Fair Channel Sharing by Wi-Fi and LTE-U Networks with Equal Priority

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Abstract. The paper is concerned with the problem Wi-Fi and LTE-U networks sharing access to a band of communication channels, while also considering the issue of fairness in how the channel is being shared. As a criteria of fairness for such joint access, α-fairness and maxmin fairness with regards to expected throughput are explored as fairness metrics. Optimal solutions are found in closed form, and it is shown that these solutions can be either: (a) a channel on/off strategy in which access to the channels is performed sequentially, or (b) a channel sharing strategy, i.e., where simultaneous joint access to the channels is applied. A criteria for switching between these two type of optimal strategies is found, and its robustness on the fairness coefficient is established, as well as the effectiveness of the fairness coefficient to control the underlying protocol of the joint access to the shared resource is managed. Finally, we note that the approach that is explored is general, and it might be adapted to different problems for accessing a sharing resource, like joint sharing of voice and data traffic by cellular carriers.

Keywords: Fairness · Maxmin fairness · LTE-U · Wi-Fi

1 Introduction

With the emergence of new wireless applications and devices, the demand being placed on limited radio spectrum has been dramatically increasing over the last decade. Developing methods by which different wireless technologies can effectively share under-utilized spectrum bands is an important step towards meeting this growing demand. Consequently, maintaining fair coexistence in unlicensed spectrum, e.g. technologies like LTE-U and Wi-Fi, with respect to throughput and latency has become a widely discussed topic by the wireless community [1], even though the specification for LTE-U has not yet been finalized.

Currently, there are only a few works dealing with LTE-U and Wi-Fi coexistence. In [2], based on simulation results it was showed that LTE system performance might be slightly affected by coexistence, whereas Wi-Fi is significantly impacted by LTE transmissions. In [3], it was pointed out that the coexistence of

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LTE and Wi-Fi needs to be carefully investigated since, as it was illustrated, Wi-Fi might be severely impacted by LTE transmissions. The performance of coexisting femtocell and Wi-Fi networks operating over a fully-utilized unlicensed band were analytically modeled in [4]. The effects of Wi-Fi channel access parameters on the performance of Wi-Fi and femtocell networks were investigated in [5]. A fair and QoS-based unlicensed spectrum splitting strategy between Wi-Fi and femtocell networks was studied in [6], and experimental results for the coexistence of Wi-Fi and LAA-LTE were presented in [7]. Modeling the coexistence of LTE and Wi-Fi heterogeneous networks was performed in [9], and a proportional fair allocation scheme for them was develop in [8].

One critical challenge facing coexistence of such technologies is having an architecture that can support dynamic spectrum management of LTE-U and Wi-Fi networks [10,11]. In [10], a system for coordinating between multiple heterogeneous networks to improve spectrum utilization and facilitate co-existence, which is built on the principles of Software Defined Networking to support logically centralized dynamic spectrum management involving multiple autonomous networks, was presented. Based on this architecture, an optimization model to maximize the aggregated Wi-Fi+LTE throughput was designed and tested in [11]. This optimization problem was divided into two steps: in the first step, based on information about networks' infrastructure and agents exploiting networks' facilities, power control optimization problems were solved to get optimal throughput for Wi-Fi only access and joint Wi-Fi+LTE access to the channels. In the second step, a throughput maxmin problem dealing with joint time division channel access was solved. Evaluation of such joint coordination showed that such a dual optimization approach might increase the aggregated Wi-Fi+LTE throughput. Our work builds upon these prior efforts, and in particular, in this paper we provide strong analytical evidence supporting this result, and consider joint coordination of Wi-Fi+LTE under a unified fairness criteria.

The organization of this paper is as follows: in Sect. 2, we first give formulation of α-fairness problem. Then, in Sect. 3, we solve it in closed form. In Sect. 4, the problem of maxmin fairness is formulated and solved. Finally, in Sect. 5, discussions are supplied.

