A Novel Constellation Shaping Method to Reduce PAPR for Rate Compatible Modulation

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Abstract. Although rate compatible modulation (RCM) achieves continuous and efficient spectrum efficiency, it has high peak to average power ratio (PAPR) because that its modulation constellation is a dense rectangular. To reduce PAPR of RCM, this paper proposes a novel constellation shaping method, namely transform from rectangular to circle (TR2C), which different to the traditional constellation shaping methods. The TR2C approach compress the points, that with higher amplitude and lower probability, into a circle, such that it can reduce the PAPR of shaping constellation. Furthermore, to deal with the problem of AWGN noise amplification in inverse transform of TR2C, we give out the geometrical method of estimating noise variance. Simulations show that, comparing with original constellation, TR2C scheme can achieve 1 dB performance gain at BER = 10^{-5} by improving the transmission power with 1.8 dB. And TR2C achieves the capacity performance close to original constellation when SRN from 5 dB to 30 dB.

Keywords: Constellation shaping · Rate compatible modulation · PAPR

1 Introduction

Rate adaptation technology is the effective means to achieve high spectral efficiency under varying channel conditions, in physical layer of modern wireless communications. In recent year, three receiver adaptation schemes [1–4] were proposed to tackle the problem, that sender needs accurate and immediate channel feedback from the receiver in traditional adaptive modulation coding technology. Rate compatible modulation (RCM) [1] is a novel rate adaptation technology based on compressive sensing (CS) [5], and it is a promising application of the emerging CS theory in wireless communications. In RCM scheme, modulated symbols are incrementally generated from information bits through weighted mapping. Therefore, rate adaptation is achieved by varying the number of modulated symbols.

RCM achieves continuous and efficient spectrum efficiency. However, its peak to average power ratio (PAPR) is very high. In modern wireless communications,

X.-L. Huang (Ed.): MLICOM 2016, LNICST 183, pp. 15–24, 2017.
DOI: 10.1007/978-3-319-52730-7_2

constellation shaping is the major mean to reduce PAPR and approach channel capacity. Constellation shaping technique partition the basic constellation into several subconstellations, so that the lower energy signals are transmitted more frequently than their higher energy counterparts [6]. Constellation shaping technique can be used in BICM and OFDM. A subset of the interleaved bits output by a binary LDPC [7] encoder are passed through a nonlinear shaping encoder, whose output is more likely to be a zero than a one. The shaping bits are used to select from among a plurality of subconstellations, while the unshaped bits are used to select the symbol within the subconstellation. Because the shaping bits are biased, symbols from lower-energy subconstellations are selected more frequently than those from higher-energy sub-constellations [8–11].

To reduce PAPR of RCM, this paper proposes a novel constellation shaping method named transform from rectangular to circle (TR2C). The TR2C approach compress the points, that with higher amplitude and lower probability distribution, into a circle, such that it can reduce the PAPR of RCM constellation. Furthermore, to deal with the problem of AWGN noise amplification in inverse transform of TR2C, we give out the geometrical method of estimating noise variance. Simulations show that, comparing with original constellation, TR2C scheme can achieve 1 dB performance gain at BER $= 10^{-5}$ by improving the transmission power. And TR2C achieves the capacity performance close to original constellation when SRN from 5 dB to 30 dB.

The rest of this paper is organized as follows. Section 2 gives out the system model based on constellation shaping, and briefly reviews the rate adaptive scheme of RCM. Section 3 presents the proposed constellation shaping methods for RCM. The simulation evaluations are included in Sect. 4. Finally, Sect. 5 concludes this paper with some discussions on future work.

2 System Model

In this section, we first describe a system model which combines RCM and novel constellation shaping. Then we we briefly review RCM and its standard decoding algorithm [1].

2.1 System Model

To reduce PAPR of RCM, we design a system model as shown in Fig. 1. Form the diagram, we can see that this communication system combines RCM component indicated in the rectangle of red dashed line and shaping component indicated in the rectangle of blue dashed line. Here, shaping component includes constellation shaping subcomponent and its inverse transform component. It is should be noted that the system only with RCM component also can work. In this paper, we only focus on shaping component.

