Edge Detection in Low Quality Medical Images

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Abstract. The edge detection is very useful for contour and segmentation. But with weak objects, this is very difficult, especially for medical images which store the information of patients. In this paper, we propose an algorithm to detect the edges of object in medical images. Our method uses Bayesian thresholding for medical images denoising and B-spline curve for smoothing. The proposed method is compared with the recent methods. The results of our method are better than the other methods.

Keywords: Edge detection · Canny · B-spline · Bayesian thresholding

1 Introduction

In the field of health, images become a useful tool. Many diseases are detected based on the medical images. Medical images provide full details inside the human body, where the naked eye cannot see. Every detail becomes highly valuable if anomalies in the images are able to be detected as soon as possible. For serious diseases, if we detect as early symptoms or abnormalities, it will be more likely to maintain the life. So, the required visible pixels are extremely important. Based on the advice from health professionals, patients will be target of methods to acquire the information of their body by the stored images. In stored medical images, the combined pixels together create the contours of an object around a specific body (eg bone, liver, blood vessels, etc…). If there are any unusual locations in the details such as size, location can be shifted as initial manifestation of the diseases, helping the diagnosis and treatment of specialists. Therefore, the detecting of edges is very necessary for contour or segmentation. If we detect many edges, we will have more information about objects and will be more comfortable for detecting contour or segmentation [3].

In the past, there are many algorithms which are proposed for edge detection such as: Sobel [9], Canny [6–8], B-spline [11–15] or in the generation types of wavelet transform as [4, 11, 16, 17]. Specially, B-spline curve is the most popular in many algorithms. The authors propose multi-scale B-spline wavelet for edge detection in [11] by multi-scale for smoothing steps. In [12], the algorithm is proposed to be B-spline Snakes by multi-scale to apply for contour detection. This idea is continued with

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Brigger [15]. The reason of applying is because of the value from the smoothing spline. On the other hand, many previous methods for edge detection are done in transforms which we must mention in wavelet of Wang [11] and scale multiplication in wavelet domain of Lei [16]. The new generation is used as discrete shearlet transform in [4]. A concept of strong and weak objects in contour is defined by Binh [17]. In [17], the contour detection based on the context aware combination in complex wavelet domain. The quality of these algorithms continues to improve. The scientists use single or double threshold, filter to show the pixels.

Most of the edge detection methods always include the removing noise steps [1, 6]. There are many causes for medical images of which quality is reduced. In case medical images have blur or noise, their sharpness will be affected. When the quality of medical images is bad, the edge detection is reduced and it is very hard to see it. This case is the weak object, and this is the hard problem for edge detection because of the gray of different levels. Although both of the blur and noise appear in the reasons of reducing the quality of medical images, the blur is not popular in medical images. Because the blur is the movement or migration, this problem is overcome by the breakdown or the quality of machinery. In [6], Canny proposes a method for edge detection. In his method, the first step is the removed noise by Gaussian filter, and all steps do not include deblurring images to prepare for edge detection. It does demonstrate that the denoising is very important to edge detection.

In this paper, we propose an algorithm for edge detection of objects in medical images. We concentrate on low-quality medical images. We present the basis of edge detection and denoising images in Sect. 2. The proposed method is presented in Sect. 3. In this section, we use Bayesian thresholding for denoising of the input medical images. Then we change the smoothing steps of Canny by using the B-spline to improve the number of edges detected. The results of experiments are compared with the recent methods in Sect. 4. And in Sect. 4, we add the concept of the weak object of [17]. Conclusions are shown in Sect. 5.

2 Background

In this section, we will present the background of edge detection and denoising images and the results of other previous methods in these fields.

2.1 Edge Detection

Edges occur on the boundary between two different regions in an image. The task of edge detection is to the important features from the edges of images, such as: lines, corners, curves, etc. This process is affected by intensity changes. And the reasons of changes are: shadows, depth, texture, or surface color. The four steps of edge detection [1] as follows:

– Smoothing: remove noise from the input images but must keep the features of edges.
– Enhancement: using the filters to improve the quality of edges.
– Detection: the thresholding is the basis of the detection.