2 Formulation of α-Fairness Problem

As a system for coordinating between multiple heterogeneous networks for the improvement spectrum utilization and to facilitate co-existence, we consider the model suggested in [11]. The core element of this system is a Global Controller (GC), which employs information about the network "ecology" and the associated access points to calculate throughput under separate or joint access between Wi-Fi and LTE networks. In this paper we add to the system an additional component, the Fairness Decision Maker (FDM) that, based on this information, finds the fair *joint* time division channel access and returns it to the GC to optimize throughput (Fig. 1).

To deal with the problem of fairly allocating the fraction of time each system can access the channel, it is necessary to employ an appropriate fairness metric.

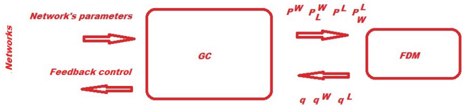


Fig. 1. Coordination between GC and FDM involves passing parameters and solutions to the optimization

A survey of different fairness concepts as used in wireless communication is given in [12]. Generally, in the formulation of fairness, there are n agents, each of which has an utility depending on its share of a common resource. The fair allocation of a common resource depends directly on the criteria for fairness being used, and maxmin is one possible criteria that is popular in the literature. We focus, however, on α-fairness, which provides a unified framework for considering a wide array of fairness concepts, such as bargaining (for $\alpha = 1$) and maxmin (for α tending to infinity). We note that α-fairness has been applied previously in the literature, in [13] it was applied to a throughput assignment problem, while in [14], it was applied to fair power control for femtocell networks. In [15], generalized α-fairness concept explored and applied to optimizing resource allocation in downlink cellular networks. In [16,17], a problem of fair resource allocation under a malicious attack was investigated for SINR (signal to interference plus noise ratio) and throughput as the user utilities.

We note that although we deal only with two agents (LTE-U and Wi-Fi networks with throughput as their utilities), the problem generalizes to a classical fairness problem since an agent's utility depends on the amount of the resource it uses (individual access to the channels) as well as on the joint resource (joint access to the channels). In this more general situation, we can also observe that increasing the coefficient of fairness to ∞ yields the maxmin criteria.

Let us formulate the problem of fairly allocating the fraction of time that Wi-Fi and LTE-U access a channel. In order to get insight into the problem, similar to what was studied in [11], we assume that the total throughput of each network is proportional to the fraction of time a technology access the channel and on whether the channel access by Wi-Fi and LTE-U is simultaneous or not. In this paper, we consider a model where there is equal right to access the channels for both Wi-Fi and LTE-U networks. To describe the problem let us introduce the following notations:

(i) q^W is the fraction of time the channel is accessed by Wi-Fi network only (Wi-Fi access mode).
(ii) q^L is the fraction of time the channel is accessed by LTE-U network only (LTE-U access mode).
(iii) q is the fraction of time the channel is accessed by both the networks simultaneously (Joint Wi-Fi and LTE-U access mode).
(iv) Without loss of generality we can assume that total time slot for access to the channel is denoted $[0, 1]$. Thus, $q^W + q^L + q = 1$, and the vector of time fractions is $\boldsymbol{q} = (q^L, q, q^W)$.

(v) P^W is the throughput of the Wi-Fi network per time unit, when the network is in Wi-Fi access mode.
(vi) P^L is the throughput of the LTE-U network per time unit, when the network is in LTE-U access mode.
(vii) P^L_W and P^W_L are the throughputs of LTE-U and Wi-Fi networks per time unit, when the system is in joint Wi-Fi and LTE-U access mode, where both networks access the channel simultaneously. It is natural to assume that the extra interference in the network reduces its throughput, i.e., $P^L_W \le P^L$ and $P^W_L \le P^W$.
(viii) $\overline{P}^W$ is the total throughput of the Wi-Fi network, i.e., $\overline{P}^W = q^W P^W + qP^W_L$.
(ix) $\overline{P}^L$ is the total throughput of the LTE-U network, i.e., $\overline{P}^L = q^L P^L + qP^L_W$.

