

Joint Power Control and Subchannel Allocation for D2D Communications Underlying Cellular Networks: A Coalitional Game Perspective

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Abstract. A coalition based joint subchannel and power allocation approach is studied to improve the performance of device-to-device (D2D) communication underlying cellular networks with uplink spectrum sharing. To exploit the spectrum reuse gain, we formulate the problem as a coalition formation game. Furthermore, a distributed coalition formation algorithm is devised to assist D2D pairs in joining or leaving a coalition. During the coalition formation process, we introduce an iterative power control method. By using this method, D2D pairs can evaluate their current coalition with D2D sum rate maximization and cellular user equipment protection. Numerical results are provided to corroborate the proposed studies.

Keywords: Coalitional game theory · Power control · Device-to-device communication

1 Introduction

Nowadays, the demand for wireless internet access witnesses a huge increment. Cisco Systems, Inc. estimates that the wireless data traffic will continue to grow exponentially and reach over 24 exabytes per month in 2019. Device-to-device (D2D), a type of proximity communication, has been proposed to work underlying existing cellular network for spectrum efficiency improvement. In a D2D pair, under the control of evolved NodeB (eNB), user equipments (UEs) communicate with each other through direct link instead of resorting to eNB's assistance. Under this network architecture, the spectrum band can be utilized simultaneously by both D2D pairs and traditional cellular pairs, and the D2D links can exploit the spectrum reuse gain without any hardware investment. As a result, D2D communication is involved as a key component in LTE-Advanced systems (Doppler et al. 2009, Lei et al. 2012) and in the fifth generation communication systems (Boccardi et al. 2014, Tehrani et al. 2014).

Despite the benefits, D2D communication may also cause two types of interference: interference to primary cellular user equipments (CUEs) and interference to other D2D pairs that use the same frequency band. Current literature mainly focus on the problems of power control (Yu et al. 2009a,b 2009, Xing and Hakola 2010, Dong et al. 2016), subchannel allocation (Yu et al. 2011, Xu et al. 2013, Xu et al. 2014) and interference management (Janis et al. 2009, Xu et al. 2010) by considering both types of interference. For example, the authors in (Yu et al. 2009) showed that proper power control can coordinate the interference to maximize the sum rate. In (Xu et al. 2013), a reverse iterative combinatorial auction based method was proposed to efficiently allocate subchannel resource to the D2D pairs, which operates in the downlink period of CUEs. These methods manage the interference from D2D to CUEs when D2D pairs operate in the downlink period.

On the other hand, fewer works have considered the interference management that occurs in the uplink transmission. Indeed, the uplink interference management is a more challenging issue because the interference control process is left to multiple UEs instead of the single eNB. Moreover, existing works (Yu et al. 2009a,b Xing and Hakola 2010, Yu et al. 2011, Janis et al. 2009, Xu et al. 2010) only considered the resource allocation and interference management under a restricted scenario where only one D2D pair coexists with one CUE.

Some recent literature (Min et al. 2011, Wang et al. 2013, Feng et al. 2013, Li et al. 2014) considered a more practical scenario with multiple D2D pairs or multiple CUEs. The authors in (Min et al. 2011) studied a case in which multiple CUEs coexist with a D2D pair and proposed a location based interference management approach. The proposed approach defined an interference limited area for D2D pair where CUEs cannot share the spectrum with the D2D pair. The authors in (Wang et al. 2013) assumed that a D2D pair can reuse the channels of multiple CUEs. They developed a suboptimal algorithm to jointly allocate the transmission power of CUEs and the D2D pair such that the throughput of the D2D pair is maximized, and the QoS of CUEs are guaranteed. The authors in (Feng et al. 2013) formulated the resource allocation problem as a system throughput maximization problem with the assumption that the resource of a CUE can be shared at most by one D2D pair. The authors in (Li et al. 2014) introduced coalitional game theory to model the subchannel allocation in D2D communication underlying uplink cellular network. However, they did not study the topic of power control for D2D pairs which may help D2D pairs further exploit the spectrum reuse gain.

