# A Fast Vision-Based Localization Algorithm for Spacecraft in Deep Space

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Abstract. Star light navigation can provide the current attitude and position of the spacecraft in deep space. However, the accuracy of stellarinertial attitude determination is degraded because of star image smearing under high dynamic condition. To solve this problem, two key work, including accuracy star extraction and fast star identification, should be done. In this paper, we bring interpolation algorithm into contiguous area pixel searching for star extraction, and get sub-pixel coordinate information of the star points. In addition, a divisional method is proposed to improve star identification algorithm speed based on Hausdorff distance. The simulation results show that work not only has accuracy identification rate but also has better recognition speed. It was used successfully in the actual projects.

Keywords: Smearing image  $\cdot$  Autonomous navigation  $\cdot$  Star extraction

### 1 Introduction

Autonomous spacecraft navigation means the spacecraft can real-time determine its own position and attitude without any other support. The key factor in achieving autonomous navigation is accurate measurement of spacecraft attitude [1]. Currently, a new generation of CMOS star sensor is used for aircraft attitude measurement because of its high precision, none attitude cumulative error, fast fault recovery capability and intelligent [2]. It can provide accurate spacecraft flight attitude to a few arc-seconds without any prior knowledge. The star pattern recognition is one of the key technologies for spacecraft autonomous navigation based on star sensor. Many scholars are committed to this research, and proposed a number of algorithms. Nowadays, typical stellar identification algorithms used commonly include polygon angular distance matching algorithm, polygon angles matching algorithm, main star identification method proposed by Bezooijen, triangular matching algorithm, quadrilateral sky autonomous star identification algorithm, the sky autonomous grid algorithm, etc. [3–5]. Most of these algorithms complete recognition based on feature extraction. As a result, these complex algorithms are slow or need large storage and have poor antiinterference ability [6]. In addition, the star starlight images in the moving star

sensor will be stretched during the exposure time and lead to will lengthen and bring smearing. The smearing images reduce the centroid extraction accuracy, making a big decline in recognition accuracy decline.

In this paper, we aim at star centroid extraction in smearing images from moving star sensor. A extraction algorithm based on Gaussian curved interpolation is proposed to improve the centroid extraction accuracy. Secondly, we use the whole star database as a standard reference set, and consider extracted centroid data as a set to be recognized. Then, the minimum Hausdorff distance between two sets is determined to identify star location. At last, the divisional strategies for whole star database are proposed to improve the computational efficiency of the recognition algorithm.

## 2 Fast Vision-Based Localization Algorithm

Algorithm Overview. Because the star is considered to be at infinity, starlight can be seen as parallel to the light. In the inertial coordinate system, if the star sensor moving along a straight line, then the stars in the star sensor imaging position is fixed, which is similar to a static star sensor. However, when the star sensor rotates, stars' position detected in the star sensor will change and result in smearing. In Fig. 1, it shows a smearing image get from a moving star sensor.



Fig. 1. Case of smearing imaging.

Therefore, for deep space spacecraft location, the smearing images must be processed to extract the stellar centroid accurately. Then, it searches in whole star database with these stellar centroid data to get the current spacecraft location. The fast vision-based localization algorithm for spacecraft, proposed in this paper, consists of three steps, as shown in Fig. 2. First, Gaussian curved interpolation method for extraction of star centroid is used to obtain sub-pixel star centroid location information. Then, according to the moving state of the aircraft, we cut apart the whole star database, and match star centroid information in the divisional database to identification. Finally, the location of the aircraft in the whole star pattern is output as a result.



Fig. 2. Flow graph of fast vision-based localization algorithm.

### 2.1 Star Centroid Estimation Algorithm Based on Gauss Curved Fitting

Due to limitations of star sensor resolution, it is difficult to obtain high-precision stellar position from the star sensor image. Thus, there is a certain precision error in the extracted star pattern position. Set star sensor FOV (Field of View) of  $100 \times 100$ , the star sensor has a resolution of  $1024 \times 1024$ , the star sensor angular resolution is approximately 36", the error of the extracted star pattern position is also close to 36". Obviously, the error of extracting star pattern does not contribute to the correct rate of star pattern recognition, but also affect navigation accuracy. Taking into account the scattering of the lens, the imaging results in stellar star sensor should be a stellar position as the center of the spot. Because the star is a point light source, under normal circumstances the brightness of spots are represented by the point spread function, energy distribution can be approximated as a Gaussian surface, and the brightness decreases as quickly away from the center position. Considering the spot size is not large, and the point spread function of the specific parameter is difficult to determine. To solve this problem, the paper studies the Gaussian surface interpolation method to obtain analytic recursive Gaussian surface parameters.

