SLNR-Oriented Power Control in Cognitive Radio Networks with Channel Uncertainty

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Abstract. The majority of existing studies on power control in cognitive radio networks focus on maximization of signal-to-interference-noise ratio (SINR), while this paper firstly introduces the *signal-to-leakage-and-noise ratio* (SLNR)-oriented power control to optimize throughput in a cognitive radio network (CRN), where massive secondary connections (SCs) and a primary user (PU) coexist with each other sharing the same frequency spectrum. Considering the practical challenge that the channel gains between SCs and PU are typically uncertain, we introduce a probabilistic interference constraint to protect the PU's transmission and reformulate it according to the *Lyapunov*'s central limit theorem (CLT). Then, we apply the convex optimization theory to solve the intractable problem by excluding the probabilistic constraint. Especially, a novel algorithm based on the first-order Lagrangian is developed where the dual variables are updated simultaneously. Furthermore, we provide numerial results using different parameter, which display that the proposed method can achieve higher throughput with much lower computational complexity comparing with the existing literature.

Keywords: Cognitive radio network · Power control · Channel uncertainty · Massive secondary connections · Signal-to-leakage-and-noise ratio

1 Introduction

Spectrum resource is more and more crowded with the ever increasing demand for wireless devices and applications. Pushed by the current severe situation, cognitive radio (CR) has drawn much attention which is a promising technique to improve the efficiency of spectrum utilization [1–3]. Specifically, in the underlay CR mode, a primary user (PU) shares the same spectrum with multiple secondary connections (SCs) in a cognitive radio network (CRN) [4]. With this concept, SCs can access the licensed spectrum used by the PU provided that

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no harmful interference beyond tolerance is introduced. Therefore, it is widely recognized that power control becomes essential for the whole system to mitigate harmful mutual interference.

As an important issue in CR systems, power control has been studied extensively in the literature. Specifically, in [5], a heuristic algorithm under OFDMA has been proposed with an assumption that channel state information (CSI) is perfectly available. However, SCs might have not been given priority to know the signal characteristics of the PU, and thus have to rely on imperfect channel estimation. Consequently, power control for CRNs must account for channel uncertainty [6,7]. In [6], the interference constraints as a probability in a power control problem under the CR scene have been considered, where the uncertainty has been in fading channels including shadowing and Nakagami fading. Power control problems for OFDMA under channel uncertainty is also studied in [7]. However, to the authors' best knowledge, all the existing studies of throughput optimization are oriented towards the signal to interference plus noise ratio (SINR), thus there are no closed form solutions on account of the coupled nature of the corresponding optimization problem. Moreover, the majority of existing studies consider the case that the number of SCs is small (up to tens) and they do not explicitly address the mutual interference between SCs in the system.

In this paper, we propose a *signal-to-leakage-and-noise ratio* (SLNR)-oriented power control method to promote the throughput capacity of the system. Meanwhile, the interference is mitigated between massive SCs (up to hundreds of more) with channel uncertainty. Notably, the so-called SLNR is originally used to design precoders in multi-user MIMO communications [8], where leakage refers to the interference caused by the signal intended for a desired user on the remaining users in a precoding scheme. Differently, in this paper, leakage means the interference generated from one SC to all other SCs. As a result, SLNR is able to measure how much power leaks to the other SCs in the CRN. More importantly, due to the coupled interference nature of the corresponding throughput optimization problems, existing solutions based on SINR do not have closed forms. Differently, the proposed SLNR-oriented power control method in this paper can circumvent the hurdles of SINR perfectly, which leads to a decoupled optimization problem and allows an analytical closed form solution. This method has been proved to be much more effective in this paper (see Sect. 4). Specifically, there are three innovations below the part:

(i) Describe an optimal problem, where channel uncertainty and interference constraints are jointly considered. Different from other power control methods, the throughput is promoted via a novel concept which optimizes the sum SLNR instead of SINR.
(ii) Introduce Gaussian approximation based on the *Lyapunov*'s central limit theorem (CLT) to offer a conservative surrogate and propose an effective power control algorithm based on first-order Lagrangian where the dual variables are updated simultaneously.
(iii) Provide numerial results using different parameter, such as the transmit power and the interference threshold of SCs, which display that our method can outperform the state-of-the-art works in the literature.