In this paper, we formulate the problem of joint subchannel and power allocation for D2D enabling system as a coalition formation game. For the devised game model, a distributed coalition formation algorithm is proposed, where each D2D pair can make decision to leave or join a coalition. Within a specific coalition, each D2D pair tries to optimize the its utility via power control. Here, the utility of each D2D pair is formulated as difference between achieved spectrum efficiency with the priced power cost. Each D2D pair evaluates its satisfaction

level on the current coalition based on its achieved utility. Our contributions are summarized as follows.

- We propose a coalition formation algorithm for D2D pairs to select the subchannel. We prove the coalition formation algorithm converges to a Nash stable partition.
- For a specific coalition, we derive a distributed iterative power control algorithm to mitigate the interference on CUEs and interference among D2D pairs. We also discuss the convergence issue for the power control algorithm.

Simulation results illustrate that the proposed scheme can increase the sum rate of both CUE and D2D pairs. Meanwhile, the proposed algorithm can also reduce the unnecessary coalition switch operations.

The rest of this paper is organized as follows. In Sect. 2, the system model is described. In Sect. 3, we model the D2D pair coalition formation game to allocate the subchannels, and a distributed algorithm is proposed. Moreover, we discuss the power control in each coalition. Simulation results are given in Sect. 4. Section 5 concludes our works.

2 System Model

We consider the uplink of orthogonal frequency division multiple access (OFDMA) based wireless network, where an eNB is located at the center of the cell and multiple UEs are distributed uniformly within the cell. This network contains two types of UEs, i.e., M CUEs and N D2D pairs where $N > M$. Let $\mathcal{M} = \{1, 2, \dots, M\}$ and $\mathcal{N} = \{1, 2, \dots, N\}$ denote the CUE set and the D2D pairs set, respectively. Moreover, the distance between two UEs in a D2D pair satisfies the constraint of D2D communication. We assume all CUEs utilize orthogonal subchannels and D2D pairs share the subchannels with CUEs. The subchannel assignment for CUEs is fixed, and multiple D2D pairs can share one subchannel with the CUE simultaneously to improve the system spectrum efficiency.

Figure 1 illustrates the existing interference under the above network setting in uplink period. We can see that there are two types of interference, e.g., interference among D2D pairs and interference between CUE and D2D pairs. For example, let us consider a case where the 1st and the 2nd D2D pairs share the same subchannel with CUE c_1 . Thus, the corresponding D2D receivers d_1^r and d_2^r are exposed to the interference from CUE c_1 . While the eNB receives interference from d_1^t and d_2^t which are the transmitters of the 1st and the 2nd D2D pairs respectively. Meanwhile, there exists interference between the 1st and the 2nd D2D pairs. CUE c_2 and the 3rd D2D pair use orthogonal subchannels, and as a result, they do not interfere with each other.

As the number of D2D pairs increases, both types of interference will become more severe. Therefore, if interference is not managed properly, the potential gain in spectral efficiency obtained by spectrum sharing will be wiped out. Motivated by this fact, we focus on power control and subchannel assignment for D2D pairs.

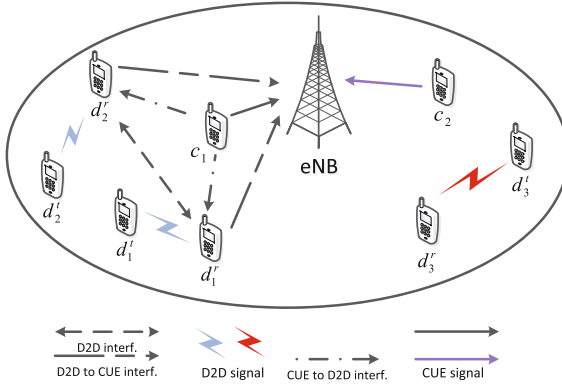


Fig. 1. Multiple D2D pairs coexist with multiple cellular users.