As shown in Fig. 3, Set  $p_0(x, y)$  is the maximum position of stars resulting from the star sensor images, coordinates (x', y'), its four adjacent gray values of  $p_1(x1, y)$ ,  $p_2(x2, y)$ ,  $p_3(x, y1)$ ,  $p_4(x, y2)$ . Pixel  $p_0(x, y)$  and neighbor pixels are constituted by a Gaussian surface, so the mathematical expression is formula 1:

$$p = Aexp(-\frac{r^2}{B}) \tag{1}$$

In this formula,  $r^2 = (x - x_0)^2 + (y - y_0)^2$ , and  $(x_0, y_0)$  corresponds to a central location of Gaussian surface, A corresponds to the maximum value of the Gaussian surface, and correspondence with magnitude. The larger the magnitude, the greater the value of A; B corresponding to the spot size of the star, the smaller the size of the star, the smaller the value of B. The above equation with four unknowns. In order to obtain analytic equations, equation parameters such as  $x_0, y_0, A, B$  must to be get. However, the above equation is a nonlinear exponential function, analytic fitting parameters is very difficult. When taking the origin of the coordinates  $(x_0, y_0)$  into consideration, the above equation containing only A, B two parameters. Logarithm of both sides of the

	<sup>p</sup> 3 <sup>(x, y</sup> 1)		
<sup>p</sup> 1 <sup>(x</sup> 1, y)	Р <sub>О</sub> (х, у)	Р <sub>2</sub> (х <sub>2</sub> , у)	
	<sup>p</sup> 4 (x, y <sub>2</sub> )		

Fig. 3. Stellar location and adjacent gray distribution.

equation, there is formula 2:

$$\ln(p) = \ln(A) - \frac{r^2}{B} \tag{2}$$

Assuming:

$$y = ln(p), x = r^2, a = -\frac{1}{B}, b = \ln(A)$$
 (3)

There is:

$$y = ax + b \tag{4}$$

Obviously, the formula 4 is a linear function. It can be obtained coefficients a, b by a linear least-squares fitting method.

Assuming

$$S = \sum_{i=1}^{n} (y_i - y)^2 = \sum_{i=1}^{n} (y_i - ax_i - b)^2$$
(5)

Taking the logarithm on both sides of formula 4 and setting logarithmic zero,

$$\begin{cases} \frac{\partial S}{\partial s} = 0\\ \frac{\partial S}{\partial b} = 0 \end{cases}$$
(6)

It will be:

$$\begin{cases}
a = \frac{k \sum y_i x_i - \sum y_i \sum x_i}{k \sum x_i^2 - (\sum x_i)^2} \\
b = \frac{\sum x_i^2 \sum y_i - \sum x_i \sum y_i x_i}{k \sum x_i^2 - (\sum x_i)^2}
\end{cases} (7)$$

In formula 7, k is the number of data. The coefficients, a and b, obtained from the above can help to restore the coefficients A and B in formula 2. Formula 7 shows that precision magnitude can be obtained by fitting Gaussian surface with a known star location coordinates. To simplify the calculations, the fitting position is the position of the center of the star with 4 or 8 adjacent pixels.

In the above calculation, determining the position of the stars becomes the key to the algorithm. Since the complex Gaussian surface, to facilitate the calculation, the Gaussian curve fitting should be done in the x and y direction respectively, then, the position of the stellar is obtained by finding the maximum value of the curve.

