Distributed Sharing of Base Stations for Greening: A Population Game Approach

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Abstract. Towards better QoSs and larger market share in highly competitive cellular network market, many mobile network operators (MNOs) aggressively invest in their base station (BS) deployment. As a result, BSs are densely deployed and incur a lot of energy consumptions, resulting in a large portion of operation cost. To save energy consumption, sharing BSs among different MNOs is a promising approach, where each user can be served from any BSs regardless of his or her original subscription, i.e., roaming. In this paper, we address the question of how many users should be roamed in a distributed manner with the goal of some sense of optimality. To answer this question, we take a population game approach, where we model flow-level dynamics of traffic and define an user association game among different MNOs. We prove that the game is an exact potential game with *'zero'-price-of-anarchy.* We develop a distributed algorithm that converges the NE (which is a socially optimal point) that can be used as a light-weight, dynamic user association algorithm.

Keywords: Greening · Base station sharing · Population game theory

1 Introduction

With increasing demands of mobile data traffic and large market competition among MNOs, most MNOs aggressively enhance their spectral efficiency by densely deploying BSs. As a result, the current BSs are densely deployed in many places, which incurs a lot of BS energy consumptions with large operating expenditures (OPEX). To save energy waste, BS sharing is a promising solution, where depending on the traffic conditions and the user locations, more energy-wise efficient association can be applied. However, without a suitable user association rule, the effects of BS sharing would not be impressive, which we address in this paper.

Main contribution. In this paper, we study user association policies under a certain roaming agreement among existing MNOs, where each MNO strategically tries to minimize their OPEX by regulating how users should behave for

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energy efficiency. The user association determines "how many users to roam?" and depends on some key factors such as roaming price and BS deployment of each MNO. The main contribution of this paper is summarized as follows:

- Determining user association in BS sharing is a very challenging problem and often losing tractability, when we consider user-level QoS. Thus, we take a population game approach, in which all users are categorized by different groups according to their adaptive modulation and coding (AMC) level and original subscription. This gives us a mathematically tractable framework with implications for a distributed user association algorithm. In our model, we take a flow-level performance (such as file-transfer delay) into account for measuring user-level QoS as done in [1].
- The challenges lying in analyzing the game are (a) the complex couplings among QoS, BS energy consumption and roaming fee, each of them depends on the other MNO's roaming price, heterogeneous BS deployment, and user distribution as well as (b) finding a distributed user association algorithm which has implementable complexity. We first show that user association population game is an exact potential game which has an NE with '*zero-price-of-anarchy*'. Next, we also study the three evolutionary dynamics of game that provably converge to the NE, and propose a distributed user association algorithm inspired by the best response dynamic, which is one of the evolutionary dynamics considered. Finally, we verify the user-level QoS and greening effects in BS sharing through numerical simulations.

Related work. User association in BS sharing is proposed in [2–7]. Most work largely relies on packet-level throughput maximization [2,5,6] with an ideal assumption, in which all MNO have identically the same BS deployment for mathematical tractability. The authors in [3] only consider power consumption and roaming fee in BS sharing regardless of user-level QoS. The scope of the study in [4] is uplink BS sharing. Our work is mainly motivated by [7], the authors first consider a flow-level performance in BS sharing under some assumptions such as Shanon capacity based channel rate and existence of synchronous clock in user association. The main difference between [7] and our works is that we consider practical channel rate according to AMC-level with asynchronous user association clock. For the single MNO, the flow-level performance is considered in [1,8,9] and our work is motivated by [8–11] in the context of multiple MNOs. The authors in [9] take a population game for user association, in which all users behave to maximize one objective of the single MNOs, while our work considers competition among multiple MNOs with roaming fee in population game.

2 Model

2.1 System Model

Network and BS sharing service. We consider a set $\mathcal{M}$ of multiple MNOs, and each of them has operating BSs denoted by a set $\mathcal{B}_m$, respectively. For simple notations, we define a set of other MNOs $-m \doteq \mathcal{M} \setminus \{m\}$, and a set of entire BSs $\mathcal{B} \doteq \bigcup_{m \in \mathcal{M}} \mathcal{B}_m$, and we abuse the notation of m, where we use $m(b)$ to indicate the MNO who owned the BS b. For the service model, we consider that each user can subscribe only one MNO, but the user can be served from any BSs irrespective of her original subscription, if her MNO pays a certain roaming fee as done in [2–7]. Also, we assume that all MNOs do not differentiate in the service priority between roaming and unroaming users.