We denote $X = [x_i^k]_{N \times M}$ as the subchannel assignment matrix where x_i^k takes either 1 or 0 to indicate whether the subchannel of k th CUE is assigned to i th D2D pair or not, $i \in \mathcal{N}$ and $k \in \mathcal{M}$. We allow a D2D pair to use only one subchannel, that is, $\sum_{k \in \mathcal{M}} x_i^k \leq 1$. Based on these assumptions, the received signal at the eNB of CUE $k \in \mathcal{M}$ and the signals at receiver of i th D2D pair underlying CUE k can be, respectively, written as

$$y_k = \sqrt{p_k H_k} s_k + \sum_{i \in D_k} x_i^k \sqrt{p_i G_i^k} s_i + n_k \quad (1)$$

and

$$z_i^k = \sqrt{p_i h_{i,i}^k} s_i + \sqrt{p_k g_i^k} s_k + \sum_{j \in D_k \setminus \{i\}} x_j^k \sqrt{p_j h_{j,i}^k} s_j + n_i^k \quad (2)$$

where s_i and p_i are the signal and the transmit power of the i th transmitter, $i \in \mathcal{M} \cup \mathcal{N}$; the terms H_k and G_i^k denote the channel gain of CUE k and the interference gain between D2D pair i to CUE k , respectively; $h_{i,j}^k$ is the channel gain between the transmitter of D2D pair i to the receiver of D2D pair j underlying CUE k ; the set D_k represents the D2D pairs share the subchannel of CUE k , $D_k \subset \mathcal{N}$, and D_k can be empty; n_k and n_i^k are the additive white Gaussian noise of CUE k and D2D pair i underlying CUE k with power N_0 .

3 Interference Mitigation as a Coalition Formation Game

In this section, we first present the coalition formation game formulation. Then, we analyze the power control issue in a specific coalition. At last, we propose a distributed coalition formation algorithm.

3.1 Coalitional Game in Partition Form

In the studied network, there are M CUEs and N D2D pairs, where D2D pairs choose to share the subchannels with CUEs to enhance the network sum rate

throughput. We allow multiple D2D pairs operating on the same subchannel of a CUE to form cooperative group, i.e., *coalition*. As a result, we denote the *coalition partition* as $\pi = \{D_1, D_2, \dots, D_M\}$, where $\bigcup_{k=1}^M D_k = \mathcal{N}$, $D_k \cap D_m = \phi$, $\forall k, m = 1, 2, \dots, M$ and $k \neq m$. Note that $D_k = \phi$ means no D2D pair reuses the subchannel of CUE k .

Based on above analysis, we can denote the received SINR for CUE k and D2D pair i as

$$\Gamma_k = \frac{p_k H_k}{\sum_{i \in D_k} x_i^k p_i G_i^k + N_0} \tag{3}$$

and

$$\gamma_i^k = \frac{p_i h_{i,i}^k}{p_k g_i^k + \sum_{j \in D_k \setminus \{i\}} x_j^k p_j h_{j,i}^k + N_0}. \tag{4}$$

Furthermore, we can calculate the throughput of UEs by the Shannon formula $r = \log_2(1 + \text{SINR})$.

Note that with the increase of D2D pairs in the coalition D_k , the interference among the CUE and D2D pairs will increase. Thus, D2D pairs will deviate from their current coalition to join another coalition for their throughput improvement. This motivates us to employ the coalitional game theory (Saad et al. 2009) to formulate the coalition switch mathematically. In this paper, we formulate the joint power and subchannel allocation as a coalition formation game in partition form with nontransferable utility.

Definition 1. A coalition formation game with non-transferable utility (NTU) for joint power and subchannel allocation in D2D communication network is defined by a pair (\mathcal{N}, V) where \mathcal{N} is the set of players¹ and V is a mapping such that for every coalition $D_k \subset \mathcal{N}$, $k = 1, 2, \dots, M$, $V(D_k)$ is a closed convex subset of \mathbb{R}^{D_k} that contains the payoff vectors that players in D_n can achieve.