If we assume constant parameter y in formula 2, then in the x direction, it will be:

$$p = Aexp(-\frac{(x-x_0)^2 + (y-y_0)^2}{B})$$
(8)

Taking the logarithm on both sides of formula 8 and bringing x1 = -1x2 = 0x3 = 1 to it, solution of this formula will be:

$$\begin{cases} B = \frac{2}{2\ln(p_2) - \ln(p_1) - \ln(p_3)} \\ x_0 = \frac{B}{4} \left( \ln(p_3) - \ln(p_1) \right) \end{cases}$$
(9)

Obviously,  $x_0$  is the star coordinates in the x direction, regardless of its size and y. Since the X-axis and Y-axis are symmetrical in the Gaussian surface relative to the coordinate origin, the star coordinate in y direction can be get in the same way, which is shown in formula 10.

$$\begin{cases} B = \frac{2}{2\ln(p_2) - \ln(p_4) - \ln(p_5)} \\ x_0 = \frac{B}{4} \left( \ln(p_5) - \ln(p_4) \right) \end{cases}$$
(10)

### 2.2 Fast Divisional Matching-Based Star Pattern Recognition Algorithm

In the basic star gallery, which stored a standard stellar parameters, their vector form is celestial coordinates which is represented by red latitude, in this paper, we use the basic celestial coordinates red latitude to identify. In the star sensor the images to be identified are two-dimensional gray-scale image, the two-dimensional X, Y coordinates with red latitude, and gray-scale image coordinates with magnitude. Usually in the identification, X, Y must be converted to red latitude. Unfortunately, due to before recognition, star sensor point is not completely sure, so that we can only get from a relative red latitude from X, Y coordinates, but can not get the absolute end, to this end, using the Hausdorff distance between the relative position of the star pattern and the satellite library as a criterion to conduct star identification.

Despite the lack of precise red longitude coordinates from the star sensor, however, depending on the structure and the relative position of the star field is kept substantially constant, set to be recognized star Pictured  $A = \{a1, ..., ak, ..., ap\}$  star standard library  $B = \{b1, ..., bj, ..., bq\}$ , the improvement of the Hausdorff distance between them is defined as:

$$H = \sum_{k} \min(d_k) \tag{11}$$

In this formula,

$$d_k = w_1(a_{1k} - (b_{1j} - b_{1i})) + w_2(a_{2k} - (b_{2j} - b_{2i})) + w_3d_{smk}$$
(12)

It represents the relative weighted distance between the k-th star, in A that to be identified, and the j-th star in star database B.  $W_i$  (i = 1,2,3) is the weight value. Usually,  $W_1$  should equal to  $W_2$ , and i is the serial number of i-th star in star database, where j is the serial number of j-th star in star database. The third item represents the changes in magnitude. Due to the magnitude of the error is relatively large, the value of  $W_3$  should be less than  $W_1$ . The distance between magnitude can be expressed as:

$$d_{smk} = |a_{3k} - b_{3j}| \tag{13}$$

As the magnitude of the change has a great impact on the star pattern recognition, especially the star sensor threshold of exposure and weak star, due to the dynamic effects of noise and star sensor, when hidden, particularly large impact on the star pattern. Taking the impact of changes in magnitude into account, Eq. 12 can be rewritten as:

$$d_k = exp\left(\frac{d_{smk}}{d_0}\right)\left(\left(a_{1k} - \left(b_{1j} - b_{1i}\right)\right) + \left(a_{2k} - \left(b_{2j} - b_{2i}\right)\right)\right)$$
(14)

In formula 11 Hausdorff distance is the sum of the minim distance between each recognized star and star database in recognized stars, when the star to be identified in the star sensor match with the star in the star database, it has the least sum of the minimum distance. This avoids noise due to individual stars appear larger mismatch problem that may arise.

When there is uncertainty about the direction of the star sensor completely, it not sure star general area in the repository. Due to the number of stars in the all star database, more recognition speed is slow. Assuming N stars in star database, calculated by the formula 11, complexity of a complete minimum H distance calculating is  $N_2$ . If the standard star database is divided into M region, with N/M stars in each region, the complexity will reduce to  $N_2/M$ . Theoretically, there is M times faster than matching in whole star database, where the bigger M can get the faster calculation speed. However, the size of each area, should be greater than that of star sensor field, and each area should have enough redundancy for matching in a subregion.

Assuming that stars in the sky is evenly distributed, and taking the unit of length to be the radius of sky, the number of stars is  $N/4\pi$ . Assuming that the area size is  $K \times K$ , and the size of the FOV of star sensors is  $L \times L$ , one area includes the area of star sensor at least, meeting  $K \times K \ge 2L \times 2$ . Planar area is approximation for each region:

$$S_{partial} \approx K^2$$
 (15)

An area which does not overlap region is:

$$S_{efficitive} \approx (K - L)^2$$
 (16)

The rest is the redundant area. Assuming that each area is equal, the integral area number is:

$$NUM = \frac{S_{full}}{S_{efficitive}} \approx \frac{4\pi}{(K-L)^2}$$
(17)

Each number of stars in the area of approximation is:

$$M = S_{partial} \times \frac{N}{4\pi} \approx \frac{N}{4\pi} K^2 \tag{18}$$

The total number of calculations is:

$$Count = M^2 \times NUM \approx \frac{N^2 K^2}{4\pi (K-L)^2}$$
(19)

For the derivation and ordering derivative equaling to zero, it can get K = 2. When K = 2, the calculation number corresponds to the minimum value with  $N^2 L^2/\pi$ .

