

# Koch Fractal-Based LED Lamp Appearance Design Method

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**Abstract.** Along with developing of LED lamps, more technical appearance design methods become possible. A lamp appearance design method based on fractal technology is researched in this paper. Firstly, the famous Koch fractal is analyzed at different generators. Then, the Koch fractal pattern is generated by the relevant parameters. Finally, the lamps appearance was designed by the Koch fractal characteristics. The design and simulation results show that the fractal model can be applied to the design of lamps appearance.

**Keywords:** Koch fractal · Lamp appearance · LED

## 1 Introduction

LED lighting has become a trend in 21st century. Especially in bedroom lighting, LED lamps will replace traditional incandescent and fluorescent lamps. LED technical features have made the lighting design of contents and modes change substantially. The new light-resource of LED promotes innovations in lighting design and development. To a certain extent, LED technique also changes our concept of lighting. We are gradually liberated from traditional, linear light limitation. Lamp designs of the language and concept can be free to play and reshape with greater and flexible space in the pursuit of creative expression of natural beauty [1, 2].

The fractal theory is a very active branch in Nonlinear Science, which mainly study irregular and rough geometric shapes in nature and non-linear systems. Existing research shows that natural scene and contours possess non-stationary, self-similarity and multiple scales characteristics. These characteristics are described by the stochastic fractal [3–10]. Therefore, Natural scenery and some graphics can be produced by fractal technique [11–13]. It can bring out a person feeling the characteristics of natural beauty.

Koch curve is a very important fractal curve in geometry, with clearly mathematical description and simple programming. In this paper, the deformed Koch curve composes of regular polygons by edges. Because of regular polygon ring closed and symmetrical, the curve changes with some parameters selecting the patterns that can make design possess natural beauty and form new light-outlook. Meanwhile, due to the geometric shapes are constructed by model it makes lamps shape with natural beauty [14, 15].

## 2 The Generation Method of Koch Fractal Curve for Lamp Appearance

Koch curve is typical of fractal curve, its structure process is through to repeatedly replacing each line with similar graphics of generator, thus graphics of each part are same with shape itself called as self-similarity, which is also an most important feature of fractal pattern, its structural process also decides the way of making the curve in computer that should be recursive method, that is function himself calls the process of himself. Transformation rules of Koch curves  $R$  is produced by fractal generator, the basic characteristics are decided by fractal generator completely. We can generate a variety of fractal graphics by some generator, the process is shown as in Fig. 1.

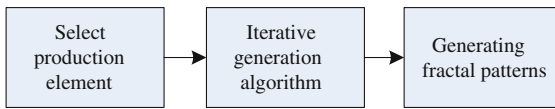


Fig. 1. Forming method of Koch fractal patterns

### 2.1 Koch Fractal Generators

#### (1) Triangle generators

Given a straight  $F_0$ , dividing the line into three fractions equally and replacing the middle fraction with other two edges of the equilateral triangle constituted of this line,  $F_1$  can be get. According to above way, we modify each piece of fig  $F_1$  in this manner, until infinitum, the ultimate  $F_n$  of the curve is finally get. It is called as the Koch curve. This process is shown in Fig. 2.

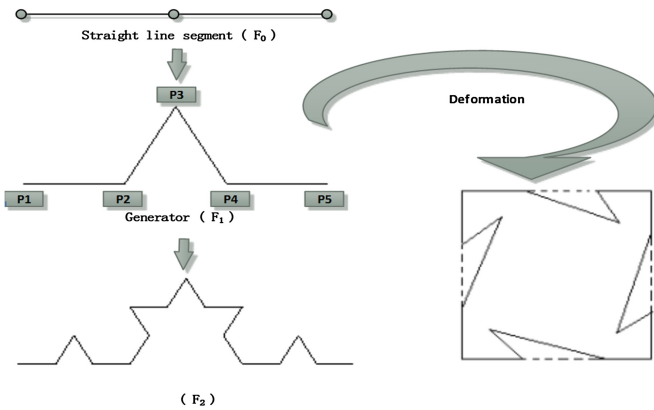


Fig. 2. Triangle generator

(2) Square generators

With the same as the construction method of the triangle Koch curve generator, the line is divided into three fractions equally. The middle fraction is replaced by the square. We can get the F1. F1 is formed substantially by triangle generator whose triangular parts are replaced by square, it is noted that the height of square should be one-third of the original line, finally we can get first order of curve constituted with five equal lines (shown as in Fig. 3a). Then, according to above way, we modify each piece of fig F1, until infinitum, we finally get the ultimate curve. It is called as the Koch curve with square generators. In this manner, if it takes a square as the original graph and divides each edge of the square into three parts, then similarly, the middle section is used to form another square generator. We can get another fractal pattern. It is shown as Fig. 3(b).