Users. We assume that there exist sufficiently many users in the cellular networks to consider the society of continuous mass of user groups called *classes*. The set of classes is denoted by $\mathcal{Q}$, and each class q has a mass denoted by d_q. We consider that each user in a class q commonly shares (i) original subscription, (ii) the set of supporting BSs, (iii) AMC-level from the supporting BSs, and (iv) traffic characteristics. Due to the need of denoting the set of classes that share original subscription, we occasionally use $\mathcal{Q}_m$, in which the original subscription of the classes ($q \in \mathcal{Q}_m$) is the MNO m. Note that the subscription of user is mutually exclusive (i.e., $\mathcal{Q}_m \bigcap \mathcal{Q}_n = \phi$, for all $m \neq n$).

Traffic, capacity and loads. We assume that users in a class q have identically independent Poisson arrival traffic with rate λ_q, and its file size is independently distributed with mean $1/\mu_q$. Therefore, the unit mass in class q generates $\gamma_q = \lambda_q/\mu_q$ traffic intensity, and the class q totally generates $\gamma_q d_q$ traffic intensity. The users in q experience same data rate c_q^b when associating with BS b. Note that data rate only depends on AMC-level between the user and BS b, thus, user group would not be located on a point but on a region. For a pair of class q and BS b, we define *system-load intensity* as $\varrho_q^b \doteq \frac{\gamma_q}{c_q^b}$, which represents the service-time-portion of traffic intensity γ_q in BS b. For a given BS b, we introduce an association vector $\boldsymbol{y}^b \doteq (y_q^b : q \in \mathcal{Q})$, in which $y_q^b \in [0, d_q]$ denotes the fraction of class q's mass that are associated with BS b, where $\sum_{b \in \mathcal{B}} y_q^b = d_q$. For notational convenience later, we also use $\boldsymbol{y} \doteq (\boldsymbol{y}^b : b \in \mathcal{B})$ to denote the entire association vector. For a given association vector $\boldsymbol{y}$, we define *system-load* in BS b as $\rho^b(\boldsymbol{y}^b) \doteq \sum_{q \in \mathcal{Q}} \varrho_q^b y_q^b$.

3 Problem Formulation: Game

We consider a population game played by all users called *user association population game (UAPG)*, in which each user has an individual payoff function regulated by the MNO that he or she subscribes, and selfishly determines associating BS. Note that user association game implicitly reflects the selfish behavior of each MNO to minimize their cost (or equivalently maximize their revenue) by regulating the subscriber's payoff function under given roaming price $\boldsymbol{k} \doteq (k_m : m \in \mathcal{M})$ by a roaming agreement among MNOs a priori. In order to show the regulation rationale, we first describe a population game and we will compare the NE of the population game to that of conventional BS sharing game played by MNO (as done in [7]) in Sect. 4.

3.1 Social Objective

In order to include the selfish behavior of MNOs, we consider a social objective of UAPG as the potential function of BS sharing game as follows.

$$\mathcal{V}(\boldsymbol{y}) = -\sum_{b\in\mathcal{B}}\Big\{ \underbrace{\phi_\alpha(\boldsymbol{y}^b)}_{(a)} + \underbrace{\eta\mathcal{E}^b(\boldsymbol{y}^b)}_{(b)} + \underbrace{k_{m(b)} \sum_{q\notin\mathcal{Q}_{m(b)}} g^b(y_q^b)}_{(c)} \Big\}, \quad (1)$$

where *(a)* is flow-level performance, *(b)* is BS power consumption, and *(c)* is roaming fee. In detail, *(a)* flow-level performance (such as file-transfer delay) is modeled by:

$$\phi_\alpha(\boldsymbol{y}^b) = \begin{cases} \frac{(1-\rho^b(\boldsymbol{y}^b))^{1-\alpha}}{\alpha-1}, & \text{if } \alpha \neq 1, \\ \log(\frac{1}{1-\rho^b(\boldsymbol{y}^b)}), & \text{if } \alpha = 1, \end{cases} \quad (2)$$