This system model includes transmitter and receiver. At transmitter, the binary data stream is divided into blocks with length N_b. Each block $\boldsymbol{b}$ is encoded by RCM encoder, and we get a real vector denoted by u. In order to make use of

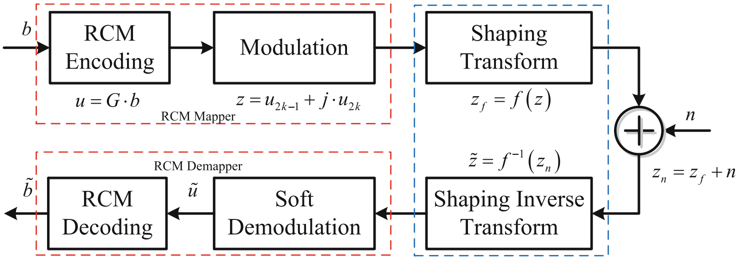

Fig. 1. System model of constellation shaping with RCM (Color figure online)

two dimensional modulation, two consecutive symbols are combined together to form In-phase and Quadrature-phase modulation symbol z, i.e., $z_k = u_{2k-1} + j \cdot u_{2k}, k = 1, 2, \cdots, N_b/2$. Then, Each complex symbol is transformed by shaping transformation that named TR2C. After passing the standard AWGN channel model, receiver gets $z_n = z_f + n$, where n is Gaussian noise vector with $n \sim N\left(0, \sigma^2\right)$. At receiver, through shaping inverse transform, soft demodulation, and RPC decoding operator, we get the estimation of $\boldsymbol{b}$.

2.2 Rate Compatible Modulation

RCM Encoding. A bipartite graph representation of RCM encoding is provided in Fig. 2. Square and circle denote symbol nodes and bit nodes, respectively. Each edge is assigned with a weight $w \in W$, where $W = \{w_1, w_2, \cdots, w_L\}$ is the weight set with length L. The bipartite graph can be represented by $G = (U, V, E)$, where $U = b_j, j = 1, \cdots, N$ is the source bits, $V = u_i, i = 1, \cdots, M$ is the set of measurements representing modulated symbols, and E defines the connection between the two sets. The RCM encoding maps binary bits into modulation symbols. Each modulation symbol is calculated by $u_i = \sum\limits_{l \in N(i)} w_{il} \cdot b_{i_l}$, where w_{il} is the weight corresponding to ${i_l}^{th}$ bit b_{i_l}, and $N(i)$ denotes the set of neighbors of the i^{th} symbol node. The RCM encoding process can be described by

$$\boldsymbol{u} = \varPhi \cdot \boldsymbol{b}, \tag{1}$$

where $\varPhi$ is a random projection matrix with dimension $M \times N$, and $\boldsymbol{u}$ is the RCM encoded symbols vector with length M.

RCM Decoding. RCM decoding uses the belief propagation (BP) algorithm like LDPC decoding. Since RCM is employing weighted sum check, probability convolution operation is used in horizontal iteration instead of $log(tanh)$ operation. The decoding algorithm of RCM is depicted as factor graph shown in Fig. 2. An edge with a weight w_{ij} denotes the connection between symbol node i and bit node j, and arrow line denotes the probability message flow. $r_{ij}^{(t)}$ defines

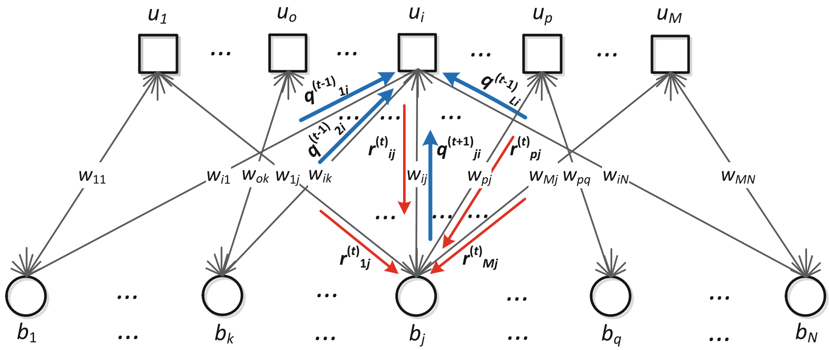

Fig. 2. Bipartite graph of ARC code.