2 System Model and Problem Formulation

Consider a CRN where a PU and massive SCs utilize the same spectrum in the underlay mode. The SCs are supposed to be randomly distributed around the PU and $\mathcal{N} = \{1, 2, \cdots, N\}$ is denoted as the set of all SCs. In addition, let p_n represent the transmit-power of the n_{th} SC. Also, let p_{max} denote the maximum transmit-power of SCs, I_{max} denote the maximum interference of SC, and $I^p_{\max}$ denote the maximum interference of PU.

The channel gain $g_{n,m}$ between the n_{th} and the m_{th} SC is known accurately [10]. Because SCs almost have no cooperation with PU through their transmissions, it is hard to precisely estimated the gain g_n^{PU} between them.

The SINR of the n_{th} SC can be obtained as follows:

$$SINR_n = \frac{p_n g_{n,n}}{\sum_{m=1,m\neq n}^{N} p_m g_{m,n} + \sigma_0^2}, \tag{1}$$

where $p_n g_{n,n}$ is the received signal power, $\sum_{m=1,m\neq n}^{N} p_m g_{m,n}$ represents the mutual interference of SCs. The whole throughput of the network is expressed as the following formulation:

$$TH_{sum} = \sum_{n=1}^{N} \log_2 (1 + SINR_n). \tag{2}$$

The SLNR of the n_{th} SC is defined as:

$$SLNR_n = \frac{p_n g_{n,n}}{\sum_{m=1,m\neq n}^{N} p_n g_{n,m} + \sigma_0^2}, \tag{3}$$

where the power of the desired signal component for SC n is given by $p_n g_{n,n}$. Meanwhile, the interference caused by SC n on SC m is given by $p_n g_{n,m}$. Therefore, $\sum_{m=1,m\neq n}^{N} p_n g_{n,m}$ represents the power leaked from SC n to all other SCs, which is the concept of *leakage* for SC n. This lies the difference from SINR.

Due to the mutual interference, the constraints of SCs are expresses as follows:

$$\sum_{m=1,m\neq n}^{N} p_m g_{m,n} = I^{(-n)} \leq I_{\max}, \ \forall n. \tag{4}$$

In addition to this, the protection for the PU is taken into consideration. That is to say, the sum interference from all the SCs to the PU is limited under a certain threshold [10]. In order to clearly quantify this, Pr [.] is defined as the outage probability, which can be written as the following expression:

$$\Pr\left[\sum_{n=1}^{N} p_n g_n^{PU} \leq I^p_{\max}\right] \geq 1 - \varepsilon, \tag{5}$$

where g_n^{PU} is a random variable which is independent and identically following a distributed exponential and the mean is θ, and the threshold of the outage probability is ε.

In this paper, in order to circumvent the hurdles of SINR, the following SLNR-oriented problem is skillfully designed to achieve the purpose of throughput maximization:

$$
\begin{aligned}
(\mathcal{P})\ \max_{\mathbf{p}=\{p_n\}} & \sum_{n=1}^{N} SLNR_n \\
\text{s.t.}\ & C1: p_n \leq p_{\max},\ \forall n \\
& C2: \sum_{m=1,m\neq n}^{N} p_m g_{m,n} = I^{(-n)} \leq I_{\max},\ \forall n \\
& C3: \Pr\left[\sum_{n=1}^{N} p_n g_n^{PU} \leq I_{\max}^{p}\right] \geq 1-\varepsilon,
\end{aligned} \tag{6}
$$

where $C1$ restricts the transmit-power of SC, $C2$ guarantees that all the SCs can coexist with each other and $C3$ focuses on the protection for the PU. As is mentioned in the previous definition of SLNR, not only does SLNR promote SCs benefit, but also can suppress the interference on others. In this way, the performance should be greatly improved, where the motivation lies. For problem (6), the challenge is that the channel gain is uncertain along with considering massive SCs' interference. In addition, there is not yet any satisfactory solution to the open issue so far. In the following part, we introduce a feasible way to tackle the difficulty with low complexity, which takes advantage of the convex optimization theory.