If $q = 0$, we call such strategy $\boldsymbol{q}$ as a *channel on/off strategy*, i.e., the networks do not access the channel simultaneously, but rather one by one. If $q > 0$, we call such strategy $\boldsymbol{q}$ as a *channel sharing strategy*, i.e., in which the networks might access the channel simultaneously. Note that, different resource sharing strategies have arisen in different network optimization problems, c.f. for channel sharing [18–21], for bandwidth scanning [22–24], for time sharing [25], and for node protection [26]. To deal with the problem of the joint access to a shared channel for LTE-U and Wi-Fi networks we apply α-fair approach, which also incorporate the maxmin approach. The considered α-fairness problem can be formulated as follows:

$$v_\alpha = \max_{\boldsymbol{q}} v_\alpha(\boldsymbol{q}), \tag{1}$$

with

$$v_\alpha(\boldsymbol{q}) = \begin{cases} \dfrac{(q^W P^W + qP^W_L)^{1-\alpha}}{1-\alpha} + \dfrac{(qP^L_W + q^L P^L)^{1-\alpha}}{1-\alpha}, & \alpha \neq 1, \\ \ln(q^W P^W + qP^W_L) + \ln(qP^L_W + q^L P^L), & \alpha = 1. \end{cases}$$

Let $\boldsymbol{q}_\alpha$ be the optimal α-fair strategy, i.e., $\boldsymbol{q}_\alpha := (q^L_\alpha, q_\alpha, q^W_\alpha) = \arg\max_{\boldsymbol{q}} v_\alpha(\boldsymbol{q})$.

3 Optimal α-Fair Strategies

In this section, Theorem 1 gives the optimal α-fair strategy $\boldsymbol{q}_\alpha$ in closed form as well as the condition for it to be either a channel on/off strategy or a channel sharing strategy.

Theorem 1. *(a) Let*

$$P^W_L/P^W + P^L_W/P^L < 1. \tag{2}$$

Then the optimal α-fair strategy $\boldsymbol{q}_\alpha = (q^W_\alpha, q_\alpha, q^L_\alpha)$ is channel on/off strategy, and it is given as follows:

$$(q^W_\alpha, q_\alpha, q^L_\alpha) = \left((P^W)^{(1-\alpha)/\alpha} \Big/ \left((P^L)^{(1-\alpha)/\alpha} + (P^W)^{(1-\alpha)/\alpha}\right), 0, 1 - q^W_L\right). \tag{3}$$

(b) Let (2) do not hold and

$$(P_W^L)^{1-\alpha}(P_L^W)^{\alpha} \le P^W - P_L^W \textit{ and } (P_W^L)^{\alpha}(P_L^W)^{1-\alpha} > P^L - P_W^L. \tag{4}$$

Then

$$(q_\alpha^W, q_\alpha, q_\alpha^L) = \left(1 - q_\alpha, \frac{(P_W^L)^{(1-\alpha)/\alpha}/(1 - P_L^W/P^W)^{1/\alpha}}{(P^W)^{(1-\alpha)/\alpha} + \left(P_W^L/(1 - P_L^W/P^W)\right)^{(1-\alpha)/\alpha}}, 0\right). \tag{5}$$

(c) Let (2) do not hold and

$$(P_W^L)^{\alpha}(P_L^W)^{1-\alpha} \le P^L - P_W^L \textit{ and } (P_W^L)^{1-\alpha}(P_L^W)^{\alpha} > P^W - P_L^W. \tag{6}$$

Then

$$(q_\alpha^W, q_\alpha, q_\alpha^L) = \left(0, \frac{(P_L^W)^{(1-\alpha)/\alpha}/(1 - P_W^L/P^L)^{1/\alpha}}{(P^L)^{(1-\alpha)/\alpha} + \left(P_L^W/(1 - P_W^L/P^L)\right)^{(1-\alpha)/\alpha}}, 1 - q_\alpha\right). \tag{7}$$

(d) Let (2) do not hold and

$$(P_W^L)^{1-\alpha}(P_L^W)^{\alpha} \ge P^W - P_L^W \textit{ and } (P_W^L)^{\alpha}(P_L^W)^{1-\alpha} \ge P^L - P_W^L. \tag{8}$$

Then $(q_\alpha^W, q_\alpha, q_\alpha^L) = (0, 1, 0)$.