#### 2.3 Divisional Strategies for Standard Star Database

By formula 19, in order to improve the speed of star pattern recognition, the standard star database should be divided into different regions. Because the star standard library is stored according to the spherical coordinates of latitude and longitude, the coverage of the latitude and longitude coordinates is not uniform to the certain star sensor in different latitude and longitude position, the star database segmentation is not uniform. Figure 4 shows the longitude and latitude area covered by star sensor.

The figure shows:

$$ab = 2ac\sin(\angle acb/2) \tag{20}$$

$$ac = oa\sin(\angle aoc)$$
 (21)



Fig. 4. Conventional diagram of star database segmentation.

So:

$$ab = 2 \times ac \times \sin(\angle acb/2) \times \sin(\angle aoc)$$
 (22)

$$ab = 2 \times oa \times \sin(\angle aob/2) \tag{23}$$

By formulas 22 and 23 it can be obtained:

$$\sin(\angle aob/2) = \sin(\angle acb/2) \times \sin(\angle aoc) \tag{24}$$

 $\angle aob$  is the FOV of star sensor expressed as  $\theta$ .  $\angle acb$  is latitude expressed as  $\alpha$ , and  $\angle aoc$  is complementary angle corresponding longitude expressed as  $90 - \delta$ . The formula 24 will change to:

$$\sin(\theta/2) = \sin(\alpha/2) \times \cos(\delta) \tag{25}$$

Along the longitude direction,  $\angle eof$  corresponding to the amount of change is the longitude of  $\triangle \alpha$ , therefore, star database can be split directly along the longitude of the star sensor based on the field size.

A star sensor's FOV is  $100 \times 100$ , and each child area overlaps. The segment of star database calculated from formula 25 as shown in Table 1. The actual interval in the table is 2 times to division by  $360^{\circ}$ .

No	Latitude range	Theoretical interval	The actual interval	Number of segments
		in degrees	in degrees	
1	$-90 \sim -70$	360	360	1
2	$-80 \sim -60$	90	180	4
3	$-70 \sim -50$	30.5116	72	10
4	$-60 \sim -40$	20.3220	48	15
5	$-50 \sim -30$	15.6731	36	20
6	$-40 \sim -20$	13.1018	30	24
7	$-30 \sim -10$	11.5669	24	30
8	$-20 \sim 0$	10.6490	24	30
9	$-10 \sim 10$	10.1559	24	30
10	0~20	10.6490	24	30
11	10~30	11.5669	24	30
12	20~40	13.1018	30	24
13	30~50	15.6731	36	20
14	40~60	20.3220	48	15
15	50~70	30.5116	72	10
16	60~80	90	180	4
17	$70 \sim 90$	360	360	1
Total				298

Table 1. Division of star database.

### 3 Related Work

Since the first CCD-based star tracker was developed by Salomon in 1976 [7]. great advancements in star identification have been made in about four decades. Many faster and more reliable methods were proposed from the 1990's [3]. Scholl proposed a method based on inter-star angles ordered by their relative brightness [8]. His method aimed at the search process acceleration with less time than the classical multi-step star identification method proposed by Baldini [9]. However, Scholl's method retains the  $O(nf^2)$ , so many faster techniques were proposed in the following years. To reduce the search time much further, a method using a "k-vector" to search the database in an amount of time independent of the size of the database [10] was proposed by Mortari. With this method, the search time for a single star-pair would be O(k). Guangjun [11] proposed method based on feature extraction in 2007, using the inter-star angles and the angle made by two stars relative to a central star, which was similar to Liebe [12]. He uses a linear database search running in O(n) time, while feature extraction time remains  $O(f \lg b)$ . In 2008, Kolomenkin [13] proposed a modification of the SLA algorithm [14] to reduce the time spent cross-checking the results of the k-vector. While the algorithm performs the cross check O(k/f) faster than the SLA which take  $O(k^2)$ -time, it calculates  $O(f^2)$  more inter-star angles, and k-vector searches, each of which takes O(k)-time, contributing an increase of  $O(kf^2)$ -time.