3 Algorithm Design

In this section, an algorithm based on Lagrangian techniques is developed to solve the problem $\mathcal{P}$. The objective function (3) is rewritten by $f(p_n) = \frac{p_n g_{n,n}}{\sum_{m=1,m\neq n}^{N} p_n g_{n,m}+\sigma_0^2}$ and obviously $f(p_n)$ is a concave function. According to [11], the objective function is also a concave function. The original optimization objective can be converted into the following form:

$$(\mathcal{P}*)\ \min_{\mathbf{p}=\{p_n\}}\ -\sum_{n} f(p_n). \tag{7}$$

Nevertheless, $C3$ does not meet the requirement of a convex function. Let $X_n = p_n g_n^{PU}$, and we assume that X_n independently follow the exponential distribution with mean $p_n\theta$. Meanwhile, assume the sum of the whole random variables is $X = \sum_n X_n$. Next, the original constraint is able to be changed into the probability form as follows:

$$\Pr\left[X = \sum_{n} X_n \leq I_{\max}^{p}\right] \geq 1-\varepsilon. \tag{8}$$

To deal with the constraint (8), the distribution of X is very important. To study the distribution of X, we use the following Lemma 1 [12] to get Gaussian approximation.

Lemma 1. *The* Lyapunov*'s central limit theorem (CLT): If* $X_1, X_2, \ldots, X_n$ *are independent of each other with mean* $E(X_k) = \mu_k$ *and variance* $D(X_k) = \sigma_k^2 > 0$*, we can obtain that*

$$Z_n = \frac{\sum_{k=1}^{n} X_k - \sum_{k=1}^{n} \mu_k}{B_n} \sim N(0,1) \tag{9}$$

and

$$\sum_{k=1}^{n} X_k \sim N\left(\sum_{k=1}^{n} \mu_k, B_n^2\right), \tag{10}$$

where $B_n^2 = \sum\limits_{k=1}^{n} \sigma_k^2$.

Generally, X can be regard as a normally distributed random variable due to massive connections. Approximately, the mean is m and the variance is σ^2:

$$\begin{aligned} m &\sim \textstyle\sum_i p_i\theta \\ \sigma^2 &\sim \textstyle\sum_i (p_i\theta)^2. \end{aligned} \tag{11}$$

As a result, the following expression is a substitute product for (8):

$$P(\mathbf{p}) = 1 - F_N(I_{\max}^p) = \frac{1}{2}\text{erfc}(\frac{I_{\max}^p - m}{\sqrt{2}\sigma}) \leq \varepsilon. \tag{12}$$

where $F_N(\cdot)$ is the cumulative distribution function (CDF) of a normal distribution and its mean is m, the variance is σ^2. Moreover, $\text{erfc}(z) = \frac{2}{\sqrt{\pi}} \int_z^{\infty} e^{-t^2} dt$. For (12), the assumption is that $f_3(\mathbf{p}) = \frac{1}{2}\text{erfc}(\frac{I_{\max}^p - m}{\sqrt{2}\sigma}) - \varepsilon$. Inspired by the scheme proposed in [10], the problem $\mathcal{P}$ can be decomposed into a sub-problem $\mathcal{P}1$ and (12):

$$\begin{aligned} (\mathcal{P}1) \quad & \min_{\mathbf{p}=\{p_n\}} \quad -\sum_n f(p_n) \\ \text{s.t.} \quad & \quad C1, C2 \end{aligned} \tag{13}$$

When a power allocation is given from $\mathcal{P}1$, we can check if it meets (12). If it does, the given power is optimal; or lower a step and check it again.

Next, we solve this minimization problem ($\mathcal{P}1$) with Lagrangian techniques. Firstly, by introducing nonnegative dual variables $\boldsymbol{\lambda} = [\lambda_1, \lambda_2, ..., \lambda_N]$ and $\boldsymbol{\mu} = [\mu_1, \mu_2, ..., \mu_N]$, the Lagrange function is given by

$$\begin{aligned} L(\mathbf{p}, \boldsymbol{\lambda}, \boldsymbol{\mu}) = &-\sum_n f_0(p_n) + \sum_n \lambda_n (p_n - p_{\max}) \\ &+ \sum_n \mu_n \left(\sum\nolimits_{m=1, m\neq n}^{N} p_m g_{m,n} - I_{max}\right). \end{aligned} \tag{14}$$

The dual function is defined as an unconstrained minimization of the Lagrangian function:

$$g\left(\boldsymbol{\lambda},\boldsymbol{\mu}\right)=\inf_{\mathbf{p}} L\left(\mathbf{p},\boldsymbol{\lambda},\boldsymbol{\mu}\right). \quad (15)$$