Denoting by $v_i(D_n)$ the payoff of D2D pair i in coalition $D_n \in \pi$, thus the coalition value set is defined as

$$V(D_k) = \{ \mathbf{v}(D_k) \in \mathbb{R}^{D_k} \mid v_i(D_k), i \in D_k \} \tag{5}$$

and the payoff of each D2D pair is

$$v_i(D_k) = r_i(p_i^*, p_{-i}^*), \forall i \in D_k \tag{6}$$

where p_i^* is the transmit power of D2D pair i , p_{-i}^* is the transmit power of other D2D pairs belonging to the same coalition as i , i.e. $-i \in D_n \setminus \{i\}$. Both of them will be determined by the power control scheme afterwards. The NTU property indicates the payoff for each D2D pair depends on the joint actions of all the D2D pairs in the coalition (Saad et al. 2009).

¹ We use the same set of D2D pairs as all the D2D pairs join the formulated game.

3.2 Power Control Within a Specific Coalition

After forming a coalition, all D2D pairs in this coalition, say $D_k \in \pi$, work cooperatively to maximize their sum rate. Meanwhile, all the D2D pairs are punished as they cause excessive interference on the CUE. That means, D2D pairs can evaluate a coalition with both sum rate maximization and CUE protection from interference. In the proposed game model, the punishment is linear to the total transmission power of D2D pairs on subchannel k . Thus, we can derive the optimal power $\mathbf{p}^*(D_k) = (p_1^*, p_2^*, \dots, p_{|D_k|}^*)^2$ of D2D pairs by solving the following optimization problem

$$\max_{\mathbf{p}(D_k)} \sum_{i \in D_k} \log_2(1 + \gamma_i^k(\mathbf{p}(D_k))) - \lambda \sum_{i \in D_k} p_i \quad (7a)$$

$$s.t. \quad 0 \leq p_i \leq p_{\max}, \forall i \in D_k \quad (7b)$$

where λ is a fixed linear price factor; constraint (7b) gives the power range.

From (3)–(4), we notice that the optimization problem (7a)–(7b) is non-convex, which can be complex to solve. As a result, we consider a low-complexity distributed iterative method to find a local optimum point. Then, optimization programming (7a)–(7b) is replaced by

$$\max_{p_i \in \mathbf{p}(D_k)} \log_2(1 + \gamma_i^k(\mathbf{p}(D_k))) - \lambda p_i \quad (8)$$

$$s.t. \quad 0 \leq p_i \leq p_{\max}, \forall i \in D_k.$$

Generally, the optimization problem (8) is convex and can be solved using a standard method (Boyd and Vandenberghe 2004). As a result, we develop Algorithm 1 to generate a sequence of transmit power for D2D pairs in D_k .

Algorithm 1. Iterative Power Control Algorithm (IPC)

- 1: All D2D initialize their power $p_i(t) = 0, \forall i \in D_k$, iteration count t , power price λ and maximum iteration number MAX .
 - 2: **repeat**
 - 3: $t := t + 1$
 - 4: Transmitter of D2D pair i estimates the interference-plus-noise level, i.e., the denominator of (4), $\forall i \in D_k$
 - 5: Transmitter of D2D pair i estimates the channel gain $h_{i,i}^k$ using the received signal power of control packet, $\forall i \in D_k$
 - 6: Transmitter of D2D pair i get the transmit power of t th iteration p_i^t by solving (8), $\forall i \in D_k$
 - 7: **until** $t > MAX$ or $\|p_i^t - p_i^{t-1}\| \leq \epsilon, \forall i \in D_k$
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Following proposition provides a sufficient condition of the convergence properties of (7a)–(7b).

² The operator $|\cdot|$ denotes the cardinality of a set.