– Localization: based on the location of each edge.

Many algorithms are proposed to detect the edges, such as: Sobel [9], Canny [6], etc but their results sometimes have no enough edges. The Fig. 1 is the results of edge detection based on the Sobel and Canny.

(a) (b) (c)

Fig. 1. The results of edge detection by Sobel and Canny method. (a) The normal image. (b) The result of edge detection by Sobel method. (c) The result of edge detection by Canny method.

Then, many methods detected in transform: wavelet transform [11, 16, 17], shearlet transform [4], etc… With transform, the authours proposed the detection based on the combination between pixels. Edge detection bases on estimating the gradient: strength, gradient direction by −90 degrees. The gradient is the two-dimensional equivalent of the first derivate and is defined as the vector:

$$G[f(x,y)] = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix} \tag{1}$$

where vector G[f(x, y)] points in the direction of the maximum rate of increase of the function f(x, y). And the direction of the gradient is defined as:

$$\alpha(x,y) = \tan^{-1}\left(\frac{G_y}{G_X}\right) \tag{2}$$

where α is measured with respect to the x axis.

The Sobel [9] use operator:

$$M_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \tag{3}$$

and

$$M_y = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix} \tag{4}$$

where M_x and M_y are approximations at (i, j).

The Canny method improved the results from Sobel method, the quality of edge detection was clearer. With the previous methods, the results of edge detection are good for strong objects. The strong objects are the objects which have the high quality and which are easy for detection by color. However, the weak objects are the objects which have noise, blur or no clarity between pixels, which is very difficult for detection. Recently, using context awareness for edge detection and smoothing contour has been proposed in [17] to give the good results. In the other methods, the authors use interval type-2 fuzzy logic to improve the quality of the Sobel technique in [19].

2.2 Denoising Images

Each image has noise to have a form:

$$g = x + n \tag{5}$$

where x is normal image, n is the noise value and g is the noisy image. The noise has many types, such as: Gaussian, speckle, salt & pepper. Most of the medical images have noising and blur because of many reasons such as environment, capture devices, technician's skills, etc [33, 34]. These reasons have reduced the quality of medical images. Consequently, enhancing medical image process is useful and necessary. However, this is very difficult for image processing.

The goal of denoising is to extrude noise details from the low quality medical images, but keep edge features. Many researches are proposed to solve this problem. The results from denoising by wavelet transform [20, 31] are very positive. In there, the authors used the threshold in decomposition domain. Calculating the threshold depends on the noise variance. Then, they based on the threshold to show or not to show the pixel details. They continued with the wavelet coefficients to reconstruct the image. The denoising results are continued to improve by discrete wavelet transform (DWT) [21, 22]. But DWT has three serious disadvantages [23]: shift sensitivity, poor directionality and lack of phase information. To overcome these disadvantages, the scientists used filters in transform such as contourlet transform [24], nonsubsampled contourlet transform (NSCT) [25, 26], ridgelet transform [27, 28], curvelet transform [29]. In contourlet domain, they used laplacian pyramid (LP) and directional filter bank (DFB) to remove the bad pixels and perform the reconstruction by discrete LP and discrete DFB. Contourlet transform is only to use 8 or 16 directions in pyramid and filter in each direction. NSCT [25, 26] includes nonsubsampled laplacian pyramid (NSLP) and nonsubsampled directional filter bank (NSDFB). In NSCT domain, the authours used multi-directional and filter in each direction. In other words, NSCT is the improvement from contourlet transform. The ridgelet transform [27, 28] is continued to improve by curves. Ridgelet transform is the first generation of curvelet transform [29] … Most domains of transforms are denoised by threshold. At first, they used hard threshold (T_{hard}) and soft threshold (T_{soft}) which are given in equations:

$$T_{hard}\left(\hat{d}_{jk}, \lambda\right) = \hat{d}_{jk} I\left(\left|\hat{d}_{jk}\right| > \lambda\right) \tag{6}$$

and

$$T_{soft}\left(\hat{d}_{jk}, \lambda\right) = \text{sign}(\hat{d}_{jk})\max\left(0, \left|\hat{d}_{jk}\right| - \lambda\right) \tag{7}$$

where $\lambda \geq 0$ is parameter wavelet, I is normal parameter value.