where the parameter $\alpha \geq 0$ characterized cost for flow-level performance. It is well known that the function represents the summation of user rate when $\alpha = 0$, and the summation of average delay when $\alpha = 2$ by [1]. For BS power consumption *(b),* we consider BS load proportional BS power consumption for each BS b, modeled by:

$$\mathcal{E}^b(\boldsymbol{y}^b) = \beta^b E^b \rho^b(\boldsymbol{y}^b) + (1-\beta^b)E^b, \quad (3)$$

where $\beta^b \in [0, 1]$ is a parameter quantifying the portion of load proportional power and E^b is maximum BS power consumption when fully utilized (i.e., $\rho^b(\boldsymbol{y}^b) = 1$). Note that BS b is ideally *energy-proportional* when $\beta^b = 1$, but, β^b ranges from 0.5 to 0.8 in practical BSs [12]. In *(c),* for a given BS b, the function $g^b(y_q^b)$ represents the summation of load and BS power consumption where the original subscription of class q is not the MNO who owns BS b (i.e., roaming traffic in class q) as follows.

$$g^b(y_q^b) = \varrho_q^b \cdot y_q^b + \eta\beta^b E^b \varrho_q^b \cdot y_q^b. \quad (4)$$

Note that $\varrho_q^b \cdot y_q^b$ represents incurred load on BS b by the amount of y_q^b mass of class q. In (1), the parameter $\eta \geq 0$ trade off flow-level performance and BS power consumption. The large η implies MNOs give higher priority to BS power consumption than flow-level performance when operating cellular networks. The value k_m, which is given by some constant K, and is unit roaming price determined by each MNO when they make an agreement on roaming. Thus, social objective (1) represents negative total costs for entire traffic service (including roaming and unroaming) in whole cellular networks.

3.2 Payoff Function

We now introduce the payoff function for a class q in our population game as follows.

$$F_q^b(\boldsymbol{y}) \doteq \underbrace{\frac{-\varrho_q^b}{(1-\rho^b(\boldsymbol{y}))^\alpha}}_{(i)} - \underbrace{\eta\beta^b E^b \varrho_q^b}_{(ii)} - \underbrace{k(b,q)\varrho_q^b(1+\eta\beta^b E^b)}_{(iii)}, \tag{5}$$

where $k(b,q)$ represents the unit roaming price of BS b's owner, when q is not the subscribers of the owner and 0 otherwise (i.e., if $q \notin \mathcal{Q}_{m(b)}$ then $k(b,q) = k_{m(b)}$, and $k(b,q) = 0$ for otherwise). The payoff function is composed of three part: (i) selfish QoS cost, (ii) BS power pricing, and (iii) roaming pricing.

(i) Selfish QoS cost: The first term describes selfish QoS cost motivated by flow-level performance cost as described in (2). Note that for $\alpha = 1$, this term represents to *conditional delay*, where the conditional delay is the expected file-transfer time that a user in class q experiences when she is associated with BS b as described in [9,13].

(ii) BS power pricing: The second term denotes the increments in BS power consumption when the unit mass of class q is associated with BS b. Note that for a user in class q, this term considered as a proportional factor of power increment when the user is associated with BS b, thus actual power increment is multiplication of user's mass x and this term (i.e., $\eta\beta^b E^b \varrho_q^b \cdot x$).

(iii) Roaming pricing: The third term corresponds to the incurred roaming fee by unit mass of class q. Similar to BS power pricing, a user in class q generates roaming fee according to her mass x with proportional to this term, if the user is associated with BS b (i.e., $k(b,q)\varrho_q^b(1+\eta\beta^b E^b) \cdot x$).

4 Equilibrium Analysis

In this section, we analyze UAPG for which we exploit the potential function of the game. Primary issues that we are interested in include the existence of NE, price-of-anarchy, and the existence of a distributed user association algorithm which converges to the NE.

4.1 Price-of-Anarchy and Existence of Equilibrium

Prior to describe price-of-anarchy and equilibrium, we first show that our game is an exact potential game with a certain potential function that gives us the insight of price-of-anarchy and the existence of equilibrium.

