Robust Topology and Chaos Characteristic of Complex Wireless Sensor Network

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Abstract. Wireless sensor networks have found their chaos characteristics in their applications, for example, in network coding, synchronizing, network security communicating, and so on. To analyze the robustness of WSN under complex status, the nonlinear dynamic equation of network connectivity changed by attack is proposed, and then the bifurcation of network dynamic connection topology is analyzed in the paper. It shows that there is a limit to predict or control the network topology in the global (or top to down design), but it can be partly overcome by inverse design (or down to top design). This analytical finding is confirmed by numerical simulations; meanwhile it presents an inverse design method that is capable of stabilizing network topology under attack.

Keywords: Chaos · Bifurcation · Robust topology · Complex wireless sensor network · Nonlinear dynamic equation

1 Introduction

Chaotic system is sensitive with its initial values; it has the statistic characteristic of white noise, ergodic characteristic of chaotic sequences, very complex fractal structure; and meanwhile is unpredictable.

The nodes, relays, routes in WSN may lose their functions when they are to be attacked. The connection status of network is very complex and unpredictable when the interferences are unknown. Most median or large scale networks have complex dynamic architecture due to their large network capacity, scalability, and mobility. For example, the robust topology is needed when network is under environment stressed, load changed and nodes to be attacked. The related research includes: Helmy studies the small-world network effects in wireless sensor network [1]; other researchers have studied the graph theory [2, 3], scaling theory [4, 5] of WSN.

The interest findings are that the network is regarded as dynamic nonlinear system, and can be classified as three kinds of related chaos. The first kind of chaos comes from the asymptotic integration of Lienard equation [6, 7]. The second kind of chaos comes from the modification of harmonic linearization and describing function (when the generalized Routh–Hurwitz conditions are satisfied) [8, 9]. The third kind of chaos comes from the strange attractors [10, 11].

J. Wan et al. (Eds.): Industrial IoT 2016, LNICST 173, pp. 155–165, 2016.
DOI: 10.1007/978-3-319-44350-8_16

In researches of robust topology of complex sensor network, paper [12] studies detecting topological holes in WSN with no localization information, for example, using a distributed scheme based on the communication topology graph. Paper [13, 14] use an algebraic topological method to detect single overlay coverage holes without coordinates based on homology theory. Paper [15] presents a heuristic method to detect holes based on the topology of the communication graph. Paper [16] presents a coordinate-free method to identify boundaries in WSNs. Paper [17] proposes a self-organization framework based on topological considerations and geometric packing arguments, to determine the boundary nodes and the topology of the whole network.

This paper focuses on robust topology and chaos characteristic of wireless sensor networks under complex status. The contributions of paper include:

(1) The paper presents dynamic nonlinear equations of network topology when they are to be attacked or they lose functions themselves in networks.
(2) The paper analyzes the chaos and Bifurcation of topology of wireless sensor networks.
(3) The paper gives the design method of robust topology of WSN under complex status.

The rest of the paper is organized as follows: Sect. 2 presents the fundament of Bifurcation equation; Sect. 3 studies the dynamic nonlinear equations and network topologies with some lose function nodes; Sect. 4 introduces the robust topology of wireless sensor network; and Sect. 5 is the conclusion of the paper.

2 Bifurcation and Robust Topology

The median and large scale wireless equipment condition monitoring system is a dynamic nonlinear system; its data stream works like Geophysical fluid, it has the characteristics of dissipative system.

Bifurcation theory studies the changes in the qualitative or topological structure of a given system. It has almost the opposite meaning of robustness, a bifurcation occurs when a small smooth change made to the parameter values (the bifurcation parameters) of a system causes a sudden 'qualitative' or topological change in its behavior. Table 1 shows different bifurcations occur in both continuous systems (described by ordinary differential equations-ODEs, delay differential equation-DDEs or Partial Differential Equations-PDEs), and discrete systems (described by maps).

More general, consider the continuous dynamical system described by the ODE.

$$\dot{x} = f(x, \mu); f : \Re^n \times \Re \to \Re^n \tag{1}$$

In Eq. (1), x is state variable, μ is control variable, $\Re$ represents that the variable is in one dimensional real space, $\Re^n$ represents that the variable is in n dimensional real space.