Proposition 1. *A D2D pair which joins a coalition with the following constraint satisfied will receive a unique payoff in the coalition*

$$\sum_{j \in D_k, j \neq i} \frac{h_{j,i}^k}{h_{i,i}^k \ln 2} < 1, \forall i \in D_k. \tag{9}$$

For detailed proof, see Appendix.

Remark 1. Proposition 1 only shows a sufficient condition of convergence. However, we also note in the simulation that the convergence has a looser constraint than (9). To make the devised algorithm robust, we introduce the maximum iteration number. When the maximum iteration number is reached, the players in the same coalition use their throughput as the payoff. The iterative process in Algorithm 1 can assist D2D pairs to evaluate a coalition.

3.3 Coalition Formation Algorithm for Joint Power and Subchannel Allocation

In the formulated game model, a D2D pair can leave its current coalition and join a new coalition. However, which coalition to choose for the D2D pair remains a challenging problem for the coalition formation game (\mathcal{N}, V) . Hence, we define the preference order for the D2D pair to overcome this obstacle.

Definition 2 (Preference Order). *The preference order for a D2D pair i is expressed as \succ_i , which is a transitive binary relation over the set of all coalitions a D2D pair i can join.*

The preference order provides a metric to compare which coalition a D2D pair prefers. Consequently, given a D2D pair $i \in \mathcal{N}$ and two coalitions D_k, D_m where $i \in D_k$ and $i \in D_m$, $D_k \succ_i D_m$ means D2D pair i prefers D_k to D_m . Since our aim is to improve the total payoff of D2D pairs, we utilize the utilitarian order (Saad et al. 2009) in this paper.

Definition 3 (Switch Rule). *Given a partition $\pi = \{D_1, D_2, \dots, D_M\}$ of D2D pair set \mathcal{D} , a D2D pair i decides to leave its current coalition $D_k, k = 1, 2, \dots, M$ and join another coalition $D_m \in \pi, D_m \neq D_k$, hence forming a new partition π' , if only if, $D_m \cup \{i\} \succ_i D_k$, here*

$$D_m \cup \{i\} \succ_i D_k \Leftrightarrow \begin{cases} \sum_{j \in D_k, \forall D_k \in \pi'} v_j \geq \sum_{j \in D_k, \forall D_k \in \pi} v_j \\ v_i(D_m \cup \{i\}, \pi') > v_i(D_k, \pi) \end{cases} \tag{10}$$

where $\pi' = \pi \setminus \{D_k, D_m\} \cup \{D_k \setminus \{i\}, D_m \cup \{i\}\}$; the operator \Leftrightarrow represents left-hand-side and right-hand-side of (10) is equivalent.

The switch rule utilizes utilitarian order. On the right side of (10), the first line implies that payoff of the newly formed partition does not decrease by switching. Meanwhile, the second line indicates the switch operation increases the total

payoff of D2D pairs. Since the switch operation of each iteration is only related to coalition D_k and D_m , the following inequations are equivalent

$$\begin{aligned} \sum_{j \in D_k, \forall D_k \in \pi'} v_j &\geq \sum_{j \in D_k, \forall D_k \in \pi} v_j \\ &\Leftrightarrow \sum_{j \in D_k \cup D_m, D_k, D_m \in \pi'} v_j \geq \sum_{j \in D_k, D_m, D_k, D_m \in \pi} v_j. \end{aligned} \quad (11)$$

Furthermore, by applying the switch rule, we present the coalition formation algorithm in Algorithm 2.

Algorithm 2. Coalition Formation With Power Control Algorithm (CFPC)

Initialization

Each D2D pair selects a subchannel randomly and creates the history set $history_i$, $\forall i \in \mathcal{N}$.

Environment discovery

Each D2D pair $i \in \mathcal{N}$ discovers potential coalitions it can join.

Coalition formation process

repeat

for $i = 1 : N$ **do**

 D2D pair i lists potential coalitions it is permitted to join, and the current partition is $\pi = \{D_1, D_2, \dots, D_M\}$.

 D2D pair i negotiates with its potential coalitions, and $v_i(D_k)$ is given in (6), a result of **IPC**.