Theorem 1. *The user association game is an exact potential game with the following potential function $V(\boldsymbol{y})$:*

$$V(\boldsymbol{y}) = -\sum_{b\in\mathcal{B}} \Big\{ \phi_\alpha(\boldsymbol{y}^b) + \eta\mathcal{E}^b(\boldsymbol{y}^b) + k_{m(b)} \sum_{q\notin\mathcal{Q}_m} g^b(y_q^b) \Big\}. \tag{6}$$

Proof. For a continuous player set (e.g., large population of player), it is suffice to show that there is a continuously differentiable function whose gradient for population is same as the payoff function of each class by [14]. The gradient of the potential function (6) for all population is given by:

$$\nabla_{\boldsymbol{y}} V(\boldsymbol{y}) = \Big(\frac{\partial V(\boldsymbol{y})}{\partial y_q^b} : q \in \mathcal{Q}, b \in \mathcal{B}\Big)$$

For all $q \in \mathcal{Q}$ and $b \in \mathcal{B}$, in the case $\alpha \neq 1$,

$$\begin{aligned}
\frac{\partial V(\boldsymbol{y})}{\partial y_q^b} &= -\frac{\partial}{\partial y_q^b}\Big[\sum_{b\in\mathcal{B}}\Big\{\frac{(1-\rho^b(\boldsymbol{y}^b))^{1-\alpha}}{\alpha-1} + \eta(\beta^b E^b \rho^b(\boldsymbol{y}^b) + (1-\beta^b)E^b) \\
&\quad + k_{m(b)} \sum_{q\notin\mathcal{Q}_{m(b)}} \varrho_q^b y_q^b + \eta\beta^b E^b \varrho_q^b y_q^b\Big\}\Big] \\
&= -\frac{\partial}{\partial y_q^b}\Big[\sum_{b\in\mathcal{B}}\Big\{\frac{(1-\sum_{q\in\mathcal{Q}}\varrho_q^b y_q^b)^{1-\alpha}}{\alpha-1} + \eta(\beta^b E^b \sum_{q\in\mathcal{Q}} \varrho_q^b y_q^b + (1-\beta^b)E^b) \\
&\quad + k_{m(b)} \sum_{q\notin\mathcal{Q}_{m(b)}} \varrho_q^b y_q^b + \eta\beta^b E^b \varrho_q^b y_q^b\Big\}\Big] \\
&= -\Big[-\varrho_q^b\Big(\frac{1-\alpha}{\alpha-1}\Big)(1-\sum_{q\in\mathcal{Q}}\varrho_q^b y_q^b)^{-\alpha} + \eta\beta^b E^b \varrho_q^b + k(b,q)(\varrho_q^b + \eta\beta^b E^b \varrho_q^b)\Big] \\
&= -\varrho_q^b\Big[\frac{1}{(1-\rho^b(\boldsymbol{y}))^\alpha} + \eta\beta^b E^b + k(b,q)(1+\eta\beta^b E^b)\Big] = F_q^b(\boldsymbol{y}).
\end{aligned}$$

For the case $\alpha = 1$,

$$\begin{aligned}
\frac{\partial V(\boldsymbol{y})}{\partial y_q^b} &= -\frac{\partial}{\partial y_q^b}\Big[\sum_{b\in\mathcal{B}}\Big\{\log\Big(\frac{1}{1-\rho^b(\boldsymbol{y}^b)}\Big) + \eta(\beta^b E^b \rho^b(\boldsymbol{y}^b) + (1-\beta^b)E^b) \\
&\quad + k_{m(b)} \sum_{q\notin\mathcal{Q}_{m(b)}} \varrho_q^b y_q^b + \eta\beta^b E^b \varrho_q^b y_q^b\Big\}\Big] \\
&= -\frac{\partial}{\partial y_q^b}\Big[\sum_{b\in\mathcal{B}}\Big\{\log\Big(\frac{1}{1-\sum_{q\in\mathcal{Q}}\varrho_q^b y_q^b}\Big) + \eta(\beta^b E^b \sum_{q\in\mathcal{Q}} \varrho_q^b y_q^b + (1-\beta^b)E^b) \\
&\quad + k_{m(b)} \sum_{q\notin\mathcal{Q}_{m(b)}} \varrho_q^b y_q^b + \eta\beta^b E^b \varrho_q^b y_q^b\Big\}\Big] \\
&= -\Big[-\varrho_q^b\Big(\frac{-1}{(1-\sum_{q\in\mathcal{Q}}\varrho_q^b y_q^b)^2}\Big)(1-\sum_{q\in\mathcal{Q}}\varrho_q^b y_q^b) + \eta\beta^b E^b \varrho_q^b \\
&\quad + k(b,q)(\varrho_q^b + \eta\beta^b E^b \varrho_q^b)\Big] \\
&= -\varrho_q^b\Big[\frac{1}{(1-\rho^b(\boldsymbol{y}))} + \eta\beta^b E^b + k(b,q)(1+\eta\beta^b E^b)\Big] = F_q^b(\boldsymbol{y}),
\end{aligned}$$

