Min-max BER Based Power Control for OFDM-Based Cognitive Cooperative Networks with Imperfect Spectrum Sensing

Hangqi Li(✉), Xiaohui Zhao, and Yongjun Xu

College of Communication Engineering, Jilin University, Changchun 130012, China
{lhq14,xuyj10}@mails.jlu.edu.cn, xhzhao@jlu.edu.cn

Abstract. In this paper, a power control (PC) algorithm for multi-user Orthogonal Frequency Division Multiplexing (OFDM)-based cognitive cooperative networks under the imperfect spectrum sensing is studied to minimize total Bit Error Rate (BER) of secondary users (SUs) under the consideration of maximum transmit power budgets, signal-to-interference-and-noise ratio (SINR) constraints and interference requirements to guarantee quality of service (QoS) of primary user (PU). And a cooperative spectrum sensing (CSS) strategy is considered to optimize sensing performance. The worst-channel-state-information (worst-CSI) PC algorithm is introduced to limit the BER of SUs, which only needs to operate the algorithm in one link that CSI is worst, while the interference model is formulated under the consideration of spectrum sensing errors. In order to obtain optimal solution, the original min-max BER optimization problem is converted into a max-min SINR problem solved by Lagrange dual decomposition method. Simulation results demonstrate that the proposed scheme can achieve good BER performance and the protection for PU.

Keywords: Cooperative transmission · Imperfect spectrum sensing · OFDM-based cognitive radio networks · The worst-CSI PC algorithm

1 Introduction

Cognitive radio (CR), as an efficient technology for next generation of wireless communication, can significantly improve spectrum utilization by dynamically detecting spectrum usage and opportunistically accessing the free frequency band [1]. Generally, power control (or resource allocation) techniques are used in CR networks (CRNs), which depends on perfect channel state information (CSI) and spectrum sensing results. In practical CRNs, there are several network types, such as traditional CRNs, cognitive relay networks, OFDM-based CRNs and multi-antenna CRNs [2].

In order to expand communication scope, cooperative technology is introduced to help primary users (PUs) or secondary users (SUs) for their communications in CRNs [3]. The earliest emergence of relay networks can be traced back to the late 1970s, proposed by Dr. Cover in [4,5]. Since cognitive relay networks

D. Noguet et al. (Eds.): CROWNCOM 2016, LNICST 172, pp. 654–667, 2016.
DOI: 10.1007/978-3-319-40352-6_54

have more advantages than traditional CRNs (i.e., non-relay network), and are suitable for actual communication scenarios (i.e., heterogeneous networks, 5G communications), in this paper, we focus on the study of power allocation problem in multiuser cognitive relay networks.

Currently, power control (PC) as an important role in the performance of CRNs can provide protection for PU and allow SUs opportunistically transmit data. In [6], a distributed PC algorithm for a multiuser CRN with multicell environments is given to address uplink interference management problem. Due to the advantage of flexible scheduling spectrum of the orthogonal frequency division multiplexing (OFDM) technology, it has been widely introduced to CRNs [7].

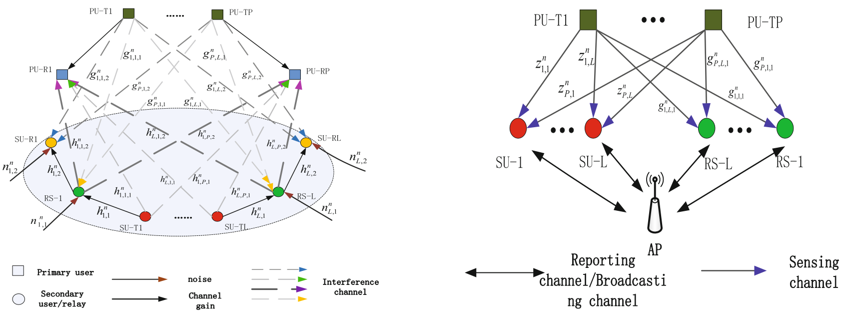
Obviously, the literatures mentioned above only consider PC problem under perfect spectrum sensing information, which may not be valid in practice due to time-varying channels, inevitable errors and uncertainties as results of imperfect spectrum sensing. In order to obtain good system performance and ensure quality of service (QoS) of SUs and PUs, it is necessary to take errors of spectrum sensing into consideration. Based on different optimization functions (e.g., minimization of total power allocation, maximization of capacity of SUs, maximization of energy efficiency, etc.), PC problems with the imperfect spectrum sensing have been studied from various network structures (e.g., traditional CRNs, OFDM-based CRNs, micro CRNs, etc.). Considering a traditional CRN with the imperfect spectrum sensing, PC problem is studied in [8]. In [9], for an OFDM-based CRN, the resource allocation is studied to maximize the overall capacity of SUs. Considering the imperfect spectrum sensing in CRNs with one primary network (PN) and many micro CRNs [10], a hybrid spectrum access strategy is proposed, where the capacity of secondary link is maximized. However, research of PC problem in cognitive relay networks under the imperfect spectrum sensing is quite few.

In this paper, a PC algorithm is proposed to minimize total bit error rate (BER) of SUs in OFDM-based cognitive relay networks under the imperfect spectrum sensing. Multiple PUs, multiple SUs and multiple relays are considered in our model. The min-max criteria is used to minimize total BER of SUs under practical constraints. We convert the original min-max BER optimization problem into an equivalent max-min signal-to-interference-and-noise ratio (SINR) problem solved by Lagrange dual decomposition.