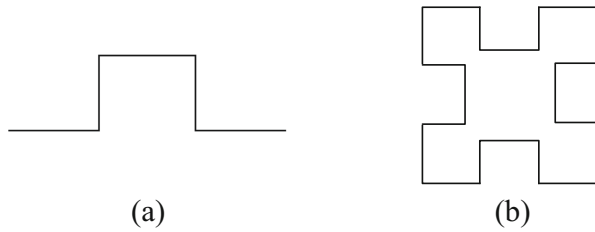


Fig. 3. Square generator

(3) Tree generators

Tree generators start from a line and marking two endpoints, as well as  $1/3$ ,  $2/3$ , then starting with  $1/3$  points and  $2/3$  points to extend out several branches, length of the branch is  $1/3$  of the initial linear, We finally get branch generators as shown in Fig. 4. The one-third point and the two-thirds points of the line is respectively shown as point 2 and 6.

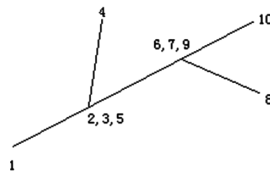


Fig. 4. Fractal generator

2.2 Koch Fractal Pattern Generating Algorithm

Koch fractal curve construction begins with a linear segment. The construction progress is shown as Fig. 5:

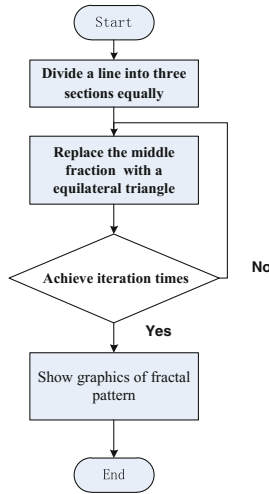


Fig. 5. Process of Koch fractal pattern generating algorithm

**(1) Segment**

Considering the process of a straight line segment (2 points) producing the first shape (5 points). In Fig. 2, assuming that P1 and P2 is the two start points of the original line segment, then we need to assert three points P1, P2 and P3. And obviously, P2 is in one-third line, P4 locates in the two-thirds line segment.

**(2) Rotation**

The location of the P3 point can be considered that P4 rotates counter-clockwise by P2 as the axis. Rotation can be achieved by an orthogonal matrix. It can be described as Eq. (1):

$$A = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix}. \tag{1}$$

According to original data, the algorithm produces 5 joint points in the Fig. 2. An array of nodes forms a  $5 \times 2$  matrix, the coordinate of the first behavior p1, the coordinate of the second, and so on, until the coordinates of p5. Matrix elements in the first column are the 5 nodes' x coordinate, the elements in the second column are the 5 nodes' y coordinate.

**(3) Iterations**

Further consideration is the regularity of node number in the process of forming Koch curve. Supposing that the k iteration produce  $n_k$  nodes, and the at time k+1 iteration,  $n_{k+1}$  nodes generates. Namely the recurrence relation between  $n_k$  and  $n_{k+1}$  is

$$n_{k+1} = 4n_k - 3. \tag{2}$$

### 2.3 Fractal Pattern of Koch

As mentioned above, the fractal function is programmed. Koch curve is generated by MATLAB. In order to present the change of points in the recursive process intuitively and dynamically, we use vector  $x, y$ . Finally it can be shown intuitively by the graph. We can see that the key step of pattern generation is algorithm design.

Triangle generator generating fractal patterns are shown as Fig. 6. In order to generate the final Koch snowflake pattern by the original lines or graphics, we should start from the definition of the Koch curve, giving a equilateral triangle that the length of side is and then adding a side  $a/3$  equilateral triangle in each center of side, determining to rotate Orthogonal matrix in algorithm analysis, considering the regularity of the number of nodes' change in the process of forming Koch curve, and infinite iterations, finally it can form the following snowflakes graphics.

The train of thought for generating fractal patterns by a rectangular generator or branch generator is basically the same, only need to adjust rotating orthogonal matrix and the corresponding part of the function. Patterns generated by square generators are shown as Fig. 7. Patterns generated by tree fractal generators are shown as Fig. 8.