(e) Let (2) do not hold and

$$(P_W^L)^{1-\alpha}(P_L^W)^{\alpha} \le P^W - P_L^W \textit{ and } (P_W^L)^{\alpha}(P_L^W)^{1-\alpha} \le P^L - P_W^L. \tag{9}$$

Then $\boldsymbol{q}_\alpha$ *is given by (5) for*

$$(P^W)^{(1-\alpha)/\alpha} + \left(\frac{P_W^L}{1 - P_L^W/P^W}\right)^{(1-\alpha)/\alpha} > (P^L)^{(1-\alpha)/\alpha} + \left(\frac{P_L^W}{1 - P_W^L/P^L}\right)^{(1-\alpha)/\alpha}, \tag{10}$$

and $\boldsymbol{q}_\alpha$ *is given by (7) if (9) does not hold.*

The case $\alpha = 0$ is a limiting case for this theorem in which we examine α tending to zero. Then, $\boldsymbol{q}_0 = (1, 0, 0)$ if $P^W \ge \max\{P_L^W + P_W^L, P^L\}$, $\boldsymbol{q}_0 = (0, 0, 1)$ if $P^L \ge \max\{P_L^W + P_W^L, P^W\}$ and $\boldsymbol{q}_0 = (0, 1, 0)$ if $P_L^W + P_W^L \ge \max\{P^W, P^L\}$.

Figures 2 and 3 illustrate the optimal α-fair fraction of time for applying the access mode for $P_L^W \in [0.1, P^W]$, $P_W^L \in [0.1, P^L]$ and $P^W = P^L = 3$ and $\alpha = 0.1, 2$. These figures illustrate that increasing the α-coefficient causes a reduction in the zone of permanent joint access to the channel, with it diminishing while α is increasing. Also, the figures illustrate the robustness of the zone for applying channel on/off strategies on the fairness coefficient.

4 Maxmin Fairness

The maxmin strategy for joint access between Wi-Fi and LTE-U networks to a shared channel in terms of maxmin throughput $\overline{P}$ can be formulated as follows: $\overline{P} = \max_{\boldsymbol{q}} \min\{q^W P^W + qP^W_L, q^L P^L + qP^L_W\}$. The following theorem gives the optimal maxmin strategy $\boldsymbol{q}$ in closed form, as well as the condition for existence of optimal channel on/off and channel sharing strategies.

Theorem 2. *(a) The maxmin solution $\boldsymbol{q}$ is a channel sharing one, i.e., $q > 0$, if and only if (2) does not hold.*

(a_1) If $P^W_L \geq P^L_W$ then $(q^W, q, q^L) = (0, 1/(1 + (P^W_L - P^L_W)/P^L), 1 - q)$;

(a_2) If $P^W_L \leq P^L_W$ then $(q^W, q, q^L) = (1 - q, 1/(1 + (P^L_W - P^W_L)/P^W), 0)$.

(b) If (2) holds then $q = 0$, i.e., maxmin solution is an channel on/off strategy, and $(q^W, q, q^L) = (P^L/(P^L + P^W), 0, P^W/(P^L + P^W))$.

This theorem shows the difference between the maxmin and the α-fair solution. Namely, permanent joint access to the channel (i.e., when $q_\alpha = 1$) cannot be a maxmin solution. Meanwhile, the more general α-fairness accepts a permanent joint access as an optimal solution.