The reminder of this paper is organized as follows. In Sect. 2, system model is described. Section 3 introduces cooperative spectrum sensing (CSS) scheme and describes the interference model. Next, PC problem with the imperfect spectrum sensing is formulated and the algorithm is given in Sect. 4. Section 5 presents some numerical results and analysis of the system performance. Finally, Sect. 6 provides the conclusion of the paper.

2 System Model

In this paper, we consider an overlay cognitive amplify-and-forward (AF) relay network with P PUs and L SUs as shown in Fig. 1.(a). The related explanation



(a) Multiuser cognitive relay networks

(b) Spectrum sensing networks with an AP

Fig. 1. System model and spectrum sensing networks (Color figure online)

is given in Table 1. The set $\mathbf{L} = \{1, 2, \cdots, L\}$ denotes the number of SUs, and $\mathbf{P} = \{1, 2, \cdots, P\}$ denotes the number of PUs, and $\forall l, j \in \mathbf{L}$, $\forall p \in \mathbf{P}$. Let SU-T and SU-R (PU-T and PU-R) denote secondary (primary) transmitter and receiver, and RS denote relay node, respectively. We assume both SUs and PUs use OFDM modulation mode, in which the total bandwidth is divided into $\mathbf{N} = \{1, 2, \cdots, N\}$ orthogonal subcarriers, and $\forall n \in \mathbf{N}$. This model is a dual-hop relay network in which time-division half duplex relays are used to help communication of SUs. The direct communications from the secondary source nodes to secondary destination nodes are not considered. Under overlay spectrum sharing scenario, multiple source nodes and relays are available to obtain spectrum information in spectrum sensing phase. Relays first assist SUs to detect vacant bands via cooperative spectrum sensing, then an access point (AP) collects local detection results reported by SUs and relays. AP takes fusion criterion and makes a global decision for data transmission as shown in Fig. 1.(b). Let V_p^n and O_p^n represent the licensed spectrum unoccupied and occupied over the subcarrier n by the p^{th} PU, respectively. $\hat{V}_p^n$ and $\hat{O}_p^n$ are used to indicate the status of the licensed spectrum estimated by secondary network.

3 Spectrum Sensing Process

3.1 Cooperative Spectrum Sensing (CSS)

Energy detector (ED) [3] is used by sensing nodes in spectrum sensing phase in order to make a decision about the spectrum occupied or unoccupied by PUs, through comparing the energy of received signal with a detection threshold. We assume that observation time spent by each subcarrier is τ/N, where τ is the observation time window on the whole licensed spectrum. And each sensing node that performs ED in a fixed bandwidth for each subcarrier is f. Therefore,

Table 1. Symbol introduction

Symbol	Specification
P_p^n	Transmit power of the p^{th} PU-T on subcarrier n
$P_{l,1}^n$	Transmit power of the l^{th} SU-T on subcarrier n
$P_{l,2}^n$	Transmit power of the l^{th} relay on subcarrier n
$h_{l,1}^n$	Channel gain of the first-hop of the l^{th} link on subcarrier n
$h_{l,2}^n$	Channel gain of the second-hop of the l^{th} link on subcarrier n
$h_{l,p,1}^n$	Channel gain of the l^{th} SU-T to the p^{th} PU-R on subcarrier n
$h_{l,p,2}^n$	Channel gain of the l^{th} relay to the p^{th} PU-R on subcarrier n
$g_{p,l,1}^n$	Channel gain of the p^{th} PU-T to the l^{th} relay on subcarrier n
$g_{p,l,2}^n$	Channel gain of the p^{th} PU-T to the l^{th} SU-R on subcarrier n
$z_{p,l}^n$	Sensing channel gain of the p^{th} PU-T to the l^{th} SU-T on subcarrier n

the time bandwidth product on each subcarrier is $f\tau/N$ [3]. Let $x_p^n(i)$ be the transmit signal from the p^{th} PU on the subcarrier n, and $\forall i \in \{1, 2, \cdots, 2f\tau/N\}$. The received signal from the p^{th} PU on the subcarrier n at the l^{th} SU-T and relay is given by

$$\begin{cases} y_{p,l,1}^n(i) = \sqrt{\alpha P_p^n} z_{p,l}^n x_p^n(i) + n_{p,l,1}^n(i) \\ y_{p,l,2}^n(i) = \sqrt{\alpha P_p^n} g_{p,l,1}^n x_p^n(i) + n_{p,l,2}^n(i) \end{cases} \tag{1}$$

where $y_{p,l,1}^n(i)$ and $y_{p,l,2}^n(i)$ are the received signal from the p^{th} PU on the subcarrier n at the l^{th} SU-T and the l^{th} relay. P_p^n is the transmit power of the p^{th} PU-T on the subcarrier n. $n_{p,l,1}^n(i)$ and $n_{p,l,2}^n(i)$ are the additive noise on the subcarrier n which are the independent zero-mean white Gaussian noise (AWGN) with power density N_0. α represents the state of the p^{th} PU on the subcarrier n, which is given by

$$\alpha = \begin{cases} 1, & O_p^n \\ 0, & V_p^n \end{cases} \tag{2}$$

When the subcarrier n is unoccupied by the p^{th} PU (i.e., V_p^n), $\alpha = 0$, otherwise $\alpha = 1$. According to energy calculation formula [11], the expressions of the received signal energy from the p^{th} PU on the subcarrier n at the l^{th} SU-T (i.e., $E_{p,l,1}^n$) and the l^{th} relay (i.e., $E_{p,l,2}^n$) are

$$\begin{cases} E_{p,l,1}^n = \sum\limits_{i=1}^{2f\tau/N} \left| y_{p,l,1}^n(i) \right|^2 \\ E_{p,l,2}^n = \sum\limits_{i=1}^{2f\tau/N} \left| y_{p,l,2}^n(i) \right|^2 \end{cases} \tag{3}$$