 D2D pair i decides to join coalition $D_k \in \pi$ based on switch rule in (10) and $D_k \notin history_i$.

end for

until No D2D pair has incentive to switch

Link level schedule

All D2D pairs in \mathcal{N} start transmit information signal afterwards.

Definition 4. A partition $\pi = \{D_1, D_2, \dots, D_M\}$ is called **Nash stable**, if and only if, $\forall i \in \mathcal{N}$, $i \in D_m \in \pi$ such that $D_m \succ_i D_k \cup \{i\}$ for all $D_k \in \pi$.

Proposition 2. Starting from any initial network partition π_0 , the coalition formation stage of the proposed algorithm always converges to a final Nash Stable partition π^* .

Proof. Starting from any initial networks partition π_0 , there are two possible results after each round of iteration: (1) the network partition is Nash stable; (2) the network partition is not Nash stable. For the first case, the iteration will terminate. For the second case, however, $\exists i \in \mathcal{N}$ with $i \in D_k$ and $D_k, D_m \in \pi$, such that $D_m \cup \{i\} \succ_i D_k$. Therefore, the D2D pair i will conduct switch operation in the next iteration. Since the total number of partition is limited (M^N in our setting) and the proposed algorithm forbids D2D pair revisiting past coalitions, thus all D2D pairs will finally converge to a Nash stable network partition.

4 Simulation Results

In this section, we provide simulation results to illustrate the performances of the proposed CFPC algorithm.

We consider N D2D pairs coexist with M CUEs. Each CUE is assigned an orthogonal subchannel. The transceiver is close enough to satisfy the maximum distance of D2D communication. The channel gain equals to $d^{-\alpha} |h|^2$, where d is the distance between the transceivers, α represents the pathloss factor. The term h denotes the complex Gaussian channel coefficient that satisfies $h \sim \mathcal{CN}(0, 1)$. We repeat the simulation 200 times and each time with the newly random-selected locations. We summarize simulation parameters in Table 1.

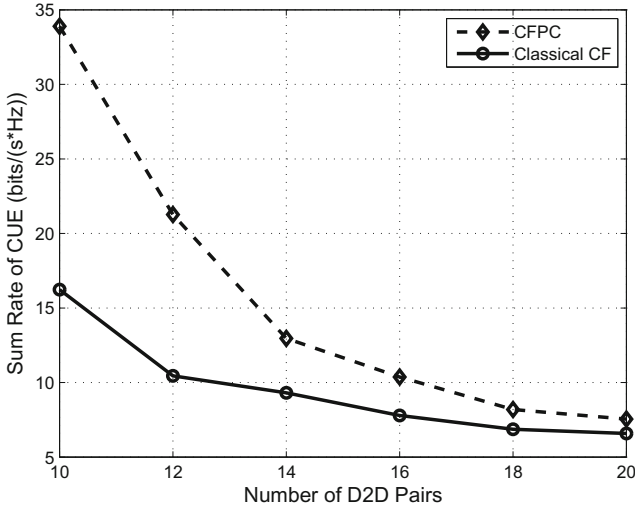
Table 1. Simulation parameters setting

Parameters	Values
Cell layout	Isolated cell, 1-sector
Cell radius	300 m
Subchannel bandwidth	180 KHz
Noise power	-174 dBm/Hz
Noise Figure	9 dB
TX power	D2D: 23 dBm in maximum, MUE: 23 dBm
Antenna gain	Device: 0 dBi, BS: 14 dBi
The maximum distance of D2D pairs	50 m
Pathloss factor, α	2