the probability message from the i^{th} symbol node to the j^{th} bit node in the t^{th} iteration. $q_{ji}^{(t+1)}$ defines the probability message from the j^{th} bit node to the i^{th} symbol node in the $(t+1)^{th}$ iteration. The RCM decoding algorithm includes initialization, horizontal decoding, vertical decoding and decision steps. The details of the RCM decoding algorithm are shown in [1].

3 Shaping Method

In this section, we firstly propose the constellation shaping method TR2C. Secondly, we present the geometrical method of estimating noise variance. Finally, we give out the modification of RCM decoding.

3.1 TR2C

The idea of RCM constellation shaping is that points in rectangle constellation is compressed into a circle via geometry transform, which is used generally in digital image processing. So, we call this approach as transform from rectangular to circle. TR2C reduces the amplitude of points with higher amplitude and lower probability, such that increase average amplitude of all constellation points. Therefore, TR2C reduces the PAPR of the whole constellation. It is noted that TR2C is different with traditional constellation shaping technology, which partitions a constellation into a few sub-constellations. The details of TR2C is following.

Assume that the original constellation point is $z = x + i \cdot y$, the point shaped by TR2C is $z_f = x_f + i \cdot y_f$, the point with awgn noise is $z_n = x_n + i \cdot y_n$, the point of TR2C inverse transform is $\hat{z} = \hat{x} + i \cdot \hat{y}$. Obviously, the 2-D coordinates of above four points are (x, y), (x_f, y_f),(x_n, y_n) and $(\hat{x}, \hat{y})$, respectively.

– TR2C Transform if $x == 0 \&\& y == 0$, then $x_f = 0, y_f = 0$. if $|x| \geq |y|$, then $r = \sqrt{1 + (\frac{y}{x})^2}$, otherwise $r = \sqrt{1 + (\frac{x}{y})^2}$. Finally, we calculate $x_f = \frac{x}{r}$ and $y_f = \frac{y}{r}$.

– TR2C Inverse Transform if $x_n == 0\&\&y_n == 0$, then $\hat{x} = 0, \hat{y} = 0$. if $|x_n| \geq |y_n|$, then $\tilde{r} = \sqrt{1 + (\frac{y_n}{x_n})^2}$; otherwise $\tilde{r} = \sqrt{1 + (\frac{x_n}{y_n})^2}$. Finally, we calculate $\hat{x} = x_n \cdot \tilde{r}$ and $\hat{y} = y_n \cdot \tilde{r}$.

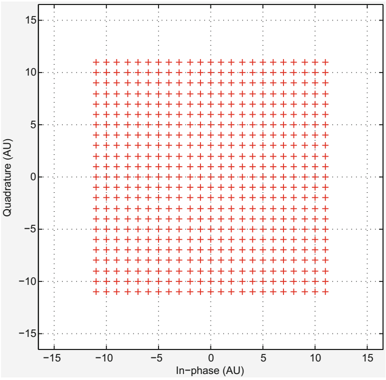

Fig. 3. Original constellation

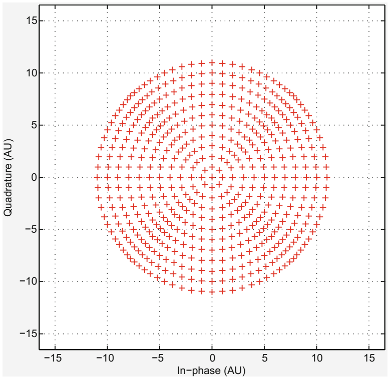

Fig. 4. Shaping constellation

Figures 3 and 4 show the original constellation and shaped constellation with a weighted set $W = \{\pm 1, \pm 2, \pm 4, \pm 4\}$, respectively. We can observe three features from the two figures. The first one is that shaped constellation is circle. The second one is that the points in first and third quadrant are symmetrical about the line $y = x$, and the points in second and fourth quadrant are symmetrical about the line $y = -x$. The last one is that the distribution of points closer to symmetrical line is dense, while the distribution of points farther from symmetrical line is sparse.