We assume that channel gains are time-invariant during the sensing phase, and suppose the decision threshold of energy detector as ε at the l^{th} SU-T and the l^{th} relay on the subcarrier n. For $\forall k \in \{1, 2, \cdots, 2L\}$, $a^n_{p,k}$ is a binary number denoting the status of comparative results. The decision criterion is

$$\begin{cases} \hat{O}^n_p, & E^n_{p,l,1} \geq \varepsilon \\ \hat{O}^n_p, & E^n_{p,l,2} \geq \varepsilon \end{cases} \tag{4}$$

$$a^n_{p,k} = \begin{cases} 1, & \hat{O}^n_p \\ 0, & \hat{V}^n_p \end{cases} \tag{5}$$

where $\hat{V}^n_p$ and $\hat{O}^n_p$ denote the sensing result of sensing node on the subcarrier n unoccupied and occupied by the p^{th} PU, respectively.

If $E^n_{p,l,1} > \varepsilon$ and $E^n_{p,l,2} > \varepsilon$, it indicates that the l^{th} SU-T and the l^{th} relay have successfully detected the presence of the p^{th} PU on the subcarrier n that satisfies the hypothesis O^n_p (the result of sensing is $\hat{O}^n_p$). Energy collected in the process of detecting status of the p^{th} PU on the subcarrier n at the sensing node in the frequency domain is denoted by $E^n_{p,k}$ which serves as a decision with the following distribution [11]

$$E^n_{p,k} \sim \begin{cases} \chi^2_{2u} & , \quad V^n_p \\ \chi^2_{2u}\left(2\gamma^n_{p,k}\right) & , \quad O^n_p \end{cases} \tag{6}$$

where u is equal to $f\tau/N$. χ^2_{2u} follows a central chi-square distribution with $2u$ degrees of freedom, and $\chi^2_{2u}\left(2\gamma^n_{p,k}\right)$ follows a non-central chi-square distribution with $2u$ degrees of freedom and a non centrality parameter $2\gamma^n_{p,k}$ [3]. And $\gamma^n_{p,k}$ is the instantaneous signal-noise ratio (SNR) of the received signal from the p^{th} PU at the k^{th} sensing node on the subcarrier n.

In order to insure the generality of the sensing, we take the spectrum sensing uncertainties into consideration so that we can derive the expressions of average detection probability, false-alarm probability, and miss-detection probability. In order to simplify the calculations, we assume that the decision threshold ε is a constant parameter.

$$P^n_{d,p,k} = \mathrm{E}[Pr(E^n_{p,k} > \varepsilon | O^n_p)] = Pr(\chi^2_{2u}(2\gamma^n_{p,k}) > \varepsilon) \tag{7}$$

$$P^n_{fa,p,k} = \mathrm{E}[Pr(E^n_{p,k} > \varepsilon | V^n_p)] = \frac{\Gamma(u, \frac{\varepsilon}{2})}{\Gamma(u)} \tag{8}$$

$$P^n_{md,p,k} = 1 - P^n_{d,p,k} \tag{9}$$

where $\mathrm{E}[\cdot]$ denotes the expectation and $Pr(\cdot)$ is the probability. $\Gamma(m, \tilde{x})$ is the incomplete gamma function given by $\Gamma(m, \tilde{x}) = \int_{\tilde{x}}^{\infty} v^{m-1} e^{-v} dv$, and $\Gamma(m)$ is the gamma function. $P^n_{d,p,k}$ and $P^n_{fa,p,k}$ denote the detection probability and the false-alarm probability. And $P^n_{md,p,k}$ denotes the probability of the miss-detection.

In the next sub-phase, sensing nodes report detection results to AP, which makes the global decision follow the OR fusion rule [3]

$$S_p^n = \sum_{k=1}^{2L} a_{p,k}^n = \begin{cases} \geq 1, & \hat{O}_p^n \\ 0 \quad , & \hat{V}_p^n \end{cases} \tag{10}$$

The decision at the k^{th} sensing node is reported to AP and expressed by $a_{p,k}^n \in \{0,1\}$ for binary phase shift keying (BPSK) modulation. S_p^n denotes a parameter that clearly identifies the state of the subcarrier n (unoccupied or occupied by the p^{th} PU). We assume that the distance between any two sensing nodes (i.e., SUs and relays) is much smaller than the distance from any sensing nodes to the primary transmitters, so that the received signal at every sensing node experiences almost identical path loss. Therefore, we can assume that we have independent and identically distributed (i.i.d.) Rayleigh fading with the instantaneous SNRs of the received signal from PUs at sensing nodes on the subcarrier n. Based on the above, we can take false-alarm probabilities $P_{fa,p,k}^n$ as identical since $P_{fa,p,k}^n$ is independent of k, and the global decision of false-alarm probability can be denoted by P_{fa}^n (i.e., $Pr(\hat{O}_p^n|V_p^n)$). In the case of the AWGN channel, the detection probabilities at the sensing nodes are independent of k, so that the detection probabilities are identical and the global decision is expressed by P_d^n (i.e., $Pr(\hat{O}_p^n|O_p^n)$). Similarly, taking the global decision of the missing-detection probability as P_{md}^n (i.e., $Pr(\hat{V}_p^n|O_p^n)$).