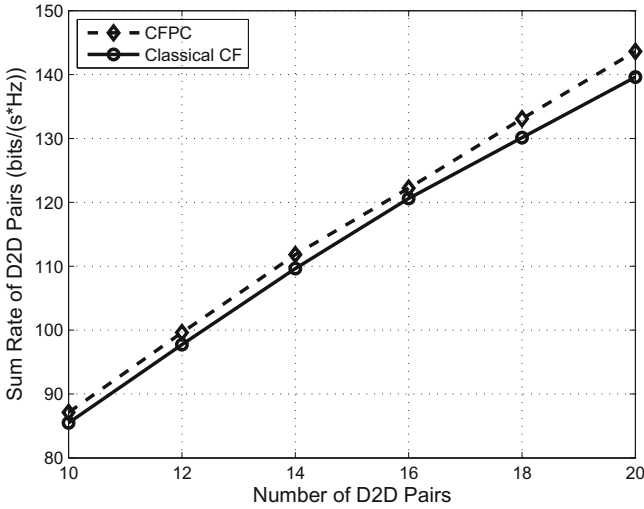
Figure 2a shows the sum rate of CUEs and Fig. 2b is the sum rate of D2D pairs. We can see that, as the number of D2D pairs increases, the sum rate of CUEs decreases and the sum rate of D2D pairs increases. When the number of CUE is fixed, more D2D pairs lead to more interference to CUEs, contributing to higher spectrum efficiency for D2D communication. Moreover, Fig. 2a and b illustrate the performance comparison of the proposed algorithm (CFPC) with the one in (Li et al. 2014) with a modification³ (Classical CF). We can see that the proposed CFPC algorithm outperforms the Classical CF in sum rate of both CUEs and D2D pairs. This is because the proposed scheme enables D2D pairs to further exploit the spectrum reuse gain by power control.

Figure 3 illustrates the fairness performance of the proposed CFPC algorithm and the Classical CF algorithm. We introduce the Jains Fairness index, which is denoted by

³ We allow the D2D pairs to switch their coalition as long as the sum rate of D2D pairs increases in exchange for sum rate of both CUE and D2D pairs rising in (Li et al. 2014).



(a) Sum rate of CUEs with varying number of D2D pairs N



(b) Sum rate of D2D with varying number of D2D pairs N

Fig. 2. Sum rate of CUEs and D2D pairs separately against varying number of D2D pairs.

$$J = \frac{\left(\sum_{i \in \mathcal{MUN}} r_i \right)^2}{(M + N) \sum_{i \in \mathcal{MUN}} r_i^2} \quad (12)$$