We consider the problem in (15) for obtaining $g\left(\lambda,\mu\right)$ with a given set of $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$. From (14), we have

$$\frac{\partial L\left(\mathbf{p},\boldsymbol{\lambda},\boldsymbol{\mu}\right)}{\partial p_n}=\frac{g_{n,n}\sigma_0^2}{\left(\sum_{m=1,m\neq n}^{N} p_n g_{n,m}+\sigma_0^2\right)^2}+\lambda_n+\mu_n\sum_{m=1,m\neq n}^{N} g_{m,n}. \quad (16)$$

Because the dual function is always convex, a gradient-type search is guaranteed to converge to the global optimum. Problem $\mathcal{P}1$ is solved via the following first-order algorithm that utilizes the gradient of $L\left(\mathbf{p},\boldsymbol{\lambda},\boldsymbol{\mu}\right)$ to simultaneously update the dual variables with constant Δ [11],

$$p_n^{k+1}=\left[p_n^k-\Delta\left(\frac{\partial L\left(\mathbf{p},\boldsymbol{\lambda},\boldsymbol{\mu}\right)}{\partial p_n}\right)\right]_{\mathbf{P}} \quad (17)$$

$$\lambda_n^{k+1}=\lambda_n^k+\Delta\left(p_n-p_{\max}\right) \quad (18)$$

$$\mu_n^{k+1}=\mu_n^k+\Delta\left(\sum_{m=1,m\neq n}^{N} p_m g_{m,n}-I_{max}\right), \quad (19)$$

where k is the iteration number and Δ is the iteration step and $[\mathbf{x}]_{\mathbf{y}}$ is the projection of $\mathbf{x}$ onto the set $\mathbf{y}$. According to [11], the above gradient update can be guaranteed to converge to the optimal dual variables as long as the sequence of scalar step Δ is chosen appropriately. Finally, the entire steps to solve the optimization problem is displayed in Algorithm 1, where Step 1 to 6 solve $\mathcal{P}1$ and Step 8 to 11 check (12). The computational complexity of the algorithm is counted as follows. Solving (17) requires solving n equations by complexity $O(N)$. (17), (18) and (19) need to update $3n$ times at the same time with k iterations. Thus the complexity of the solution is measured by $O(3KN)$.

4 Simulation Results and Analysis

In Table 1, we list the key system parameters which are used in the simulations. The presented results are acquired via 1000 independent tests. In order to evaluate the performance of our proposed method, an optimal power allocation method oriented to SINR in [14] is introduced as a benchmark scheme.

Figure 1 depicts the sum throughput of different methods as a function of the density of SCs. The first observation from Fig. 1 is that the traditional SINR-oriented power control incurs a performance loss compared to the proposed SLNR-oriented power control with channel uncertainty. This phenomenon can

Algorithm 1. The power control algorithm with channel uncertainty

1: **Initialization:** Set the parameters $k = 0$, $\mathbf{p}^{(0)} = \{p_n^0\}$,$\Delta > 0$, ε, γ, δ.
2: **for** $k = 1, 2, ...$ **do**
3: for each user, calculate $p_n^* = \left[p_n^k - \Delta\left(\frac{\partial L(\mathbf{p},\boldsymbol{\lambda},\boldsymbol{\mu})}{\partial p_n}\right)\right]_{\mathbf{P}}$.
4: Update λ_n^{k+1} and μ_n^{k+1} according to (18), (19).
5: Update $p_n^{k+1} = p_n^*$.
6: **end for** Until $\left|\mathbf{p}^{k+1} - \mathbf{p}^k\right| \leq \delta$.
7: If $P(\mathbf{p}) \leq \varepsilon$, end; or, step into 8.
8: **for** $m = 1, 2, ..., M$ **do**
9: Set $\mathbf{p}^{(k)} = \mathbf{p}^*(t)$.
10: $\mathbf{p}^{(m+1)} = \mathbf{p}^{(m)} - \gamma$.
11: **end for** Until $P(\mathbf{p}) \leq \varepsilon$.

Table 1. System parameters used in simulations.

Parameter	Value	Comments
σ_0^2	-100 dBm	Noise power
$p_{\max}$	20 dBm	The maximum power of the SC
$I_{\max}, I_{\max}^p$	-120 dBm	The interference threshold value of SC, PU
Δ	0.15	The iteration step
γ	0.02	The power step
δ	10^{-4}	The accuracy of power
ε	0.1	The threshold value of the outage probability

be attributed to the fact that not only does SLNR promote SCs benefit, but also can suppress the interference on others. In this way, the performance is greatly improved. Furthermore, we can also observe that both curves have a tendency to decline when the number of SCs becomes large, which is due to the fact that as the density increases, the mutual interference among SCs may result in several SCs out of work, considering the constraint $C2$ in the optimization.