After that, they used stationary, cycle-spinning, shiftable, steerable wavelet, Bayesian thresholding, etc. and combined between transform and threshold in [30, 32–34]. Although the results of previous methods are spectacular, the time processing is slow and the complicacy is very high. Medical images lacking information is the big problem because they must adapt to many filters or thresholds.

3 Improved the Quality of Edge Detection by Canny Combined with B-Spline in Bayesian Thresholding

In this section, we propose a method to improve the Canny technique by consolidating the quality of objects and making the smoothing. We increase the sharpness of objects

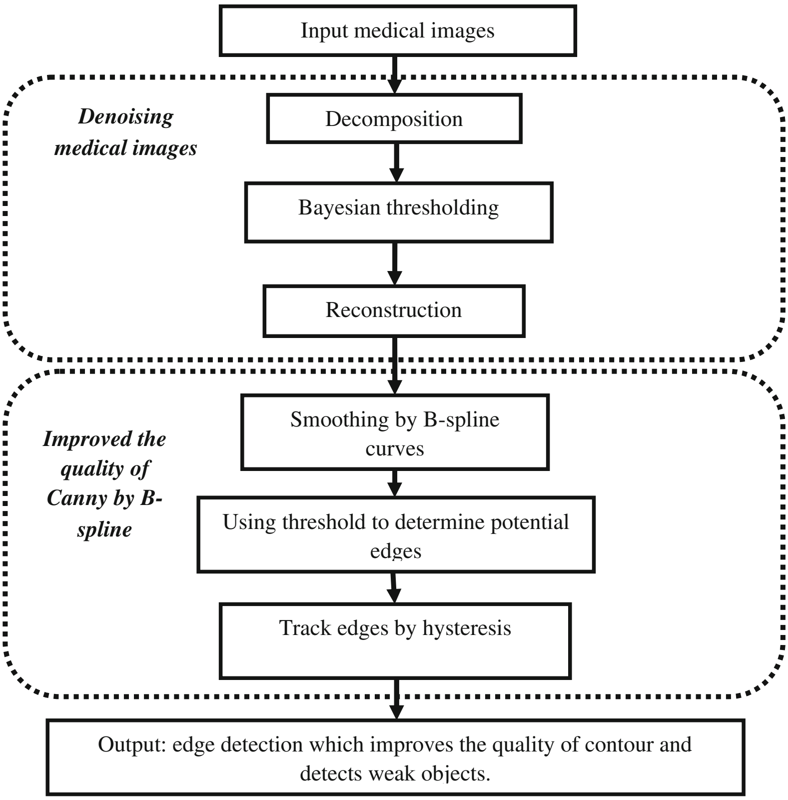


Fig. 2. The proposed method

in medical images based on the denoising image. In here, our idea uses Bayesian thresholding to remove the noise out of the medical images. Our method includes: after decomposition, we remove noise details from input images by Bayesian thresholding. The output of denoising is the medical image which has strong objects. The objects have strong edges, clear and comfortable for edge detection. If we have the good results from edge detection process, we will give more contour for medical images. These results will also help segmentation process. Then, we apply B-spline for the smoothing step. Our method is given clearly in Fig. 2.

3.1 Denoising in Wavelet Domain

In medical images, if we use many thresholdings or threshold without fixing, the results of denoising will be "too smooth". If medical images are "too smooth", they will lose several pixels. Although medical images have the high quality, the doctors will not see the details of bone or blood vessels clearly. Hence, we must propose an algorithm which is not only to denoise but also to keep features.