which completes the proof.

Lemma 1. *The potential $V(\boldsymbol{y})$ is a concave function in $\boldsymbol{y}$.*

Proof. The functions, $V(\boldsymbol{y})$, $\phi_\alpha(\boldsymbol{y}^b)$, and $\mathcal{E}^b(\boldsymbol{y}^b)$ are convex functions in $\rho^b(\boldsymbol{y}^b)$, respectively, and $\rho^b(\boldsymbol{y}^b)$ is a weighted (ϱ_q^b) linear combination of $\boldsymbol{y}^b$. Thus, $V(\boldsymbol{y})$ and $\phi_\alpha(\boldsymbol{y})$ become convex functions in $\boldsymbol{y}$ by convex-preserving operation. The function $g^b(y_q^b)$ is definitely a convex function in y_q^b as described in (4). Thus, $V(\boldsymbol{y})$ is a concave function in $\boldsymbol{y}$ (by inversed sign) due to the property of convex preserving on summation.

Theorem 2. *User association game has an NE which has zero price-of-anarchy.*

Proof. The association vector $\boldsymbol{y}$ is bounded by the mass of each class q (i.e., $d_q \in [0, d_q]$). Thus, there is a global maximal point on the range of association vector, and the point is an NE by well know property of potential game [14], in which the NE should satisfy KKT conditions for a maximizer of the potential function $V(\boldsymbol{y})$. Zero price-of-anarchy is also easily verified by the potential function. Since the potential function (6) is exactly equal to the social objective as described in (1) and KKT conditions are necessary and sufficient condition for a global maximizer in concave function, the NE satisfying KKT conditions should be a global maximizer of the social objective.

Note that there could be multiple NEs in UAPG, because the NE only implies an assigned amount of population for all pairs of user groups and BSs, and the assigned population would be achieved by various user associations when we consider identical users who share the traffic characteristics and AMC-level in each class.

Rationality for MNOs. As we mentioned earlier, for all classes in MNO m (i.e., $q \in \mathcal{Q}_m$), m regulates the class q's payoff function to maximize their economical revenue (or minimize cost) in our game, while the MNO m hopefully behaves like the game, directly played by MNOs as done in [7]. Note that our game and the game played by MNOs have the exactly same potential function (i.e., equivalent game) for an arbitrary unit roaming price. Thus, the payoff function (5) is rational to each MNO, and implicitly considers the selfish behaviors of all MNOs for maximizing their revenue.

4.2 Evolutionary Dynamics and Distributed Association Algorithm

Developing a distributed algorithm for user association is important in practice, because, if it exists, high energy-efficiency can be achieved with low-cost operations of the networks. In this subsection, we propose a distributed user association algorithm, motivated by an *evolutionary dynamic* that converges to the NE in population game. For convenience in understanding, we first introduce three well-known evolutionary dynamics [15], Replicator dynamic, Brown-von Neumann-Nash (BNN) dynamic, and best response dynamic with the definitions and the convergence properties, and then, propose a distributed user algorithm that can work practical cellular networks.

