better than that of LS, but is far less than that of the optimal estimator. This is because WPM does not use the cyclic prefix (CP), and the CP is the main method to suppress the inter-carrier interference (ICI) in OFDM system. Therefore, the ICI caused by the frequency selective fading affects the performance of the system seriously, even if the channel estimation has been done, it still can't remission the decline of the performance completely.

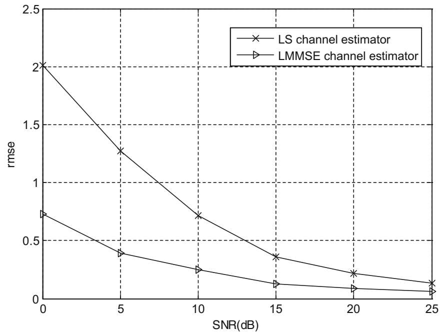

Fig. 2. The estimation error of the SLSM based typical estimators

The increase of estimation error caused by time delay and the multipath energy is shown in Figs. 4 and 5. In the simulation, two path channels were used, and the channel model was denoted as follow:

$$r_f[n] = h_0 x_{0,1}[n] + h_m x_{0,1}[n-m] + v[n] \tag{15}$$

where (15) is the particular form of (10). The time delay $\tau_m = mT$ and the ratio of the energy between the main path and the second path was defined as

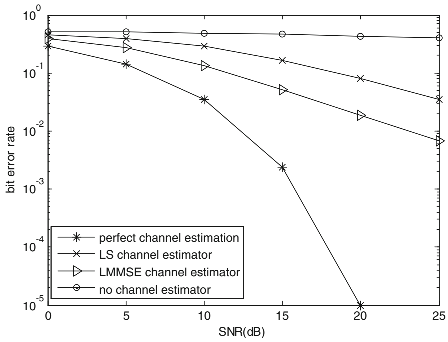

Fig. 3. BER performance of WPM system with SLSM based channel estimators in selective fading channel

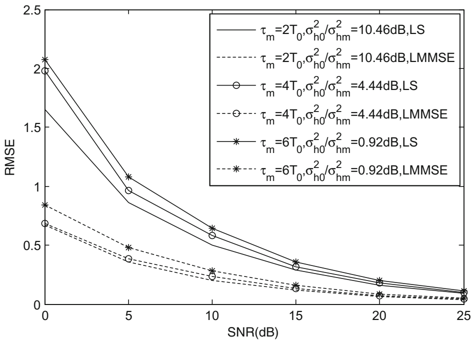

Fig. 4. The estimation error of SLSM based typical channel estimators with different time delay τ_m and $\sigma_{h_0}^2/\sigma_{h_m}^2$

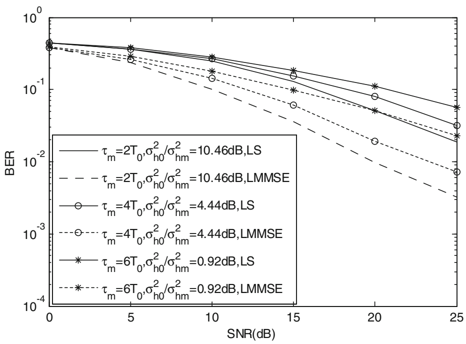

Fig. 5. BER performance of WPM system using SLSM based channel estimators in selective fading channel with different time delay τ_m and $\sigma_{h_0}^2/\sigma_{h_m}^2$

$$\sigma_{h_0}^2/\sigma_{h_m}^2 = 10\lg\left(\frac{|h_0|^2}{|h_m|^2}\right) \tag{16}$$

where h_0 was the main path.

From the simulation results we can find that, with the increase of the delay and the decrease of energy ratio, the estimation error increases and the system performance declines. The above results indicate that the SLSM based LMMSE estimation method can improve the system performance significantly, which has proved the good matching between the LMMSE and the proposed model.

6 Conclusion

The investigation shows that the nonlinear of the WPM fading signal model in frequency selective fading channel is the significant limitation to the using of the LMMSE algorithm. Therefore, the SLSM has been established and the LMMSE estimator using SLSM is employed to improve the system performance. The key features of the SLSM are: (i) It is obtained by using the orthogonality principle of the wavelet packet function and the characters of the frequency selective fading channel. (ii) The linear form of the SLSM satisfies the applicable condition of the LMMSE estimator.

The theoretical and simulation results demonstrated that the proposed SLSM has solved the mismatch problem between the nonlinear of the signal model and the linear estimator, and the SLSM based LMMSE channel estimators can significantly improve the WPM system performance in frequency selective fading environment.

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