A local bifurcation occurs at (x_0, μ_0) if the Jacobian matrix df_{x_0,μ_0} has an eigenvalue with zero real part.

Table 1. Bifurcation types and their discription

Type	Continuous systems: differential equation ('x' is state variable, 'μ' is control variable.)	Discrete systems: difference equation ('x' is state variable, 'μ' is control variable.)
Pitchfork bifurcation	$\dot{x} = \mu x - x^3$	$x_{n+1} = (\mu + 1)x_n - x_n^3$
Tangent bifurcation	$\dot{x} = \mu - x^2$	$x_{n+1} = \mu + x_n - x_n^2$
Trancritical bifurcation	$\dot{x} = \mu x - x^2$	$x_{n+1} = (\mu + 1)x_n - x_n^2$
A hysteresis bifurcation	$\dot{x} = \mu + \gamma x - x^3$	$x_{n+1} = \mu + (\gamma + 1)x_n - x_n^3$
Symmetry breaking bifurcation	$\dot{x} = \mu + \gamma x - x^3$	$x_{n+1} = \mu + (\gamma + 1)x_n - x_n^3$
Horf bifurcation	$\frac{dx}{dt} = -\gamma \cdot y + x[\mu - (x^2 + y^2)]$ $\frac{dy}{dt} = \gamma \cdot x + y[\mu - (x^2 + y^2)]$	$x_{n+1} = y_n$ $y_{n+1} = \gamma \cdot y_n(1 - x_n)$
Period doubling bifurcation	$\frac{dx}{dt} = -y - z$ $\frac{dy}{dt} = x + 0.2y$ $\frac{dz}{dt} = 0.2 + xz - \mu z$	$x_{n+1} = 4\lambda x_n(1 - x_n)$

3 Dynamic Nonlinear Equations and Lose Function Node Network Topology

This section has two parts: part one discusses the nonlinear dynamic models of complex wireless sensor networks; part two analyzes the network connection topology which has failure nodes.

3.1 The Nonlinear Wireless Sensor Networks Transmitting Model

The data transmitting equation of wireless sensor networks can be described as Eq. (2).

$$\sum_{j=1}^{K} \sum_{i=1}^{k} \mathrm{X}_j(x_i) = \mathrm{Y} \tag{2}$$

In Eq. (2), there is the number of K distributed wireless nodes transmitting data, and X_j represents any one of K nodes. In a certain period, every one of X_j has k data sequence $x_1, ..x_k$. And Y is the transmitted data sum of a determined time period.

From the view of Eq. (2), the data stream of network is transmitting and receiving data sequence from different nodes. An ideal wireless monitoring system should have high accuracy, dynamic network topology, less missing data, and so on. Because the communication environment can influence data transmitting process, and this interferences factor is random, then the real receiving data can be supposed to be a function of Eq. (2). Then we have constraint Eq. (3).

$$\sum_{j=1}^{\mathrm{K}}\sum_{i=1}^{k}\mathrm{X}_j(x_i)-f(\sum_{j=1}^{\mathrm{K}}\sum_{i=1}^{k}\mathrm{X}_j(x_i))<y_{cs} \tag{3}$$

Here y_{cs} is limit of transmitting data error or missing data number.

In multi hop communications of wireless sensor networks, the transmitting node X_j and data sequence x_i in their transmitting process should change, miss, disorder, and delay.

Meanwhile nodes of wireless sensor networks have limited resource and calculate ability, the Eq. (3) can be rewritten as (4). And Λ is influence factor, F is influence function of corresponding node; p is influence possibility of the every different data sequence. Paper proposes that the received data is often to be transmitted in good communication path, and then obtain Eq. (4). As in a certain period, the transmitted data sum may be different with different link path and different start time, so $Y_{i'j'}$ in Eq. (4) mean first node j' and first data sequence number i'. So, more generally, Y in Eq. (2) can be replaced by Y_{ij}.

$$\begin{cases} F(Y_{i'j'}) = \sum\limits_{j=j'}^{K+j'}\sum\limits_{i=i'}^{i'+k'} \Lambda\cdot \mathrm{X}_j(x_i) \\ \prod\limits_{j'=1,..k} p(F(Y_{i'j'})) = \min \end{cases} \tag{4}$$

So paper considers the link chain of network, especially the relationship of next link node communication status variable x_{n+1} with current communicate node status variable x_n.