As presented in Sect. 2.2, there are many transforms for denoising images. The process for denoising in the above methods is similar to:

(i) Analyze the input image into various types of domain.
(ii) Use filter (single or multi direction) and threshold by calculating the detail coefficients.
(iii) Compare the detail coefficients with threshold values given and make the coefficient values closer to 0. This step removes the impact of the existence of images.
(iv) Reconstruct the image from the changing coefficients (the inverse transform).

Therefore, the transforms have many filters and thresholdings, especially contourlet transform and NSCT. The contourlet transform and NSCT will lose many details of medical images because of the fixing with filters or thresholds. Ridgelet and curvelet transform also gives good result for denoising images; but it takes much time and it is slow.

Depending on these features, we are to use Bayesian threshold for calculating coefficients for denoising. We base on the coefficients calculated by Bayesian thresholding to reconstruct the result image. As a result, the medical images will not be fixed with many thresholds or filters. They will keep many edges and features necessary for edge detection process. The denoising is also to change from weak objects to strong objects in case images have noise. Our denoising process includes:

(i) Doing the decomposition and calculating sigmahat.
(ii) Calculating the sigmahat value. These values include: the estimate noise variance σ and signal variance σ_s. They can be obtained by equation:

$$\sigma=\left(\frac{\text{median}\left(\left|w_{i,j}\right|\right)}{0.6745}\right)^2 \tag{8}$$

$$\sigma_s = \sqrt{\max(\sigma_w^2 - \sigma^2, 0)} \tag{9}$$

with

$$\sigma_w^2 = \frac{1}{n^2} \sum\nolimits_{i,j=1}^{n} w^2(i,j) \tag{10}$$

where $w_{i,j}$ is the lowest frequency coefficient after performing transformations.

(iii) The calculation of the thresholds by equation:

$$\mathrm{Threshold}_{\mathrm{Bayes}} = \begin{cases} \dfrac{\sigma^2}{\sigma_s}, \sigma^2 < \sigma_s^2 \\ \max\{|A_m|\}, \sigma^2 \geq \sigma_s^2 \end{cases} \tag{11}$$

(iv) When reconstructing the image based on the Bayesian thresholded coefficients. If the value of pixel detail coefficients is less than thresholding then the result is 0. Else the result is array Y, where each element of Y is 1 if the corresponding element of pixel is greater than zero, 0 if the corresponding element of pixel equals zero, −1 if the corresponding element of pixel is less than zero.

After this process, the quality of the medical images are improved. From the results, the strength of pixels is increased and the edges are also more powerful. After denoising, our algorithm continues to do smoothing by B-spline which is presented in the next section.

3.2 Improved the Quality of Edge Detection by Canny Based on the Combination with B-Spline Curve

Many methods based on the masks such as Sobel and the proposed method takes advantage of the structure in local images. However, with the previous methods, the weak objects also cause difficulty. The Canny edge detection algorithm gives the good results in weak objects [7]. This method includes 5 steps:

(i) Apply Gaussian filter to smooth the image in order to remove the noise.
(ii) Find the intensity gradients of the image.
(iii) Apply non-maximum suppression to get rid of spurious response to edge detection.
(iv) Apply double threshold to determine potential edges.
(v) Track edge by hysteresis. Finalize the detection of edges by suppressing all the other edges that are weak and not connected to strong edges.