#### (1) The triangle generator

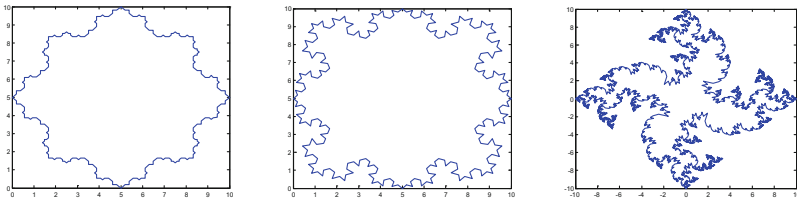


Fig. 6. Patterns generated by triangle generators

#### (2) The square generator

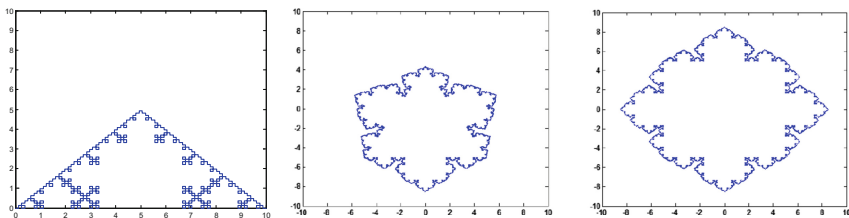
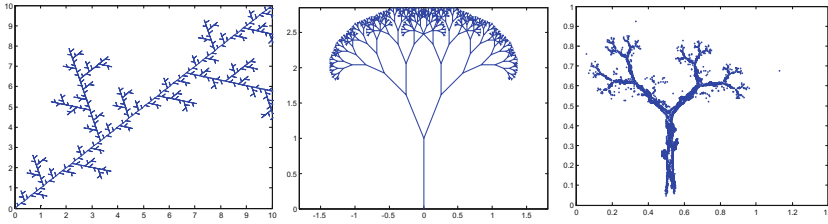


Fig. 7. Patterns generated by square generators

**(3) The tree fractal generator**



**Fig. 8.** Patterns generated by tree fractal generators

**3 Method of Lamp Outlook-Design Based on Koch Fractal Patterns**

The outlook-design of lamps is based on the Rhinoceros software modeling and apply KeyShot software in simulated lighting image in this paper. Rhinoceros software is suit at modeling of product appearances. Therefore, it is always being used in industrial design is an interactive ray tracing and rendering programs with the full domain, without complicated setting, it can produce 3D rendered image (Fig. 9).



**Fig. 9.** Modeling and Simulation of lighting model based on Koch fractal

Imitating the natural shape is one of the simplest and most direct methods of lighting design. It not only affects people’s aesthetic consciousness, but also promotes a natural, eco-design, which retains the spiritual and natural breath of lamps. The generation of fractal graphics relies on iterative function, which can express infinite subtle structure, if the computer accuracy is not restricted, it can infinitely magnify the boundary of fractal graph and a region in the graph to show a new structure element. From the overall visual effect, we can see that pattern generated by the fractal graph is with more abstract, art, regularity than the traditionally manual drawing pattern, which make up the traditional decorative pattern. Eventually we will apply fractal graphics to the structure of lamps’ shape, this is a process to transform the flat pattern into three-dimensional pattern, in this process, and different three-dimensional methods will produce different modes of lamps, which give us as a wider space to play in the mode design. The LED light-source has a small size and many other features. Thus, the lamp shape can break the limits of traditional lamps to design a variety of lamps and the reflective and casted type of lamps can be perfectly applied to lamps that use Koch curve as a prototype.

Reflective mode is the light provided through reflector lighting for residents. Using secondary optical design, we make the light spot from light-source form selected curve

shape of Koch curve. And then put the light-source into lamps so that all of the light can be casted onto the reflector. Then, reflectors cast the light onto the ground. Light is provided for residents or decorative use.

Casted pattern is directly using the light-source through the outer shade to cast onto the ground to provide lighting for the residents. It can be achieved in two methods one way is to filter out of the Koch curve made as lamp covers, then the light-source through to the shell light interior. This way generally uses in the production of ceiling lamps. The other way is through the use of LED light source characteristics of small volume of LED light-source to arrange the Koch as selected and finally show the curve outline. This method should generally be used for ceiling lamps.

#### 4 Design Results of LED Lamps Combined with Koch Fractal Pattern

As mentioned above, Koch curve patterns is an input on the Rhinoceros software, multiple linear functions is used to draw the simulation. Then, we can get the outlook of lamps, through to the operation of stretch. Finally, lamps are attached to materials in KeyShot the simulation is shown as Figs. 10, 11.