Figure 4 illustrates zones (P^L_W, P^W_L) for applying optimal maxmin channel sharing and on/off strategies, and the switching lines between them. The dotted domain depicts the zone where the optimal solutions for both models coincide

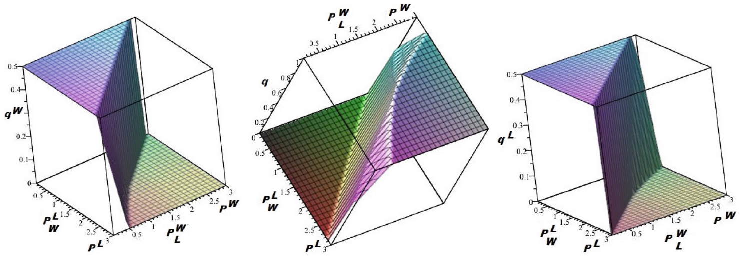


Fig. 2. The optimal α-fair time fractions q^W (left), q (center) and q^L (right) for $P^W_L \in [0.1, P^W]$, $P^L_W \in [0.1, P^L]$ and $P^W = P^L = 3$ and $\alpha = 0.1$

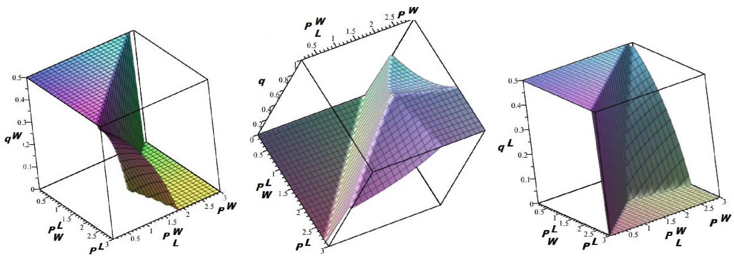
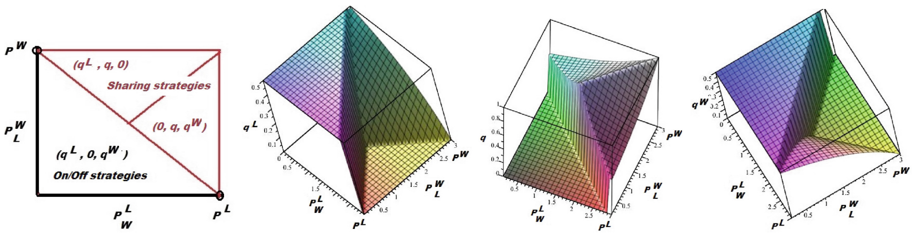


Fig. 3. The optimal α-fair time fractions q^W (left), q (center) and q^L (right) for $P^W_L \in [0.1, P^W]$, $P^L_W \in [0.1, P^L]$ and $P^W = P^L = 3$ and $\alpha = 2$.

with each other. Also, Fig. 4 illustrates the optimal time fractions q^L, q and q^W as a function of throughput in Wi-Fi and LTE-U access mode $P^W_L \in [0.1, P^W]$ and $P^L_W \in [0.1, P^L]$ for $P^W = P^L = 3$. In the zone of using such a mode we have that

(a) if $P^W_L > P^L_W$ then the time fraction q to use such mode is increasing in P^L_W and decreasing in P^W_L. Wi-Fi access mode is not used at all, while the LTE-U access mode is decreasing in P^L_W and increasing in P^W_L;
(b) if $P^W_L < P^L_W$ then the frequency q to use such a mode is increasing in P^W_L and decreasing in P^L_W. LTE-U access mode is not used at all, meanwhile Wi-Fi access mode is decreasing in P^W_L and increasing in P^L_W.

Thus, in the zone of joint access mode, the network with higher throughput yields longer access to the channel to the network with lower throughput. If either P^L_W or P^W_L becomes too large, the optimal maxmin access to the channel switches to a channel on/off strategy.