Our idea is to remove noise by the Bayesian thresholding in the previous section. We do not use Gaussian filter because the noise types in medical images are complex. When we use threshold in this step, the quality of different levels is improved. After denoising medical images, we use a B-spline curve to smooth the image. A B-spline curve is similar to Bézier curve, but B-spline curve gives more information. A Bézier curve is a parametric curve. In graphics, Bézier curves are used to model smooth curves. From the set of n + 1 control points, knot vector of m + 1 knots and a degree p. But must satisfy: m = n + p + 1. The B-spline curve of degree p ($N_{i,\,u}(u)$) defined by these control points and knot vector U is:

$$C(u) = \sum\nolimits_{i=0}^{n} N_{i,p}(u)P_i \tag{12}$$

where n + 1 control points $P_0, P_1, \ldots, P_n$ and a knot vector $U = \{u_0, u_1, \ldots, u_m\}$.

$N_{i,u}(u)$ looks like $B_{n,\,i}(u)$. The basis function of Bézier bases on the number of control points, but the degree of a B-spline basis function is an input the degree of a B-spline basis function is an input.

B-spline with the smoothing function $\beta_{2^{-1}}^{n+1}$ and $\beta_{2^{-1}}^{n+2}$ uses 2^{-1} level.

$$\psi^n(x) = \frac{d}{dx}\beta_{2^{-1}}^{n+1}(x) = 4(\beta^{n+1})^{(1)}(2x) \tag{13}$$

or

$$\psi^n(x) = \frac{d^2}{dx^2}\beta_{2^{-1}}^{n+2}(x) = 8(\beta^{n+2})^{(2)}(2x) \tag{14}$$

where n is the order of wavelet transform.

We define the desirable geometric characteristics of B-Spline curves and surfaces of degree p is defined as:

$$N_{i,0}(u) = \begin{cases} 0 \ if \ u_i \le u \le u_{i+1} \\ 1, \ otherwise \end{cases} \tag{15}$$

$$N_{i,p} = \frac{u - u_i}{u_{i+p} - u_i} N_{i,p-1}(u) + \frac{u_{i+p+1} - u_i}{u_{i+p+1} - u_{i+1}} N_{i+1,p-1}(u) \tag{16}$$

After the smoothing step, the Kernel is applied for determining gradients of the medical images. The value of gradients is also known as the edge strengths. This value is calculated by Euclidean distance measure, and it will be used to define edges to be shown. Nonetheless, these edges will be converted to “sharp” edges in the non-maximum suppression step. In each pixel of the gradient image, we focus on the gradient direction < 45^0 and 8-connected neighborhood. The direction of edges is shown by equation:

$$\theta = \arctan\left(\frac{|G_y|}{|G_x|}\right) \tag{17}$$

where G_x and G_y are the gradients in the x and y directions respectively.

If the edge strength between neighborhood and pixel in the positive is not the largest, the algorithm will be removed. From the shown edges, the weak objects are improved by denoising and non-maximum suppression step. Thus, the number of strong objects is higher.

4 Experiments and Results

As mentioned in Sect. 3, we improve the quality of Canny algorithm by Bayesian thresholding to remove noise and B-spline at the smoothing step. This idea remove the usage of double threshold and filter of the previous Canny method. Specially, we also change the Gaussian filter used at the smoothing step of Canny. We choose B-spline for smoothing after removing noise by Bayesian thresholding because this threshold is based on wavelet coefficients. B-spline also makes the curves very clear. This is necessary for edge detection in medical images.

The proposed method uses Bayesian thresholding to remove noise of which medical images have weak objects to be improved. Doctors who apply our idea into the edge detection more have more basic information about their patients. We test in medical images which have strong objects and weak objects. The strong and weak object are defined in [17] as: "the strong object is an object of which boundaries are clear and the weak object is defined as an object of which boundaries are blurred". In here, we base on the concept from [35]: "the blur details include noise details". Consequently, we add the concept of the weak object as: the weak object is an object of which boundaries are blurred and noised.