Figure 2 discloses the sum throughput versus the SC's interference threshold level I_{max} as a function of the transmission power limit, where the number of SCs is invariable. From the figure, the sum throughput has a uptrend with the increase in transmission power. Taking $I_{max} = 10^{-10}$ as an example, it can be observed that more robust performance can be approached as the transmission power increases. Nevertheless, two conditions, the interference between SCs and the power increasing, have bind effects mutually. So the sum throughput will reach saturation state when the power increases to a certain extent. The phenomenon can be seen more noticeably when $I_{max} = 10^{-12}$ and $I_{max} = 10^{-13}$. Furthermore, the power increasing would be greatly limited if the interference threshold is too small such as $I_{max} = 10^{-15}$, and thus the sum throughput is almost invariable.

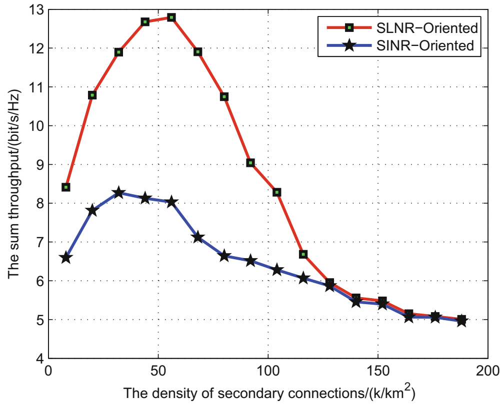

Fig. 1. The sum throughput comparison of the SLNR-oriented and SLNR-oriented power control schemes.

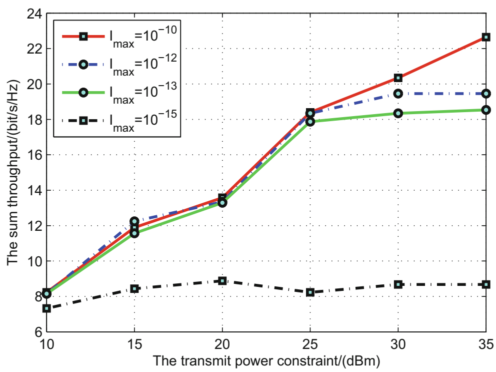

Fig. 2. The sum throughput of the CR system versus the transmission power limit for different I_{max}.

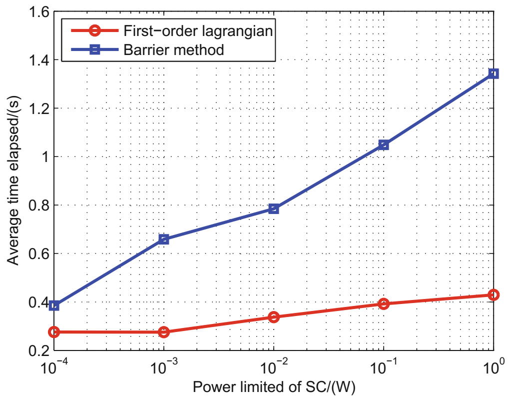

Fig. 3. Average time elapsed as a function of SC's power limit.

Figure 3 shows the average time elapsed as a function of SC's power limit. The commonly-used barrier method in the literature and the adopted first-order lagrangian method are compared. From the analysis results given in Sect. 3, we know that the adopted first-order lagrangian method has a linear complexity of $O(3KN)$. Addition to this, the results in Fig. 3 show that the time cost of our method is not only the lower one, but also it varies trivially as the SC's power limit increases.

5 Conclusion

In this paper, we have proposed a SLNR-oriented power control method to promote the throughput capacity of the system. Meanwhile, the interference is mitigated between massive SCs with channel uncertainty. Gaussian approximation based on the *Lyapunov*'s central limit theorem has been used to offer a conservative surrogate. Moreover, we have developed an effective power control algorithm based on first-order Lagrangian where the dual variables are updated simultaneously and the solution is amenable. Simulation results have validated the effectiveness of our proposed algorithm. As one future work, a subject of extension to more general channel models including correlation or feedback delay will be investigated.

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