$$P_{fa}^n = 1 - \prod_{k=1}^{2L}(1 - P_{fa,p,k}^n) \approx 1 - (1 - P_{fa,p,k}^n)^{2L} \tag{11}$$

$$P_{md}^n = \prod_{k=1}^{2L} P_{md,p,k}^n \tag{12}$$

$$P_d^n = 1 - P_{md}^n \tag{13}$$

Considering the error probability (i.e., P_e^n) of the reporting channel on the subcarrier n, we change the expression of the miss-detection probability as

$$P_{md}^n = \prod_{k=1}^{2L} [P_{md,p,k}^n(1 - P_e^n) + (1 - P_{md,p,k}^n)P_e^n] \tag{14}$$

3.2 SINR Expression (AF Protocol)

A dual-hop communication link is considered. The first hop instantaneous SINR on the subcarrier n is denoted by $SINR_{l,1}^n$, the second hop is $SINR_{l,2}^n$. For AF protocol [12], the expression of equivalent SINR of SU link is

$$\begin{aligned} SINR_{l,eq}^n &= T(SINR_{l,1}^n, SINR_{l,2}^n) \\ &= \tfrac{SINR_{l,1}^n SINR_{l,2}^n}{SINR_{l,1}^n + SINR_{l,2}^n + 1} \end{aligned} \tag{15}$$

where

$$T(x, y) = \frac{xy}{x + y + 1} \tag{16}$$

where $x = SINR_{l,1}^n$, $y = SINR_{l,2}^n$.

3.3 Interference Constraint

In order to guarantee the QoS of PUs, the transmit power of SUs and relays should be probably controlled. Since there is half-duplex scheme at relay nodes, the interference to the p^{th} PU in each hop can be written as

$$I_{SP_p} = \sum_{l=1}^{L} \sum_{n=1}^{N} Pr\left(O_p^n\right) P_{md}^n P_{l,1}^n |h_{l,p,1}^n|^2 \tag{17}$$

$$I_{RP_p} = \sum_{l=1}^{L} \sum_{n=1}^{N} Pr\left(O_p^n\right) P_{md}^n P_{l,2}^n |h_{l,p,2}^n|^2 \tag{18}$$

where $Pr(O_p^n)$ is a probability that the subcarrier n is occupied by the p^{th} PU. I_{SP_p} and I_{RP_p} are the interference produced by all SU-Ts and all relay transmitters, which must be limited by the interference temperature (IT) constraints.

4 Proposed Algorithm

The BER expressions at SU-R for multiple quadrature amplitude modulation (MQAM) (19) or multiple phase shift keying (MPSK) modulation (20) [13] over the AWGN channel are written as

$$BER_{l,MQAM}^n = \frac{4}{b}\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3b(SINR_{l,eq}^n/b)}{M-1}}\right) \tag{19}$$

$$BER_{l,MPSK}^n = \frac{2}{b} Q\left(\sqrt{2b \times (SINR_{l,eq}^n/b)\sin^2(\frac{\pi}{M})}\right) \tag{20}$$

where $Q\left(\bar{x}\right) = \frac{1}{\sqrt{2\pi}} \int_{\bar{x}}^{\infty} e^{-\frac{w^2}{2}} dw$ is a Gaussian Q-function. $b = \log_2 M$, M is the number of bits of the modulation symbols.

In this paper, a worst-channel-state-information (worst-CSI) PC algorithm is presented to limit total BER of SUs, which only needs to operate algorithm in one link that CSI is worst, while keeping the interference leakage to PUs below the IT level, and the maximum transmit power of SU and the relay below certain thresholds. Here we introduce the SINRs at SU-R and relay in order to guarantee the requirement for each hop. Thus, the optimization problem is formulated as **OP1**

$$\begin{aligned}
\mathbf{OP1} \quad & \min_{P_{l,1}^n, P_{l,2}^n} \quad \max_{\forall l} \quad BER_l^n \\
s.t. \quad & C1: 0 \leq \sum_{n=1}^{N} P_{l,1}^n \leq P_{l,1}^{max}, \quad \forall l \\
& C2: 0 \leq \sum_{n=1}^{N} P_{l,2}^n \leq P_{l,2}^{max}, \quad \forall l \\
& C3: SINR_{l,1}^n \geq SINR_{l,1,th}^n, \quad \forall l, \forall n \\
& C4: SINR_{l,2}^n \geq SINR_{l,2,th}^n, \quad \forall l, \forall n \\
& C5: \sum_{l=1}^{L} \sum_{n=1}^{N} Pr(O_p^n) P_{md}^n P_{l,1}^n |h_{l,p,1}^n|^2 \leq I_{p,th}, \quad \forall p \\
& C6: \sum_{l=1}^{L} \sum_{n=1}^{N} Pr(O_p^n) P_{md}^n P_{l,2}^n |h_{l,p,2}^n|^2 \leq I_{p,th}, \quad \forall p
\end{aligned} \tag{21}$$

where $P_{l,1}^{max}$ and $P_{l,2}^{max}$ are the maximum power budgets of SU-T and relay. $SINR_{l,1,th}^n$ and $SINR_{l,2,th}^n$ are the SINR thresholds at the relay and SU-R. $I_{p,th}$ is the interference threshold prescribed by the p^{th} PU receiver. C1 and C2 represent the transmit power constraints of the secondary system. C3 and C4 are the SINR constraints to keep basic communication requirements of SUs. C5 and C6 denote the IT constraints at the source and relay nodes. Since the objection of **OP1** is a monotonic function about the equivalent SINR (i.e., $SINR_{l,eq}^n$), so **OP1** can be converted into

$$\begin{aligned}
\mathbf{OP2} \quad & \max_{P_{l,1}^n, P_{l,2}^n} \quad \min_{\forall l} \; SINR_{l,eq}^n \\
s.t. \quad & C1 \sim C6
\end{aligned} \tag{22}$$

therefore, the original optimization problem (i.e., **OP1**) becomes a worst-CSI SINR maximization problem (i.e., **OP2**). The criterion about selecting the worst-CSI user is given by

$$\left|h_{l,1}^n\right|^2 \left|h_{l,2}^n\right|^2 \leq \left|h_{j,1}^n\right|^2 \left|h_{j,2}^n\right|^2 \tag{23}$$