3.2 Noise Variance Estimation

Figure 5 describes the change process of constellation points in different stages of shaping transform. If shaping transform is operated on Z, we can get formula (2).

$$Z_f = f(Z) = \frac{Z}{r}. \tag{2}$$

After Z_f through AWGN channel, we can get its noise version

$$Z_n = Z_f + n, \tag{3}$$

where $n \sim CN\left(0, 2\sigma_n^2\right)$. Then, shaping inverse transform is operated on Z_n, we get

$$\tilde{Z} = f^{-1}(Z_f) = \frac{\tilde{r}}{r} Z + \tilde{r} n. \tag{4}$$

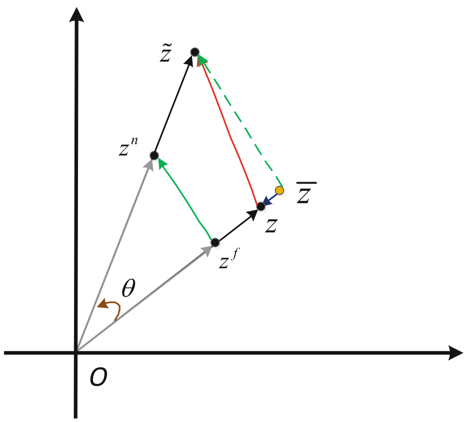

Fig. 5. The change process of constellation points in different stages of shaping transform

After simple deformation for formula (4), we finally have

$$\tilde{Z} = Z + \left(\frac{\tilde{r}}{r} - 1\right) Z + \tilde{r}n, \tag{5}$$

where the second item of right side in formula (5) is cause by transform and AWGN noise, The third item of right side in formula (5) is the scale of AWGN noise.

In Fig. 5, $z_f \rightarrow z_n$ denotes AWGN noise, $z \rightarrow \tilde{z}$ denotes the real noise because of AWGN noise and inverse transform of TR2C. We can observe two features from the Fig. 5. Firstly, The transform and inverse transform of TR2C are not change the phase of constellation point, and only change its amplitude. Therefore, z_f and z are on the line $\boldsymbol{Oz}$, z_n and $\tilde{z}$ are on the other line $\boldsymbol{O\tilde{z}}$. Secondly, AWGN noise is the factor that result in the change of phase and amplitude of constellation points. Especially, the inverse transform of TR2C amplify the noise, and it introduces the error of transform in communications system based on RCM.

How to estimate accurately noise variance is a key step in TR2C. Because that it is hard to give out the closed-form expression of probability distribution function (PDF) of noise, we propose geometrical method to estimate approximately the noise variance of scaling up. In practice, we only make use of received symbols vector z_n and AWGN noise variance σ_n^2 to estimate. The geometrical method of estimating noise variance includes two steps. Firstly, according to $\tilde{z}$, we construct triangle $\Delta_{\bar{z}O\tilde{z}}$ similar with triangle $\Delta_{z_n O z_f}$, as shown in Fig. 5. Then, using the proportional relationship among edges in similar triangles, we can approximately calculate the variance of real noise, i.e., $\bar{z} \rightarrow \tilde{z}$.

After analysis, we find that our estimation method abandons the noise expressed by $\bar{z} \rightarrow z$, which corresponding to the second item of right side of formula (5). Furthermore, the variance of AWGN affects the accuracy of estimating for real noise variance. The AWGN noise variance greater, the estimation of real noise variance is more inaccurate. On the contrary, the AWGN noise variance smaller, the estimation is more accurate.