(a) (b) (c) (d)

Fig. 4. (a) Zones of applying maxmin sharing and on/off strategies, and the optimal time fractions q^W (b), q (c) and q^L (d) for $P^W_L \in [0.1, P^W]$, $P^L_W \in [0.1, P^L]$ and $P^W = P^L = 3$.

5 Discussion

This paper examined the problem of Wi-Fi and LTE-U networks sharing access to a channel in support of each network's throughput needs. As a criteria for how the two networks jointly access the channel, we formulated the sharing using the α-fairness over expected throughput criteria. Maxmin access to the channel results as a limit case for the considered criteria as the fairness coefficient tends to infinity. It was shown that the optimal solution can be either (a) channel on/off strategies, i.e., in which access to the channel is performed sequentially, or (b) channel sharing strategies, i.e., in which simultaneous joint access to the channels is possible. A criteria for switching between these two types of optimal strategies was found, and the robustness of this criteria to the fairness coefficient was established. It was shown that the α-fair solution tends to a maxmin solution as α tends to infinity, and that a strategy supporting permanent joint access cannot be optimal for the maxmin problem, yet it can be optimal for the α-fairness problem. In particular, we showed that the fairness coefficient can be an efficient tool to control the protocol that determines what fraction of time Wi-Fi and LTE-U networks are dedicated to use the channel by themselves or the fraction of time they simultaneously use a channel. Finally, we note that the suggested scheme for access to a joint resource is general, and its realization depends on the implementation of the global controller, who is in charge for processing all the data on networks' facilities and users's demands associated with a specific problem. For Wi-Fi and LTE-U networks, a model of the global controller was developed in [11] and the model-based results were partially validated via experimental evaluations using USRP based SDR platforms on the ORBIT testbed. Numerical modelling in [11] showed significant gains in both Wi-Fi and LTE performance under the global controller's moderation. A goal of our future work is to adapt the formalism presented to the problem of multiple cellular providers sharing access to a communication medium in support of different classes of users during a disaster scenario.

A Appendix I: Proof of Theorem 1

To find the optimal $\boldsymbol{q} = bq_\alpha$ define the Lagrangian $L_\omega(\boldsymbol{q}) = v_\alpha(\boldsymbol{q}) + \omega(1 - q^W - q - q^L)$. Thus, $\boldsymbol{q}$ is the optimal probability vector then the following conditions has to hold:

$$\frac{P^W}{(q^W P^W + q P^W_L)^\alpha} \begin{cases} = \omega, & q^W > 0, \\ \leq \omega, & q^W = 0, \end{cases} \tag{11}$$

$$\frac{P^L}{(q^L P^L + q P^L_W)^\alpha} \begin{cases} = \omega, & q^L > 0, \\ \leq \omega, & q^L = 0, \end{cases} \tag{12}$$

and

$$\frac{P^W_L}{(q^W P^W + q P^W_L)^\alpha} + \frac{P^L_W}{(q^L P^L + q P^L_W)^\alpha} \begin{cases} = \omega, & q > 0, \\ \leq \omega, & q = 0. \end{cases} \tag{13}$$

Thus, the boundary strategies $\boldsymbol{q} = (1, 0, 0)$ and $\boldsymbol{q} = (0, 0, 1)$ cannot be optimal.

First we find the condition when the rest boundary strategy $\boldsymbol{q} = (0, 1, 0)$ can be optimal. Substituting it into (11)–(13) implies that the following condition has to hold: $\omega = (P^W_L)^{1-\alpha} + (P^L_W)^{1-\alpha} \geq \max\{P^L/(P^L_W)^\alpha, P^W/(P^W_L)^\alpha\}$. This condition is equivalent to (8), and (d) follows.

Let us pass to finding channel sharing optimal strategy, i.e., with $q > 0$. Then, either $q^W = 0$ or $q^L = 0$.