If the channel gain of two hops can satisfy (23), we regard the l^{th} SU as the worst-CSI user. **OP2** is not convex due to the constraints C3 and C4. In order to simplify theoretical analysis, we take C3 and C4 on reciprocal, such as

$$C3: \frac{1}{SINR_{l,1}^n} \leq \frac{1}{SINR_{l,1,th}^n} \tag{24}$$

$$C4: \frac{1}{SINR_{l,2}^n} \leq \frac{1}{SINR_{l,2,th}^n} \tag{25}$$

i.e.,

$$\frac{\frac{N_{l,1}^n}{|h_{l,1}^n|^2} + \sum_{p=1}^{P} P_p^n \frac{|g_{p,l,1}^n|^2}{|h_{l,1}^n|^2}}{P_{l,1}^n} \leq \frac{1}{SINR_{l,1,th}^n} \tag{26}$$

$$\frac{\frac{N_{l,2}^n}{|h_{l,2}^n|^2} + \sum_{p=1}^{P} P_p^n \frac{|g_{p,l,2}^n|^2}{|h_{l,2}^n|^2}}{P_{l,2}^n} \leq \frac{1}{SINR_{l,2,th}^n} \tag{27}$$

where $N_{l,1}^n$ and $N_{l,1}^n$ denote the additive noise power at the l^{th} relay and SU-R.

Define

$$\begin{cases} F_{l,1}^n = \frac{N_{l,1}^n}{|h_{l,1}^n|^2} \\ F_{l,2}^n = \frac{N_{l,2}^n}{|h_{l,2}^n|^2} \end{cases} \tag{28}$$

$$\begin{cases} G_{p,l,1}^n = \frac{|g_{p,l,1}^n|^2}{|h_{l,1}^n|^2} \\ G_{p,l,2}^n = \frac{|g_{p,l,2}^n|^2}{|h_{l,2}^n|^2} \end{cases} \tag{29}$$

Then the equivalent SINR is

$$SINR_{l,eq}^n = \frac{a_l^n P_{l,1}^n b_l^n P_{l,2}^n}{a_l^n P_{l,1}^n + b_l^n P_{l,2}^n + 1} \tag{30}$$

where a_l^n and b_l^n are given by

$$\begin{cases} a_l^n = \frac{1}{F_{l,1}^n + \sum\limits_{p=1}^{P} P_p^n G_{p,l,1}^n} \\ b_l^n = \frac{1}{F_{l,2}^n + \sum\limits_{p=1}^{P} P_p^n G_{p,l,2}^n} \end{cases} \tag{31}$$

Further more, to make the equivalent SINR tractable, we adopt the following approximation [14]

$$SINR_{l,eq}^n \approx \frac{a_l^n P_{l,1}^n b_l^n P_{l,2}^n}{a_l^n P_{l,1}^n + b_l^n P_{l,2}^n} \tag{32}$$

Define $P_{l,1}^n = x_1$, $P_{l,2}^n = x_2$, $t = \frac{1}{SINR_{l,eq}^n}$, then

$$t = \frac{1}{SINR_{l,eq}^n} = \frac{1}{b_l^n}\frac{1}{x_2} + \frac{1}{a_l^n}\frac{1}{x_1} \tag{33}$$

Therefore, **OP2** can be rewritten as

$$\begin{aligned} &\mathbf{OP3} \quad \min_{x_1,x_2} \quad \max_{\forall l} \quad t \\ &s.t. \quad C1: 0 \le \sum_{n=1}^{N} x_1 \le P_{l,1}^{max}, \quad \forall l \\ &\qquad C2: 0 \le \sum_{n=1}^{N} x_2 \le P_{l,2}^{max}, \quad \forall l \\ &\qquad C3: \tfrac{1}{a_l^n}\tfrac{1}{x_1} \le \tfrac{1}{SINR_{l,1,th}^n}, \quad \forall l, \forall n \\ &\qquad C4: \tfrac{1}{b_l^n}\tfrac{1}{x_2} \le \tfrac{1}{SINR_{l,2,th}^n}, \quad \forall l, \forall n \\ &\qquad C5: \sum_{l=1}^{L}\sum_{n=1}^{N} Pr(O_p^n) P_{md}^n x_1 |h_{l,p,1}^n|^2 \le I_{p,th}, \quad \forall p \\ &\qquad C6: \sum_{l=1}^{L}\sum_{n=1}^{N} Pr(O_p^n) P_{md}^n x_2 |h_{l,p,2}^n|^2 \le I_{p,th}, \quad \forall p \end{aligned} \tag{34}$$

Now **OP3** is a convex problem which can be solved by the dual decomposition method [15].