Replicator dynamic. One of the best known dynamic in evolutionary game is the replicator dynamic, and its definition is as follows.

$$y_q^{b,t+1} = T_q^b(\boldsymbol{y}^t) \doteq y_q^{b,t}\Big(F_q^b(\boldsymbol{y}^t) - \frac{1}{d_q}\sum_{b\in\mathcal{B}} y_q^{b,t}F_q^b(\boldsymbol{y}^t)\Big), \tag{7}$$

where $\boldsymbol{y}^t$ is the social-state $\boldsymbol{y}$ at time step t, and the term $(F_q^b(\boldsymbol{y}^t) - \frac{1}{d_q}\sum_{b\in\mathcal{B}} y_q^{b,t}F_q^b(\boldsymbol{y}^t))$ is the excess payoff of strategy b in class q. Under replicator dynamic, a user randomly selects an opponent in the same class and changes her strategy to the strategy of opponent, if the payoff of the opponent strategy is higher than her own with a probability proportional to the payoff difference.

BNN dynamic. The definition of BNN dynamic is as follows.

$$y_q^{b,t+1} = T_q^b(\boldsymbol{y}^t) \doteq d_q k_q^b(\boldsymbol{y}^t) - y_q^b\sum_{b\in\mathcal{B}} k_q^b(\boldsymbol{y}^t), \tag{8}$$

where $k_q^b(\boldsymbol{y}^t) = \max\{F_q^b(\boldsymbol{y}^t) - \frac{1}{d_q}\sum_{b\in\mathcal{B}} y_q^{b,t}F_q^b(\boldsymbol{y}^t), 0\}$. In BNN dynamic, each user randomly chooses a strategy i and changes her strategy to i with a probability proportional to strategy i's excess payoff, if the payoff of i exceeds the payoff of her own at every updating strategy epoch.

Best response dynamic. In best response dynamic, each user selects her strategy that maximizes her payoff function for a given social-state $\boldsymbol{y}$ as follows.

$$y_q^{b,t+1} = T_q^b(\boldsymbol{y}^t) \doteq \arg\max_{b\in\mathcal{B}} F_q^b(\boldsymbol{y}^t) \tag{9}$$

Note that a user selects exact one pure strategy in best response dynamic, however, when we consider a class, in which infinitesimal users individually select their strategies, best response (9) behaves like a mixed strategy in population game.

Convergence. It is well-known by [14], a dynamic, satisfying *positive correlation (PC)* and *noncomplacency (NC)* conditions, converges to the NE in potential game which has a smooth potential function. The first condition, PC, states that payoff and drift rate of strategy in dynamic are positively correlated (i.e., weak-monotonicity in dynamic). The details of PC is $T(\boldsymbol{y})\cdot F(\boldsymbol{y}) \doteq \sum_{q\in\mathcal{Q}}\sum_{b\in\mathcal{B}} T_q^b(\boldsymbol{y})F_q^b(\boldsymbol{y}) > 0$, whenever $V(\boldsymbol{y}) \neq 0$. For every trajectory of dynamic, the condition PC implies that (i) the potential function is weakly-increasing (i.e., $\frac{d}{dt}V(\boldsymbol{y}^t) = \nabla_{\boldsymbol{y}^t}V(\boldsymbol{y}^t)\cdot\boldsymbol{y}^t = T(\boldsymbol{y}^t)\cdot F(\boldsymbol{y}^t) \geq 0$), (ii) there is zero-drift for a stationary point (i.e. $T(\boldsymbol{y}^t) = 0$ whenever $\frac{d}{dt}V(\boldsymbol{y}^t) = 0$). Thus, all trajectories satisfying PC provably converge to a stationary point. However, all stationary points would not be NEs, where the points are either local maximizer or boundary of potential function. The condition, NC, guarantees that a stationary point should be a NE of potential game. By the studies in [14,15], it is verified that BNN and best response dynamic satisfy both PC and NC, but replicator dynamic only satisfies PC in the potential game. For more detail, we refer the readers to [14,15].

Distributed user association algorithm. Low signaling overheads is important in practice. In UAPG, the best response dynamic seems to require less information than the others. In detail, best response dynamic only requires social state $\boldsymbol{y}$, but the others require additional information such as average payoff and opponent's payoff. Thus, we propose a distributed algorithm motivated by best response dynamic.