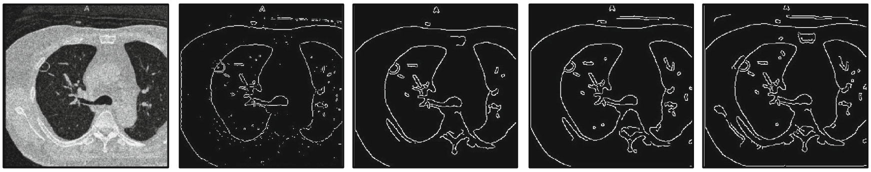
The results of our proposed method are compared with the other methods such as: Sobel [9], Canny [8], Cuckoo Search [18]. We have many edges more than other methods. Our dataset is the medical images collected in many hospitals. There are more than 1000 medical images, and in many sizes: 256×256, 512×512, 1024×1024. We test many medical images from this dataset. In here, we show some cases when we test and compare them. In Fig. 3, we test the medical image which has a strong object.

(a) (b) (c) (d) (e)

Fig. 3. The results of edge detection by other methods with strong object. (a) The strong object in the original medical image. (b) Edge detection by Sobel method [9]. (c) Edge detection by Canny method [8]. (d) Edge detection by Cuckoo search method [18]. (e) Edge detection by the proposed method.

In this case, Fig. 3(a) is the original medical image. The result of Sobel for edge detection is Fig. 3(b), the result of Canny for edge detection is Fig. 3(c) and of Cuckoo search is Fig. 3(d). In Fig. 3(e) is the result of our proposed method. We can see that the number of edge detection by the proposed method is better than the result in Fig. 3(b), (c) and (d).

In Fig. 4, we test the medical image which has a weak object. Although we state that a weak object is an object of which boundaries are blurred and noised; the blur is a problem easily overcome by the technician skill or the quality of machine. In case noise is added to medical images to create a weak object, noise details is more popular than blur details. In this case, we use Gaussian noise and Gaussian blur added to medical images with the variance noise is 0.00005 and the values of point spread function of Gaussian blur. The reason of our selection is Gaussian is the plus noise and blur into pixels of medical images. The plus noise and blur is popular in medical images.



(a) (b) (c) (d) (e)

Fig. 4. The results of edge detection by other methods with weak object. (a) The weak object in a noised medical image. (b) Edge detection by Sobel method [9]. (c) Edge detection by Canny method [8]. (d) Edge detection by Cuckoo search method [18]. (e) Edge detection by the proposed method.

The result of Sobel is Fig. 4(b), Canny and Cuckoo search alternate to be Fig. 4(c) and (d). We see that there are more edges detected by our method, Fig. 4(e), than by other methods.

From the results from Figs. 3, 4 and many other test cases, we conclude that the results of the proposed method are better than Sobel method [9], Canny method [8] and Cuckoo search method [18] in two cases: strong object and weak object. The number of edge detection which can be seen by the naked eye is higher than by other methods. Why we choose the number of edge detection. This is an unaccepted problem, because the number of edge detection is expected to be much higher. However, in medical images, they can be the foretoken of serious diseases. We try to help for the doctors in diagnosis or treatment process as much as possible.

Our experiment in edge detection for medical images should not use many filter or threshold to remove bad pixels to avoid the loss of information. We improve the quality of boundaries by threshold in the first steps of Canny, make them smoothier and give more information by B-spline or other curves. If the relationship between neighborhood and pixel, the edge strength, is not the largest, the algorithm can be removed. However, we can consider it if the strength is larger than more eight connected to the neighborhood.

5 Conclusions

Edge detection is the key for contour and segmentation. If the number of edges detection is higher, the result of contour or segmentation is better. In the previous edge detection algorithms, weak objects are a challenge. In this paper, we propose a method to strengthen the quality of weak objects in order to improve the results of edge detection. Our idea is to reconstruct the medical images which have noise. Each pixel is reconstructed by Bayesian thresholding, then we make them smoothier by B-spline curves and improve them with Canny technique. The results of the proposed method are compared with other methods such as: Sobel [9], Canny [8] and Cuckoo Search [18] method. We test with the dataset which is collected from many hospitals. The comparison of the results shows that the proposed method detects more edges than other methods.

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