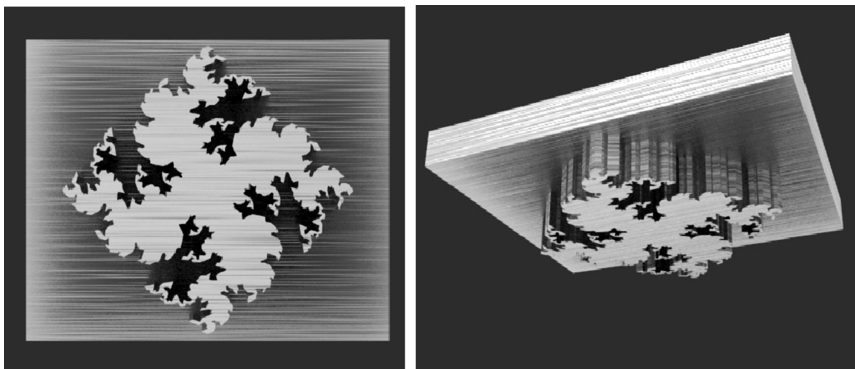


Fig. 10. Simulated lamp A

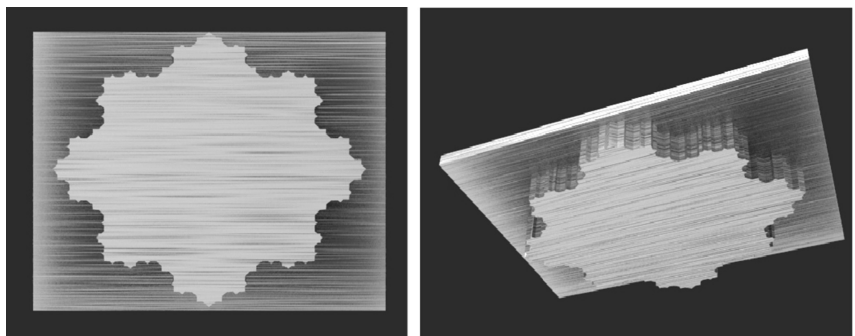


Fig. 11. Simulated lamp B

## 5 Conclusions

The fractal theory describes the feature of the natural scenery. It can construct the fractal pattern, while Koch fractal is a classic fractal pattern. This paper applies the characteristics of Koch fractal to design appearance of lighting. Design results show that the Koch fractal patterns can be used to design appearance, and fractal theory is suitable for designing appearance.

## References

1. Cheng, Z., Han, C.: Household LED design methods and development prospect of. *Sci. Technol. Econ. Market* **11**, 81–82 (2007)
2. Fu, Z.: Discussion on development trend of modern lighting design. *Stage Design* **7**, 164–165 (2013)
3. Mandelbrot, B.B.: *The Fractal Geometry of Nature*, pp. 79–95. Freeman, San Francisco (1982)
4. Yue, S., Wan, D., Lu, J.: Application research on fractal graphics in packaging design. *Packag. Eng.* **10**, 67–69 (2011)
5. Ruderman, D.L.: Origins of scaling in natural images. *Vis. Res.* **37**, 3385–3398 (1996)
6. Al-Hamdan, M., Cruise, J., Rickman, D., et al.: Effects of spatial and spectral resolutions on fractal dimensions in forested landscapes. *Remote Sens.* **2**, 611–640 (2010)
7. Al-Hamdan, M.Z., Cruise, J.F., Rickman, D.L., et al.: Characterization of forested landscapes from remotely sensed data using fractals and spatial autocorrelation. *Adv. Civ. Eng.* **2012**, 1–15 (2012)
8. Ghosh, J.K., Somvanshi, A.: Fractal-based dimensionality reduction of hyperspectral images. *J. Indian Soc. Remote Sens.* **36**, 235–241 (2008)
9. Falconer, K.J.: *Fractal Geometry Mathematical Foundations and Applications*, pp. 122–125. Wiley, New York (1990)
10. Pentland, A.P.: Fractal-Based Description of Natural Scenes. *Pattern Anal. Mach. Intell.* **6**, 661–674 (1984)
11. Ueda, Y.: *The Road to Chaos*, pp. 55–74. Aerial Press, Santa Cruz (1992)
12. Reichl, L.E.: *The Transition to Chaos*, pp. 28–67. Springer, New York (1992)
13. Schuster, H.G.: *Deterministic Chaos*, 2nd edn, pp. 78–92. Physik-Verlag, Weinheim (1988)
14. Xu, Y., Zhao, X.: Koch snowflake curve of production and its important conclusions. *J. Changchun Teach. Univ.* **2**, 6–8 (2003)
15. Qiu, W., An, N., Qi, X.: Koch fractal image generation based on MATLAB algorithm. *Comput. Digit. Eng.* **8**, 100–101 (2010)