$$x_{n+1} = f(x_n, u(m), \theta) \tag{5}$$

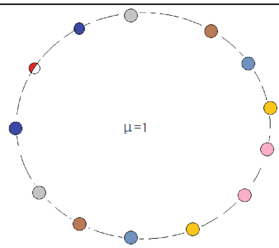
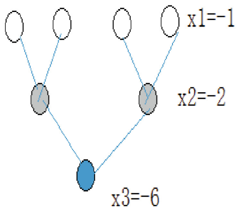
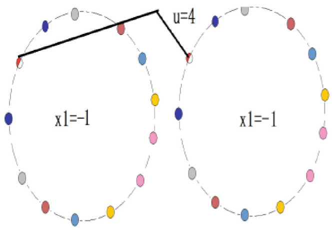
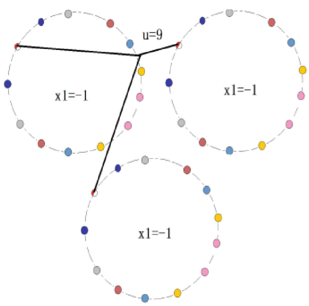
In Eq. (5), $x \in \Re^n$, its initiate value is x_1, control variation $u \in \Re^m$, and θ is uncertain parameter.

It is obvious that dynamic network topology is complex, and its delay, packet loss rate make (5) have characteristic of nonlinear equation. Without loss of generality, the paper will analyze its dynamic process using the geometric method.

3.2 The Analysis of Network Connection Topology of Failure Nodes Based on the Differential Equation

If a full function node (for example relay or route node) lose its function abruptly, it should directly influence the topology of WSN, and then influence the correctness of monitoring data.

Table 2. Some bifurcation examples of the network connection status of failure nodes

topology	network connection status	Control parameters μ and cannot connect nodes in different status x_i	difference equation of network connect status
bidirectional ring		$\mu = 1;$ $x_1 = -1; \rightarrow x_2 = -1;$ $x_3 = -1;$ $\rightarrow$ .....Tangent bifurcation	$\dot{x} = \mu - x^2$ or $x_{n+1} = \mu +$
binary tree		$\mu = 0;$ $x_1 = -1; \rightarrow x_2 = -2;$ $x_3 = -6;$ $\rightarrow$ .....Tangent bifurcation	$\dot{x} = \mu - x^2$ or $x_{n+1} = \mu +$
Two bidirectional ring having two sub network		$\mu = 4;$ $x_1 = -2; \rightarrow x_2 = -2;$ $x_3 = -2;$ $\rightarrow$ .....Tangent bifurcation	$\dot{x} = \mu - x^2$ or $x_{n+1} = \mu +$
Three bidirectional ring having two sub network		$\mu = 9;$ $x_1 = -3; \rightarrow x_2 = -3;$ $x_3 = -3;$ $\rightarrow$ .....Tangent bifurcation	$\dot{x} = \mu - x^2$ or $x_{n+1} = \mu + x$

Then, the paper discusses the bifurcation of network. First, x_1 is defined as the number of nodes that cannot be connected in initiate status (or lowest layer), in next status (or next layer) the number of nodes that cannot be connected is defined as x_2, …, in the end status (sink layer) this is defined as x_n. And then paper defines the control parameter is μ (its physical meaning is characteristics parameter of network topology). Table 2 discusses some examples of failure point network connection status and their differential equations.

Notice, the bidirectional ring can form spiral, and the MESH network can form bidirectional ring, they all have the self like structure. This is why the paper uses the Tangent bifurcation to describe the failure point network connection equation.

4 Robust Topology in Complex Wireless Sensor Network Having Disconnected Nodes

4.1 Chaos in Complex Topology Network Communication

At the beginning of this section, paper presents some definitions.

Definition 1: network connection missing degree *n*. It means in a topology, if the number of nodes which have lost their function (can not communicate to other nodes) is n, then the number of nodes in this network topology that cannot transmit their test data to the data center is $n + 1$, then define the network connection missing degree is n. For example, in the figure, two bidirectional ring having two sub network of Table 1, although, physically only one node lose function, but in logically, same network control parameter make self like structure lose simultaneously (for example, in one work mode: the allocated time slot is same for subnetwork to work simultaneously in large TDMA network or other network).