Let $q^L = 0$. Then, (11)–(13) turn into the following conditions

$$P^W/(q^W P^W + q P^W_L)^\alpha = \omega, \tag{14}$$

$$P^L/(q P^L_W)^\alpha \leq \omega \tag{15}$$

and

$$P_L^W/(q^W P^W + qP_L^W)^\alpha + P_W^L/(qP_W^L)^\alpha = \omega. \tag{16}$$

By (14),

$$q^W P^W + qP_L^W = \left(P^W/\omega\right)^{1/\alpha}. \tag{17}$$

Since $q^W + q = 1$ then $(1-q)P^W + qP_L^W = \left(P^W/\omega\right)^{1/\alpha}$. So,

$$q = \left(1 - (P^W)^{(1-\alpha)/\alpha}/\omega^{1/\alpha}\right) / \left(1 - P_L^W/P^W\right). \tag{18}$$

By (14) and (16),

$$\frac{P_L^W}{P^W}\omega + \frac{P_W^L}{(qP_W^L)^\alpha} = \omega. \tag{19}$$

Thus,

$$q = \frac{(P_W^L)^{(1-\alpha)/\alpha}}{(1 - P_L^W/P^W)^{1/\alpha}\omega^{1/\alpha}}. \tag{20}$$

By (18) and (20),

$$\frac{1 - (P^W)^{(1-\alpha)/\alpha}/\omega^{1/\alpha}}{1 - P_L^W/P^W} = \frac{(P_W^L)^{(1-\alpha)/\alpha}}{(1 - P_L^W/P^W)^{1/\alpha}\omega^{1/\alpha}}. \tag{21}$$

Thus,

$$\omega^{1/\alpha} = (P^W)^{(1-\alpha)/\alpha} + \left(\frac{P_W^L}{1 - P_L^W/P^W}\right)^{(1-\alpha)/\alpha}. \tag{22}$$

Thus,

$$q = \frac{\dfrac{(P_W^L)^{(1-\alpha)/\alpha}}{(1 - P_L^W/P^W)^{1/\alpha}}}{(P^W)^{(1-\alpha)/\alpha} + \left(\dfrac{P_W^L}{1 - P_L^W/P^W}\right)^{(1-\alpha)/\alpha}}. \tag{23}$$

It is clear that $q > 0$. Since, $\boldsymbol{q}$ is the probability vector we have to find only the condition for q being less or equal to 1. By (23), it is equivalent to

$$\frac{(P_W^L)^{(1-\alpha)/\alpha}}{\left(1 - P_L^W/P^W\right)^{1/\alpha}} \le (P^W)^{(1-\alpha)/\alpha} + \left(\frac{P_W^L}{1 - P_L^W/P^W}\right)^{(1-\alpha)/\alpha}. \tag{24}$$

The last inequality is equivalent to

$$(P_W^L)^{1-\alpha}(P_L^W)^\alpha \le P^W - P_L^W. \tag{25}$$

Finally we have to find the condition that (15) holds. Substituting q from (20) into (15) implies (b).

The case $q^W = 0$ as well as the case $q = 0$, $q^W > 0$ and $q^L > 0$ can be considered similarly, and (a) and (c) follow. To deal with (e) denote by $\boldsymbol{q}_b$ and

$\boldsymbol{q}_c$ the optimal strategies given by (b) and (b). The previous analyze yields that in (d) the optimal strategy is $\boldsymbol{q}_b$ if $v_\alpha(\boldsymbol{q}_b) > v_\alpha(\boldsymbol{q}_c)$, and it is $\boldsymbol{q}_c$ if $v_\alpha(\boldsymbol{q}_b) < v_\alpha(\boldsymbol{q}_c)$.