First, we give a Lagrange function with the Lagrange multipliers $\lambda_{l,1}$,$\lambda_{l,2}$,$\lambda_{l,3}^n$,$\lambda_{l,4}^n$,$\lambda_{p,5}$,$\lambda_{p,6} \geq 0$ as follows

$$\begin{aligned}
&L(t, \{\lambda_{l,1}\}, \{\lambda_{l,2}\}, \{\lambda_{l,3}^n\}, \{\lambda_{l,4}^n\}, \{\lambda_{p,5}\}, \{\lambda_{p,6}\}) \\
&= t + \sum_{l=1}^{L} (\lambda_{l,1}(\sum_{n=1}^{N} x_1 - P_{l,1}^{max})) \\
&\quad + \sum_{l=1}^{L} (\lambda_{l,2}(\sum_{n=1}^{N} x_2 - P_{l,2}^{max})) \\
&\quad + \sum_{l=1}^{L} (\sum_{n=1}^{N} \lambda_{l,3}^n(\tfrac{1}{a_l^n}\tfrac{1}{x_1} - \tfrac{1}{SINR_{l,1,th}^n})) \\
&\quad + \sum_{l=1}^{L} (\sum_{n=1}^{N} \lambda_{l,4}^n(\tfrac{1}{b_l^n}\tfrac{1}{x_2} - \tfrac{1}{SINR_{l,2,th}^n})) \\
&\quad + \sum_{p=1}^{P} (\lambda_{p,5}(\sum_{l=1}^{L}\sum_{n=1}^{N} Pr(O_p^n)P_{md}^n x_1 |h_{l,p,1}^n|^2 - I_{p,th})) \\
&\quad + \sum_{p=1}^{P} (\lambda_{p,6}(\sum_{l=1}^{L}\sum_{n=1}^{N} Pr(O_p^n)P_{md}^n x_2 |h_{l,p,2}^n|^2 - I_{p,th}))
\end{aligned} \tag{35}$$

The dual problem of the Lagrange function (35) is

$$\begin{aligned}
&D(t, \{\lambda_{l,1}\}, \{\lambda_{l,2}\}, \{\lambda_{l,3}^n\}, \{\lambda_{l,4}^n\}, \{\lambda_{p,5}\}, \{\lambda_{p,6}\}) \\
&= \sum_{l=1}^{L} (\sum_{n=1}^{N} \min_{x_1,x_2} \; L_l^n(t, \lambda_{l,1}, \lambda_{l,2}, \lambda_{l,3}^n, \lambda_{l,4}^n, \{\lambda_{p,5}\}, \{\lambda_{p,6}\})) \\
&\quad - \sum_{l=1}^{L} (\lambda_{l,1}P_{l,1}^{max}) - \sum_{l=1}^{L} (\lambda_{l,2}P_{l,2}^{max}) \\
&\quad - \sum_{l=1}^{L} (\sum_{n=1}^{N} \lambda_{l,3}^n \tfrac{1}{SINR_{l,1,th}^n}) - \sum_{l=1}^{L} (\sum_{n=1}^{N} \lambda_{l,4}^n \tfrac{1}{SINR_{l,2,th}^n}) \\
&\quad - \sum_{p=1}^{P} (\lambda_{p,5}I_{p,th}) - \sum_{p=1}^{P} (\lambda_{p,6}I_{p,th})
\end{aligned} \tag{36}$$

Define L_l^n as a function of x_1 and x_2

$$\begin{aligned}
&L_l^n(t, \lambda_{l,1}, \lambda_{l,2}, \lambda_{l,3}^n, \lambda_{l,4}^n, \{\lambda_{p,5}\}, \{\lambda_{p,6}\}) \\
&= t + \lambda_{l,1}x_1 + \lambda_{l,2}x_2 + \lambda_{l,3}^n \tfrac{1}{a_l^n}\tfrac{1}{x_1} + \lambda_{l,4}^n \tfrac{1}{b_l^n}\tfrac{1}{x_2} \\
&\quad + x_1 \sum_{p=1}^{P} \lambda_{p,5} Pr(O_p^n) P_{md}^n |h_{l,p,1}^n|^2 \\
&\quad + x_2 \sum_{p=1}^{P} \lambda_{p,6} Pr(O_p^n) P_{md}^n |h_{l,p,2}^n|^2
\end{aligned} \tag{37}$$

Since the primal problem in (34) is convex, strong duality holds, the dual problems can be solved by an iterative manner using the gradient projection method [15]. The Lagrange multipliers in (35) can be updated by the sub-gradient method [15].

By the Karush-Kuhn-Tucker (KKT) conditions, the optimal transmit power $P_{l,1}^n$ and $P_{l,2}^n$ at SU-T and relay can be calculated by $\frac{\partial L_l^n}{\partial x_1}=0$ and $\frac{\partial L_l^n}{\partial x_2}=0$, such as

$$P_{l,1}^{n}{}^* = x_1^* = \sqrt{\frac{\frac{1}{a_l^n}(1+\lambda_{l,3}^n)}{\lambda_{l,1}+\sum_{p=1}^{P}\lambda_{p,5}Pr(O_p^n)P_{md}^n|h_{l,p,1}^n|^2}} \tag{38}$$

$$P_{l,2}^{n}{}^* = x_2^* = \sqrt{\frac{\frac{1}{b_l^n}(1+\lambda_{l,4}^n)}{\lambda_{l,2}+\sum_{p=1}^{P}\lambda_{p,6}Pr(O_p^n)P_{md}^n|h_{l,p,2}^n|^2}} \tag{39}$$

Finally, taking the optimal solutions $P_{l,1}^{n}{}^*$ and $P_{l,2}^{n}{}^*$ into (19) and (20) respectively, the optimal BER can be calculated.