Definition 2: the folded node, the independent node, the terminal node. The degree (or valency) of a vertex of a graph is the number of edges incident to the vertex. If in degree and out degree of a network is not equal, and the network connection missing degree is 1, the node is folded node.

If in degree and out degree of a network is not equal, and the network connection missing degree is 1, the node is independent node.

Have only in degree, the node is terminal node.

Theory 1: if the network connection missing degree were 1, the transform change of topology of this network can form ‘smale horseshoe’. It has characteristics of local unpredictability by different initial value.

Investigate a network which the protocol of communication is Zigbee, wireless HART and so on; the topology of network is formed as below:

- Ideal MESH construction is made by all independent nodes, for example, the full function nodes;
- Tree cluster network is often made by relay nodes, terminal node and sink node; for example, ZIGBEE

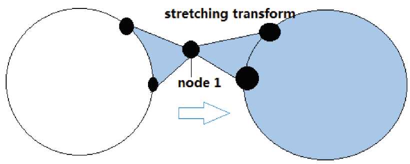
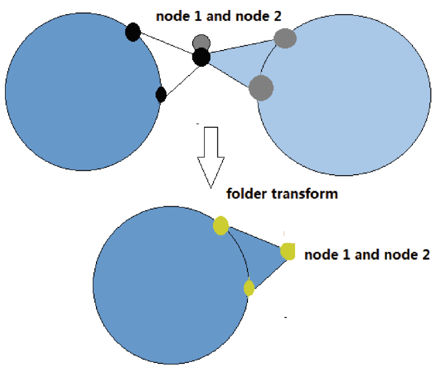
- if a network have arbitrarily topology, for example two bidirectional ring, MESH, and so on, then the network is complex network.

Table 3 shows if the network connection missing degree is 1, then the topology of two bidirectional ring can be stretched and folder, so the two bidirectional ring can do ‘smale horse shoe transformation’, then the communication of complex wireless sensor network having disconnected nodes may have chaos phenomenon.

In detail, a complex wireless sensor network which has disconnected nodes, the communications of nodes have the following properties:

- *it is sensitive to initial conditions; the initial disconnect nodes in different nodes have large difference. For example, in the network missing connection degree is 1, if there is already have one node lose function, then, the next place of lose function node can have very difference and unprediction communication status, so is next node, and so on.*
- *it is topologically mixing; and*
- *it has dense periodic orbits. These two properties is complex network communication native properties, for the orbit or communication path is very complex, it can be topologically mixing, and has dense periodic orbits.*

Table 3. The network topology tranform with a network missing connection degree is 1

Two bidirectional ring stretching transform(though ‘node 1’)	
Two bidirectional ring folder transform(though ‘node 1’ and ‘node 2’)	

4.2 Deployment and Communication in Complex Network

Theory 2: The binary tree topology which is constrained by the communication distance cannot deploy arbitrary shape of 2D in large scale network.

Prove of theory 2: Here use the method of proof by contradiction. And propose, the deployment has no other information.

First, select an infinitesimal (for example square) of a network, if the theory 2 were correct, then in Fig. 1, the number of nodes in a square should be 5, and if four vertex add center point, the edge ‘l1’ and ‘l2’ (link path from node 0 to node 2, and link path from node 1 to node 2) break binary tree topology. This is contradiction with propose.

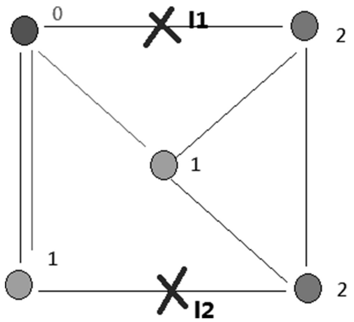


Fig. 1. The demo picture of link path of square

Second, prove the arbitrary shape of 2D can be deployed by square infinitesimal. It is obvious that regular graph can be formed by square infinitesimal. So here only need considers the odd boundary of arbitrary shape. As a special example, suppose wireless sensor node consist of the auxiliary sensor module and the communication module, and the sensor module is independent with communication module. Then the odd boundary of arbitrary shape can be omitted by deploying auxiliary sensor module in this area.

As a deduction of theory 2, the communication distance determined cluster-tree topology network is random branch network.