3.3 Modification of RCM Decoding

There is two places that should be modified in standard RCM decoding algorithm. The one is that real noise variance is should be estimated via geometrical estimation method. The another one is that each soft demodulation symbol uses itself noise variance to compute Eq. (6) in horizontal iteration.

$$P(Y_{ij}) = \left\{ \bigotimes_{m \in R_i \backslash j, l_m \neq l} P(w_{l_m} x_m) \right\} \bigotimes P(n_i), \tag{6}$$

where $\bigotimes$ is the convolution of PDFs. The distribution of weighted variables should be $P(w_{l_m} x_m = 0) = q_{mi}^{(t-1)}(x_m = 0)$, $P(w_{l_m} x_m = w_{l_m}) = q_{mi}^{(t-1)}(x_m = 1)$, and $P(n_i) \sim \mathcal{N}(0, \sigma^2)$ for zero-mean Gaussian channel.

4 Simulation Results

In this section, we present the performance of proposed TR2C method in terms of PAPR, BER and capacity metrics.

In PAPR simulation, we use Eqs. (7) and (8) to evaluate the PAPR performance of TR2C. PAPR is calculated by

$$PAPR\ (dB) = 10\log_{10}(\frac{\max\limits_{0 \le i \le N} \{|x(i)|^2\}}{\frac{1}{N}\sum\limits_{i=1}^{N} |x(i)|^2}), \tag{7}$$

where x_i is modulation symbol, N is the length of modulation symbol vector. Furthermore, complementary cumulative distribution function (CCDF) is calculated by

$$\begin{aligned} CCDF(PAPR) &= \Pr(PAPR \succ PAPR_0) \\ &= 1 - (1 - e^{-PAPR_0})^N, \end{aligned} \tag{8}$$

where $PAPR_0$ is the threshold value of PAPR, and N is the number of count. The simulation runs 10^6 frames random data. Figure 6 shows PAPR performance comparison between original and shaped constellation of W. In this figure, "W" indicates the original constellation, and "$TR2C - W$" indicates the shaped constellation. We can observe that "$TR2C - W$" achieves significant gain compare with "W". "$TR2C - W$" reaches maximum PAPR gain 1.8 dB at CCDF $= 10^{-4}$.

In BER simulation, we test TR2C at transmission rates as 1 *bps*. The channel SNR is from 5 dB to 15 dB. The size of bit block is 400. The BER is calculated after transmitting 10^5 bit blocks. Figure 7 shows the BER performance comparison between original constellation and shaping constellation. From the results

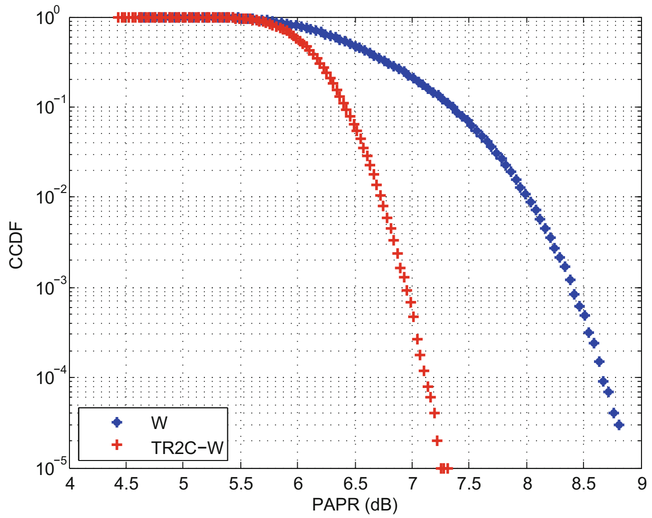

Fig. 6. PAPR performance comparison between original constellation and shaped constellation

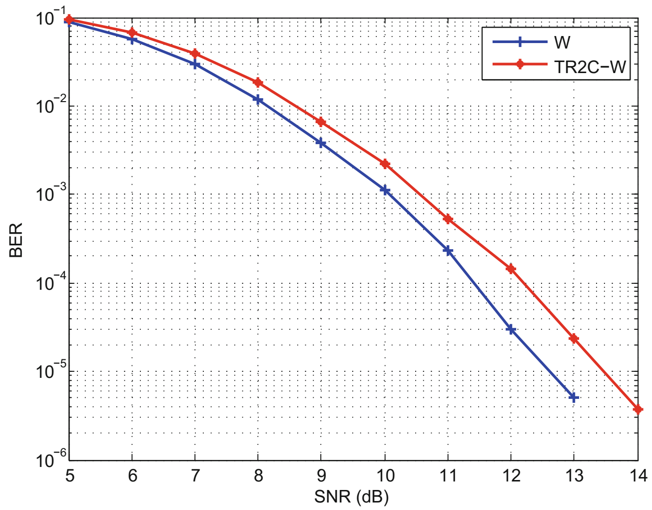

Fig. 7. BER performance comparison between original constellation and shaped constellation