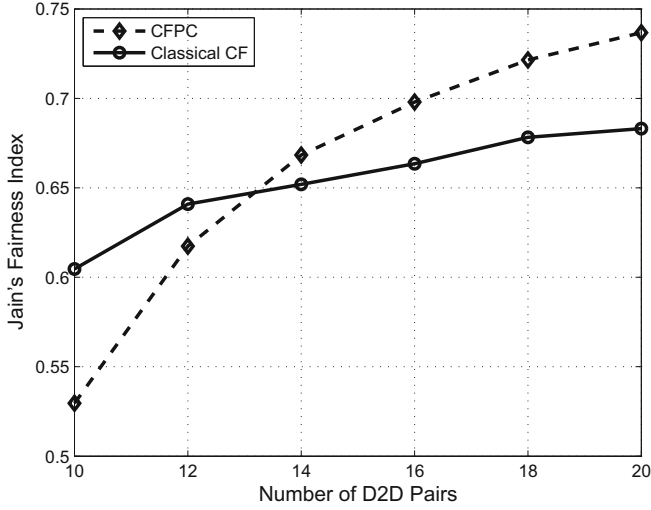


Fig. 3. Jain's Fairness index against the variation of number of D2D pairs

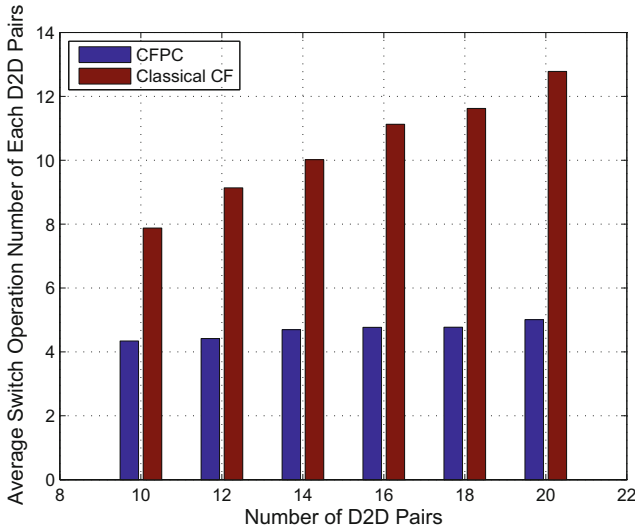


Fig. 4. Average number switch operation of each D2D pair as the number of D2D pairs increases.

as the metric to quantize the fairness. We observe that the Classical CF algorithm offers improved system fairness compared with the proposed scheme when the number of D2D pairs is small. The reason is that power control can lower the transmit power of D2D transmitters, thus can benefit the CUEs significantly compared with that of Classical CF. However, the proposed scheme offers better system fairness compared with the Classical CF algorithm when the number of

D2D pairs is large. That is because the proposed CFPC algorithm can reduce the interference to CUEs as the number of D2D becomes large.

Figure 4 shows that the iteration number grows as the number of D2D pairs increases. However the rate of increasing for the CFPC algorithm is smaller than the Classical CF algorithm. Notice that the iteration number of the Classical CF is significantly larger than that of the CFPC algorithm, because the Classical CF algorithm sacrifices switch operation for fairness. However, the proposed CFPC algorithm had the potential to mitigate the interference in the studied system. This phenomenon becomes more and more obvious as the number of D2D pairs increases; therefore, the proposed CFPC algorithm can obtain a better fairness when the number of D2D pairs is large.

5 Conclusion

We investigated the joint subchannel and power allocation problem for D2D communication underlying cellular networks. We formulated the problem as a coalition formation game. For the devised game model, a distributed coalition formation algorithm was proposed, where each D2D pair can make decision to leave or join a coalition. We also allowed D2D pairs within the same coalition to optimize their transmit power. Simulation results illustrated that the proposed scheme can increase the sum rate of both CUE and D2D pairs. Meanwhile, the proposed algorithm can also reduce the unnecessary switch operations.

A The Proof of Proposition 1

Proof. Let $\mathbf{p}(1)$ and $\mathbf{p}(2)$ be two different power allocation vectors. The solution to (8) can be shown as

$$p_i(m) = \left[\frac{1}{\lambda} - \frac{\sum_{j \in D_k \setminus \{i\}} p_j(m) h_{j,i}^k + N_0 + p_k g_i^k}{h_{i,i}^k} \right]_0^{p_{\max}} \quad (13)$$

where the operator $[x]_0^{p_{\max}}$ denotes the value of x is within $[0, p_{\max}]$, and the $m = 1, 2$.

For a fixed price factor λ , the difference for (13) with different power vector $\mathbf{p}(1)$ and $\mathbf{p}(2)$ is derived as

$$\begin{aligned} |p_i(1) - p_i(2)| &\leq \left| \sum_{j \in D_k, j \neq i} \frac{h_{j,i}^k}{h_{i,i}^k \ln 2} (p_j(1) - p_j(2)) \right| \\ &\leq \left(\sum_{j \in D_k, j \neq i} \frac{h_{j,i}^k}{h_{i,i}^k \ln 2} \right) \left| \sum_{j \in D_k, j \neq i} (p_j(1) - p_j(2)) \right| \\ &< \left| \sum_{j \in D_k, j \neq i} (p_j(1) - p_j(2)) \right|. \end{aligned} \quad (14)$$

From (14), we prove that (13) is a non-expansive operator; therefore, we conclude that the iterative procedure will converge to the unique fixed point (Miao et al. 2011, Theorem 3) when $\sum_{j \in D_k, j \neq i} \frac{h_{j,i}^k}{h_{i,i}^k \ln 2} < 1, \forall i \in D_k$.

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