4.3 Inverse Design of Robust Topology Network

As there are many bifurcation and chaos phenomenon in complex wireless sensor network. Then one cannot design the robust complex wireless sensor network in general or from top to down, owing to their unpredicted properties.

To design a robust topology network, here reference the local network design. For example, when we design network local Ethernet, we do not concern the whole world internet network, we only concern the local network. We design a network from down to top, or do inverse design.

From the view of this, the robust topology of network is in a very small local area, to form bidirectional ring, for example to form redundancy ring in cluster head that can be robust topology in complex wireless sensor network.

5 Simulation and Results

5.1 Phase Plot of Communication Having Random Disconnected Nodes

In Fig. 2, the communication area is changed when disconnected node changed, the communication plot with the lose node changed, but as circle, the whole area can all be covered (except the disconnected node itself). This is a explanation of network missing connection degree.

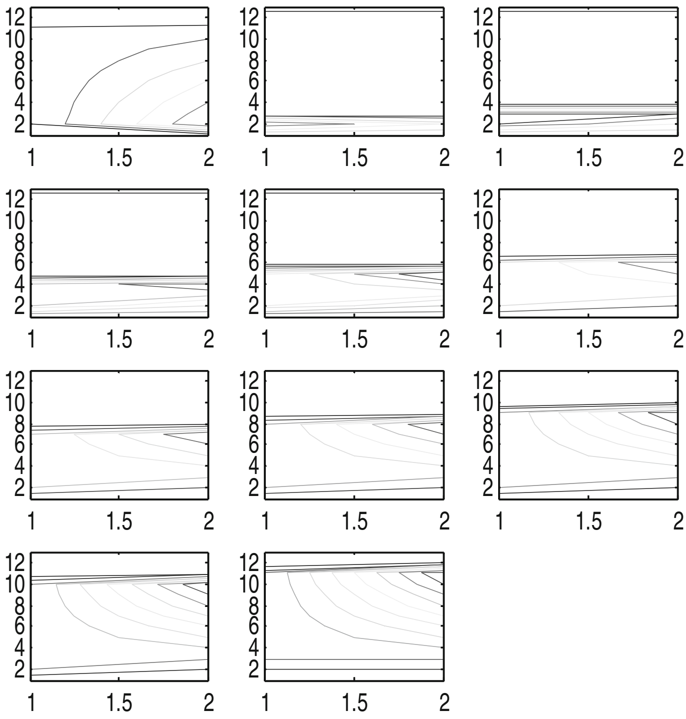


Fig. 2. The connection status of 12 nodes bidirectional ring which have a random lose funtion node

5.2 Eddy Phase Plot of Four Connection Nodes

In Fig. 3, there are four bidirectional nodes in topology, the phase plot of its communication area. It is obvious, the ring phase interspaces have cross connected phenomenon, have circulation pattern.

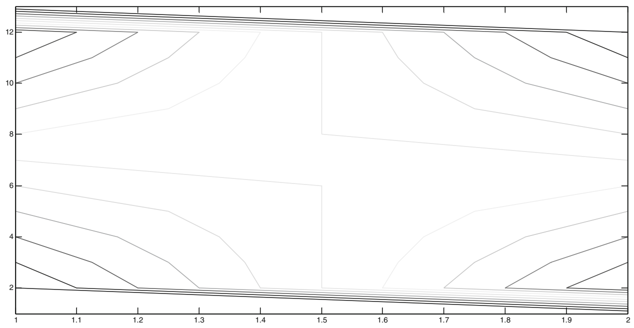


Fig. 3. The connection status of 4 bidirectional ring nodes

In Fig. 3, there are four bidirectional nodes in topology, the phase plot of its communication area. It is obvious, the ring phase interspaces have cross connected phenomenon, have circulation pattern.

6 Conclusion

To analyze the robustness of WSN under complex status, the nonlinear dynamic equation of connectivity network topology is introduced, and then the fractal of dynamic wireless sensor network topology is analyzed in the paper.

The analysis shows that there is a limit that to predict or control the network topology in the global (or top to down), but it can be partly overcome by inverse design (or down to top). This analytical finding is confirmed by numerical simulations.

Acknowledgment. This article is supported by "China-Canada Joint Research Project" (project number: 2009DFA12100) and Major Project of Education Department in Sichuan (14ZA0172). For support, contributions to discuss we would like to thank the rest of MCC (mobile computer center) of UESTC.

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