The computational complexity can be roughly analyzed as follows. The optimal solutions of power allocation in an OFDMA network requires exhaustive search to find an optimal subcarrier allocation scheme for SUs. Since the number of subcarriers is N, the computational complexity at subcarrier allocation phase is $\mathcal{O}(N)$. Since there are P pairs PUs, the computational complexity of outer loop requires a complexity of $\mathcal{O}(P)$. In order to calculate the Lagrange multipliers $\lambda_{p,5}$ and $\lambda_{p,6}$, we should evaluate whether the interference power at PU receiver is below the interference threshold, i.e., $\sum_{l=1}^{L}\sum_{n=1}^{N}Pr(O_p^n)P_{md}^n x_1|h_{l,p,1}^n|^2 \le I_{p,th}$ and $\sum_{l=1}^{L}\sum_{n=1}^{N}Pr(O_p^n)P_{md}^n x_2|h_{l,p,2}^n|^2 \le I_{p,th}$, which introduce $\mathcal{O}(I\cdot D)$, where $\mathcal{O}(D)$ is complexity of finding $P_{l,1}^{n}{}^*$ and $P_{l,2}^{n}{}^*$ under the conditions of convergence respectively. Therefore, the total computational complexity of the proposed algorithm is the sum of complexities of the aforementioned steps as $\mathcal{O}(N)\mathcal{O}(P)\mathcal{O}(I\cdot D)$=$\mathcal{O}(NPID)$, where I is the number of iterations in algorithm.

5 Numerical Results

In this section, we present numerical results to show the effectiveness of the proposed algorithm. We assume that there are four SUs and relays (i.e., L= 4), one PU (i.e., P= 1), and four subcarriers (i.e., N= 4), and each SU occupies one subcarrier. Similar to [16], the normal values of the interference channel gains $h_{l,p,1}^n$, $h_{l,p,2}^n$, $g_{p,l,1}^n$ and $g_{p,l,2}^n$ are selected from the interval (0,0.3) respectively. The normal values of the channel gains $h_{l,1}^n$ and $h_{l,2}^n$ are randomly chosen from the interval (0,1) respectively. We set the target SINR on each subcarrier at SU-R and relay is $SINR_{l,1,th}^n/SINR_{l,2,th}^n$= 3 dB. The maximum transmit power of each SU-T and relay is $P_{l,1}^{max}/P_{l,2}^{max}$= 1.5 mW. We also assume that $Pr(O_p^n)$ is same for every subcarrier, e.g., $Pr(O_p^n)$=0.1. The background noise power on each subcarrier is assumed to be identical and equal to 0.01 mW, i.e.,

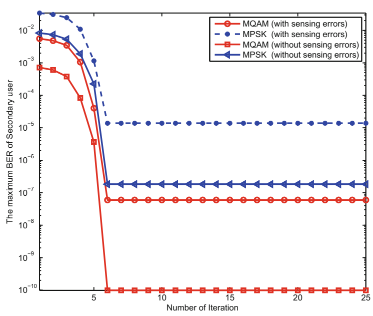


Fig. 2. Convergence of maximum BER under $P_{md}^{n} = 0.1$ and $Pr(O_p^n) = 0.1$.

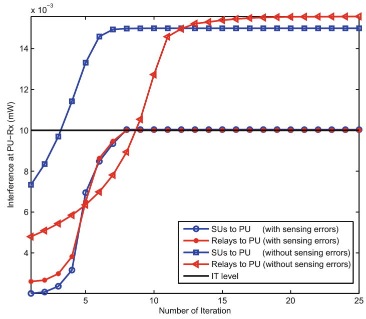


Fig. 3. Convergence of interference under $P_{md}^{n} = 0.1$ and $Pr(O_p^n) = 0.1$.

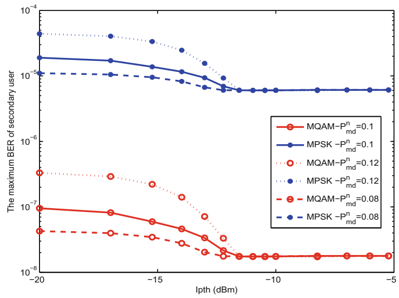


Fig. 4. Maximum BER under different $I_{p,th}$ and P_{md}^{n} under $Pr(O_p^n) = 0.1$.

Table 2. Maximum BER at SU-R for different P_{md}^{n}

Modulation form	$P_{md}^{n} = 0.08$	$P_{md}^{n} = 0.10$	$P_{md}^{n} = 0.12$
BPSK	1.102e-5	1.898e-5	4.412e-5
QPSK	9.401e-4	1.247e-3	1.936e-3
16PSK	9.777e-2	1.010e-1	1.064e-1
2QAM	4.264e-8	9.523e-8	3.309e-7
4QAM	9.401e-4	1.247e-3	1.936e-3
16QAM	6.618e-2	6.610e-2	7.367e-2

$N_{l,1}^{n}$=$N_{l,2}^{n}$=0.01 mW [7]. The simulation results are presented from Figs. 2, 3 and 4 and Table 2.

Figure 2 shows that the maximum BER performance of selected SU link. The BER of the proposed algorithm for the given IT level $I_{p,th} = 0.01$ mW is higher than that of the PC algorithm without sensing errors, while providing the protection of PU when SUs share spectrum opportunistically. From Fig. 2, we can see that the maximum BER under the proposed algorithm for both MPSK and MQAM modulation quickly converges to the stable point, and the optimization goal is achieved by minimizing the maximum BER of the worst-CSI channel to limit the total BER of SUs. Briefly, the purpose of minimizing the BER of the system is obtained by adjusting transmit power of SU-T and relay, which improves the performance and insure the QoS of SUs.