Note that, by (17) and (19),

$$v_\alpha(\boldsymbol{q}_b) = \frac{1}{1-\alpha}\left(\frac{P^W}{\omega}\right)^{(1-\alpha)/\alpha} + \frac{1}{1-\alpha}\left(\frac{P_W^L}{(1-P_L^W/P^W)\,\omega}\right)^{(1-\alpha)/\alpha}. \quad (26)$$

Substituting (22) into (26) yields

$$(1-\alpha)v_\alpha(\boldsymbol{q}_b) = ((P^W)^{(1-\alpha)/\alpha} + (P_W^L/(1-P_L^W/P^W))^{(1-\alpha)/\alpha})^\alpha.$$

By symmetry, $v_\alpha(\boldsymbol{q}_c)$ can be found, and the result follows. ■

B Appendix II: Proof of Theorem 2

The maxmin problem is equivalent to the following LP problem

$$\begin{aligned}&\text{maximize } \nu,\\ &q^W P^W + qP_L^W \geq \nu, qP_W^L + q^L P^L \geq \nu,\ q^W + q + q^L = 1, q^W, q, q^L \geq 0.\end{aligned} \quad (27)$$

Let for while the component q of the strategy $\boldsymbol{q}$ be fixed and optimal, while Then, component q^W and q^L might vary. Since $q^W + q^L = 1 - q$ the optimal q^W can be found as a solution of the following problem:

$$\begin{aligned}&\text{maximize } \nu,\\ &q^W P^W + qP_L^W \geq \nu, qP_W^L + (1-q)P^L - q^W P^L \geq \nu, q^W \in [0, 1-q].\end{aligned} \quad (28)$$

First we look for the optimal channel sharing strategy, i.e., when $q > 0$. Figure 5 illustrates that the solution of LP problem (28) can be found as an intersection of the corresponding lines.

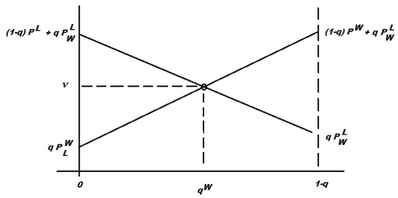


Fig. 5. Solution of the LP problem

Thus, the channel sharing solution holds if and only if $(1-q)P^L + qP_W^L > qP_L^W$ and $(1-q)P^W + qP_L^W > qP_W^L$. These inequalities are equivalent to

$$(1-q)P^L > q(P_L^W - P_W^L) \text{ and } (1-q)P^W > q(P_W^L - P_L^W). \quad (29)$$

Since either $P_L^W \geq P_W^L$ or $P_W^L > P_L^W$, then one of the conditions (29) always hold. Without loss of generality we assume that

$$P_L^W \geq P_W^L. \tag{30}$$

Then, conditions (29) are equivalent to

$$q \leq 1/(1 + (P_L^W - P_W^L)/P^L). \tag{31}$$

Let us switch on to finding the optimal ν and q^W. By Fig. 5 and (28),

$$q^W P^W + qP_L^W = qP_W^L + (1-q)P^L - q^W P^L = \nu. \tag{32}$$

Thus,

$$q^W = (P^L - (P^L + P_L^W - P_W^L)q)/(P^L + P^W). \tag{33}$$

Thus, by (30), q^W is decreasing in q, and

$$\nu = (P^W P^L + (P^L P_L^W + P^W P_W^L - P^L P^W)q)/(P^W + P^L). \tag{34}$$

So, (33) and (34) give channel sharing solution of (27) for a fixed q, and (31) is the condition such solution holds. Note that, by (34), ν is increasing in q if $P^L P_L^W + P^W P_W^L > P^L P^W$, and ν is decreasing in q otherwise. Thus, if (30) holds, then the channel sharing solution exists ($q > 0$) if and only if $P^L P_L^W + P^W P_W^L > P^L P^W$, and then $q = 1/(1 + (P_L^W - P_W^L)/P^L)$. Substituting this q into (33) implies $q^W = 0$. Thus, $q^L = 1 - q$. The case of the channel on/off optimal strategy can be considered similarly, and the result follows. ∎

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