On the other hand, some non- dimensional algorithms and recursive star identification methods are proposed to improve the robustness of star identification. Rousseau computes the attitude for each star triangle with the sine of star-triangle interior angles, and the final analytic time of is  $O(kf \lg f \lg n)$  [15]. Samaan reduced the recursive mode time was to speed the selection of stars for recursive star identification [16]. One of his methods uses the Mortari's Spherical Polygon-Search (SP-Search) [17,18], which uses a k-vector 3 times to find the stars within calculated x, y, and z ranges in inertial space. Each of the three database searches takes O(k) time, while the cross-comparison takes  $O(k^3)$ -time. The another of his methods uses the Star Neighborhood Approach (SNA) which takes O(b)-time to find candidate stars, if b stars are identified. It is uncertain how many successive iterations would be necessary to ensure that all the stars in the given field of view have been found, other than it is most likely bounded by O(fb).

### 4 Performance Evaluation

Using of statistical simulation, the extraction accuracy is analyzed by comparing Gaussian surface fitting with centroid method in [3].

Simulation Condition 1: The pixels position of theoretical centroid is (4.7, 4.1), with  $\sigma = 1$ , and fitting spot size is  $3 \times 3$  pixels. The result of 100 times on average is shown in Fig. 5.



Fig. 5. Different SNR results of two different methods in simulation 1

Simulation Condition 2: The pixels position of theoretical centroid is (4.5, 4.5), with  $\sigma = 1$ , and fitting spot size is  $3 \times 3$  pixels. The result of 1000 times on average is shown in Fig. 6.



Fig. 6. Different SNR results of two different methods in simulation 2.

The above results indicate that the window is calculated for participating quantized pixel size. When the centroid position is different, the distribution of image within the window is asymmetric, whereby the accuracy is different. When the SNR is relatively small, the centroid extracting accuracy is limited because of the image of an asymmetric distribution in the window. On the other hand, noise is mainly restricted in Gaussian fitting, so Gaussian fitting has higher accuracy than the centroid method. Therefore, for the establishment of high-precision imaging model, using Gaussian fitting method is a better choice.

Simulation Condition 3: Under different conditions, a simulation of fast divisional matching-based star pattern recognition algorithm was done in whole star database with the magnitude between 0 and 6. In order to facilitate experiments and showing, we choose an arbitrary area  $60^{\circ} \times 60^{\circ}$  as a reference star pattern, and arbitrarily select FOV of  $100 \times 100$  regions to map to star sensor image. In recognition process, star sensor image is randomly generated.



Fig. 7. Results of star pattern matching with random sensor image.

Figure 7 shows the results of star pattern matching. The y-axis represents the H distance, and the x-axis represents the serial number of stars. The matching results can be obtained by the serial number in the figure. According to the maximum of the star sensor possible noise, when experimenting, set the noise of latitude to 36 arc-second, the noise of magnitude to 0.5. It can be clearly seen from the figure that H distance is suddenly reduced when the star sensor image matches the star pattern. On the other hand, if the sensor image does not match the reference star pattern, H distance is relatively large and random. Figure 7(b) shows a matching result of a meteor interference with 1 arc-minute of latitude noise and 1 of magnitude noise. It can be seen from the figure, even under harsh conditions, the algorithm can identify the correct result.

# 5 Conclusion

Based on the analysis of star sensor imaging principle, this paper proposes a minimum use of star sensor relative space position Hausdorff distance map recognition method. This method is based on the spatial structure of stars similar principles, avoiding the complex feature extraction algorithm, but takes full advantage of all the information obtained by the star sensor. Experimental results show that the minimum matching method based on the Hausdorff distance, it is possible to obtain better recognition accuracy. For in the absence of prior knowledge, identify areas for full Star slower problem, the paper presents the basic methods star database partitions, partition star identification, so that the recognition speed is improved. Acknowledgments. This research was supported in part by the NSF of China (Grant No.61272470, 61305087); the Fundamental Research Funds for the Central Universities, China University of Geosciences (Wuhan) (Grant No. CUGL120284, CUGL120289, CUG120114).

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