From Fig. 3, we find that our PC algorithm under the imperfect spectrum sensing can guarantee the interference power is always below the IT level, whereas the PC algorithm without the sensing errors results in the actual received interference power exceeds the allowable region. Combining Fig. 2 with Fig. 3, we get a conclusion that the proposed algorithm can well provide the protection for PU at the cost of its BER increases.

In Fig. 4, we depict the maximum BER versus the IT level from $I_{p,th}$=−20 dBm to $I_{p,th}$=−5 dBm of our proposed algorithm for different P_{md}^n. Figure 4 shows that the maximum BER performance against $I_{p,th}$ and P_{md}^n for MPSK and MQAM (M =2) modulation. For a given P_{md}^n, for example, P_{md}^n=0.1, the maximum BER of SUs first decreases as the increasing interference power constraint and then keep flat because of the maximum transmit power constraint. What's more, we find that the BER performance under different P_{md}^n of our proposed algorithm is same when $I_{p,th}$ is large, for example, larger than −12 dBm, and the BER performance for P_{md}^n=0.08 is the best of three when $I_{p,th}$ is low. In fact, from another perspective, the interference power constraint stands for the distance, with the increasing distance between SU and PU, more transmit power is allocated to achieve lower BER.

Table 2 shows that the maximum BER versus different P_{md}^n, for the given $I_{p,th}$, for example, $I_{p,th}$=0.01 mW, and the transmission data for MPSK and MQAM (M =2, 4 and 16) modulation. From Table 2, we know that the spectrum sensing requirement is improved from P_{md}^n=0.12 to P_{md}^n=0.08 for the given modulation methods (i.e., MPSK and MQAM), and the maximum BER of the system decreases accordingly. The reason is that, with an improved spectrum sensing requirement, a spectrum hole would be detected more accurately thus less interference occurs between the primary network and the secondary network, resulting in decreased BER for the secondary transmission. Furthermore, we also find that the maximum BER of the system increases with the increase of the number of bits of the modulation symbols.

6 Conclusion

In this paper, we have investigated the issues on BER problem under the imperfect spectrum sensing in cognitive relay networks. A PC algorithm under maximum transmit power constraints, SINR constraints and interference constraints to guarantee the QoS of PUs is proposed to minimize total BER for all SUs. We find that the maximum BER of the secondary system decreases as the decreasing miss-detection probability. Besides, the proposed algorithm can well protect the communication of PU though there is a little BER increase of the secondary system at the price. In our future research, the PC optimization problem with the introduction of more complicated channels in the underlay cognitive relay networks will be conducted.

Acknowledgment. The work of this paper is supported by National Natural Science Foundation of China under grant No. 61571209.

References

1. Haykin, S.: Cognitive radio: brain-empowered wireless communications. IEEE J. Sel. Areas Commun. **23**(2), 201–220 (2005)
2. Xu, Y.J., Zhao, X.H., Liang, Y.-C.: Robust power control and beamforming in cognitive radio networks: A survey. IEEE Commun. Surveys Tuts. **17**(4), 1834–1857 (2015)
3. Letaief, K.B., Zhang, W.: Cooperative communications for cognitive radio networks. Proc. IEEE **97**(5), 878–893 (2009)
4. Cover, T.M., Gamal, A.E.: Capacity theorems for the relay channel. IEEE Trans. Inf. Theory **25**(5), 572–584 (1979)
5. El, G.A., Cover, T.M.: Multiple user information theory. Proc. IEEE **68**(12), 1466–1483 (1980)
6. Rasti, M., Hasan, M., Le, L.B., Hossain, E.: Distributed uplink powercontrol for multi-cell cognitive radio networks. IEEETrans. Commun. **63**(3), 628–642 (2015)
7. Xu, Y.J., Zhao, X.H.: Robust power control for underlay cognitive radio networks under probabilistic quality of service and interference constraints. IET Commun. **8**(18), 3333–3340 (2014)
8. Wu, Y., Tsang, D.H.: Joint bandwidth and power allocations for cognitive radio networks with imperfect spectrum sensing. Wireless Per. Commun. **57**(1), 19–31 (2011)
9. Wang, S., Zhou, Z.H., Ge, M., Wang, C.: Resource allocation for heterogeneous multiuser ofdm-based cognitive radio networks with imperfect spectrum sensing. In: IEEE INFOCOM, pp. 2264–2272 (2012)
10. Tan, X., Zhang, H., Hu, J.: Capacity maximisation of the secondary link in cognitive radio networks with hybrid spectrum access strategy. IET Commun. **8**(5), 689–696 (2014)
11. Digham, F.F., Alouini, M.-S., Simon, M.K.: On the energy detection of unknown signals over fading channels. IEEE Trans. Commun. **55**(1), 21–24 (2007)
12. Liu, Z., Yuan, H., Li, H., Guan, X., Yang, H.: Robust power control for amplify-and-forward relaying scheme. IEEE Commun. Lett. **19**(2), 263–266 (2015)
13. Xu, X., Bao, J., Cao, H., Yao, Y., Hu, S.: Energy efficiency based optimalrelay selection scheme with a ber constraint in cooperative cognitive radionetworks. IEEE Trans. Veh. Technol., January 2015. doi:10.1109/TVT.2015.2389810
14. Ge, M., Wang, S.: Energy-efficient power allocation for cooperative relaying cognitive radio networks. In: IEEE Wireless Communications and Networking Conference (WCNC), pp. 691–696, April 2013
15. Boyd, S., Vandenberghe, L.: Convex Optimization. Cambridge Univ. Press, Mar, Cambridge (2004)
16. Setoodeh, P., Haykin, S.: Robust transmit power for cognitive radio. Proc. IEEE **97**(5), 915–939 (2009)