in Fig. 7, we can observe that original constellation has better BER performance then shaped constellation. Comparing with "W", "$TR2C - W$" suffers from a performance loss about 0.8 dB at BER = 10^{-5}. The BER performance loss is increase slightly with the improvement of SNR. But the loss can be accepted

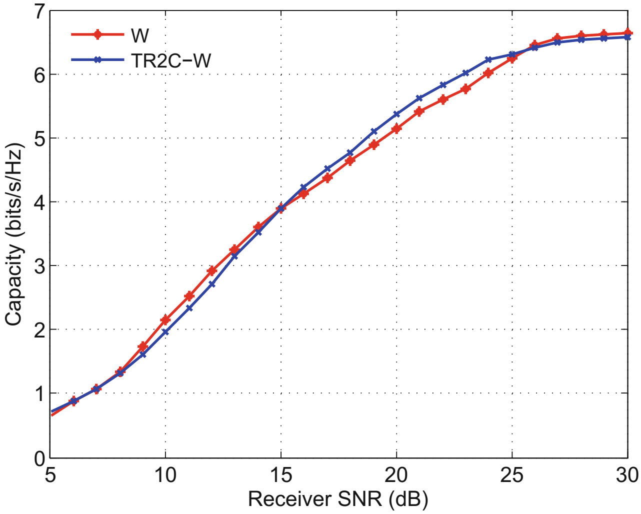

Fig. 8. Capacity performance comparison between original constellation and shaped constellation

relative to PAPR gain. And, the overall performance of our proposed system can be improved by improving the transmission power.

Finally, we evaluate the capacity performance of TR2C. Figure 8 shows the results under AWGN channel models. In this figures, the x-axis is receiver SNR. We run simulation on every integer SNR from 5 to 30 dB. At each SNR, 1000 data frames are transmitted, each with size 400 bits. The y-axis in figures are goodput. It is computed by transmission rate times 1-PER for 802.11a modulation and coding schemes. From Fig. 8, we can observe that shaped constellation suffers from a little capacity performance loss compare with original constellation when SNR $\leq$ 15 dB. Specially, the maximum loss gain is 0.2014 $bits/s/Hz$ at SNR = 12 dB. This is conformity with the BER performance comparison shown in Fig. 7. In other hand, shaped constellation achieves a little capacity performance gain when SNR in $[15, 26]$ dB, and the maximum capacity gain is 0.236 $bits/s/Hz$ at SNR = 23 dB. On the whole, TR2C scheme achieves the capacity performance that close to original constellation when SRN from 5 dB to 30 dB.

5 Conclusion

This paper presents an novel constellation shaping method TR2C to reduce PAPR for RCM, and gives out the geometrical method of estimating noise variance. TR2C can achieve better overall system performance via improving transmission power, and achieves the capacity performance that close to original RCM in SNR range [5,30] dB. One thing to be noted is that our proposed shaping scheme makes the Euclidean distance of points in constellation smaller. In the future, we will work on the optimization design of our proposed constellation shaping method. Especially, we will focus on the design that can enlarge the Euclidean distance of constellation points, improve capacity performance and reduce PAPR for RCM.

Acknowledgment. The authors would like to thank the anonymous reviewers. This work was supported in part by the Nature Science Foundation of China (No. 61305052), the NSF of Jiangxi province of China (No. 20142BAB217007), the Science and Technology Plan Funding of Jiangxi province of China (No. 20151102040042), and Science and Technology Projects of education bureau of Jiangxi province of China (No. GJJ150984, No. GJJ151001).

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