Distributed user association algorithm

BS algorithm. For every changes in user association, each BS b updates $\rho^{b,t}$ as follows.

$$\rho^{b,t} = \sum_{q \in \mathcal{Q}} \varrho_q^b \cdot y_q^b, \tag{10}$$

and exchange $(\rho^{b,t}, k_{m(b)})$ to all BS in the neighboring BS set, denoted by $\mathcal{N}(b)$. After exchanging the information, each BS broadcasts $\boldsymbol{\rho}^{\mathcal{N}(b),t}, \rho^{b,t}$ and $\boldsymbol{k}$ to all (associated) users.

User algorithm. For a user in some class q, at every *association clock*[a] ticking, the user associates with a BS that satisfies following:

$$\underset{b \in \{i\} \cup \mathcal{N}(i)}{\arg\max} \; -\varrho_q^i \Big\{ \frac{1}{(1-\rho^{i,t})^{\alpha}} + \eta\beta^i E^i + k(i,q)(1 + \eta\beta^i E^i) \Big\}, \tag{11}$$

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[a]We consider each user has an individual clock for determining user association. In detail, this clock would be implemented by many ways such as Poisson clock, and flow arrival and departure time.

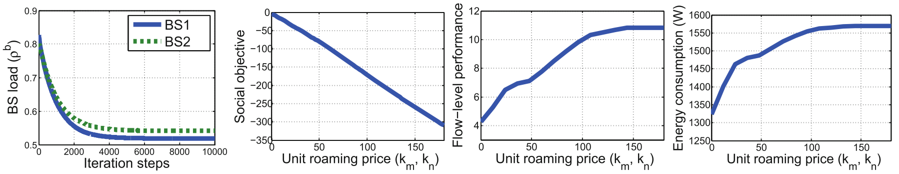
Theorem 3. *The distributed user association algorithm converges to the NE.*

Proof. Our algorithm is a practical version of best response dynamic which satisfies both (i) PC and (ii) NC.

5 Numerical Analysis

In this section, we verify the greening effects of our algorithm inspired by the analysis in the population game framework. In all simulations, we consider a duopoly market (i.e., 2 MNOs denoted by m and n) in 0.5 Km by 0.5 km square area, in which MNO m and n have 1 BS denoted by BS1 and BS2, respectively, where BS1 and BS2 are located at (0,0) and (0.5,0.5)[1], respectively. We consider that users are uniformly distributed in the square area while generating homogeneous traffic requests. For data rate c_q^b, we refer to the pairs of data rate

[1] The unit of axis is km.



(a) Convergence ($k_m = k_n = 1$) (b) Social objective ($\alpha = 2$) (c) Flow-level performance ($\alpha = 2$) (d) Total BS power consumptions ($\alpha = 2$)

Fig. 1. Various results of BS sharing

and AMC-level in Mobile WiMAX [16]. We consider the case when all MNOs adopt a same unit roaming price (i.e., $k_m = k_n$) for roamed traffic due to the symmetry property in unit roaming price under symmetric BS deployment and identical user characteristics.

We first verify the convergence of our algorithm (see Fig. 1(a)). In the figure, the initial points are the BS loads of conventional non-BS sharing and each BS load rapidly decays from the initial point until it converges with iterations. In Figs. 1(b), (c) and (d), we show the impact of our algorithm in terms of potential function, flow-level performance, and BS power consumptions according to given unit roaming price $\boldsymbol{k}$. The result in Fig. 1(b) shows that social objective is maximized when each MNO assigns zero-unit roaming price and it decreases as $\boldsymbol{k}$ increases due to the raised roaming price. As shown in Figs. 1(c) and (d), flow-level performance (i.e., delay when $\alpha = 2$) and total BS power consumptions are increased by expensive roaming price, and finally converge to that in conventional non-BS sharing, since no one is interested in roaming when highly expensive roaming price (e.g., $k_m = k_n \geq 150$) is applied.

6 Conclusions

In this paper, we studied BS sharing under a fixed roaming price using a population game-theoretic approach, and we proposed a practical user association algorithm motivated by an evolutionary dynamic, which is best response dynamic. We further demonstrated that a significant amount of delay and of energy consumption would be reduced by the proposed algorithm.

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