Efficient Power Allocation Approach for Asynchronous Cognitive Radio Networks with FBMC/OFDM

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Abstract. In this paper, we address the problem of power minimization under rate constraint for a multi-carrier-based underlay cognitive radio (CR) network. In fact, both primary users (PUs) and secondary users (SUs) employ either orthogonal frequency-division multiplexing (OFDM) or filter bank based multi-carrier (FBMC). The problem is formulated as a non-cooperative interference pricing-based game (NPBG) in order to circumvent the coupling primary interference constraint and also to propose distributed solutions. We provide a sufficient convergence condition to a Nash-equilibrium (NE) point for the modified water-filling algorithm. Moreover, we propose a distributed algorithm that always converges to a unique NE of the NPBG. Simulation analyses are then provided to demonstrate the efficiency of our proposed distributed algorithms. Furthermore, the simulations enhance the advantages of using FBMC as a modulation technique if compared to OFDM.

Keywords: FBMC · OFDM · Cognitive radio · Game theory · Resource allocation

1 Introduction

New paradigms that can enable efficient spectrum utilization emerge to anticipate shortages of radio spectrum in wireless networks that face increasing number of demand from users. Cognitive radio (CR) is proposed as an appealing technology capable of not only improving the spectrum utilization, it can also enhance efficiency of spectrum sharing in wireless networks. This can be done by dynamically allocate radio spectrum by permitting unlicensed users; the secondary users (SUs) to access the frequency band of the primary users (PUs).

There exist two paradigms for the operation of CR technology: opportunistic and concurrent spectrum access [1–3]. In an opportunistic spectrum access scenario, SUs are permitted to communicate only when the PUs are detected to be inactive. On the contrary, SUs are allowed to transmit simultaneously with the PUs in a concurrent spectrum access scenario, provided the quality of service (QoS) of the PUs is not degraded by the activity of the SUs. In this paper, we mainly focus on the second paradigm.

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Multi-carrier modulation techniques such as the orthogonal frequency division multiplexing (OFDM) are eligible for the physical layer of CR networks [4]. For CR networks that experience asynchronous transmission due to lack of cooperation among the users, OFDM may sacrifice data rate transmission because of imperfect time and frequency synchronization. Its high spectral efficiency makes filter bank multi-carrier (FBMC) an alternative to conventional OFDM for transmission over CR networks.

For asynchronous multi-carrier-based CR networks, judicious resource allocation is required to mitigate the effect of inter-carrier interference. The problem of resource allocation for asynchronous underlay CR networks employing FBMC and OFDM was greatly studied over the past decade [5–7]. In [5], the authors addressed the downlink resource allocation for a CR network consisting of a single PU and a single SU. The same scenario as [5] was investigated in [6], yet by considering the uplink case. In [7], Shaat et al. proposed a modified water-filling solution to the problem of downlink rate maximization multi-cell CR network.

In this paper, we investigate the problem of sum power minimization subject to rate constraint for downlink asynchronous underlay CR networks with FBMC and OFDM. To the best of our knowledge, no other research group has addressed this issue. Motivated by [8,9], we formulate the problem as a non-cooperative interference pricing-based game (NPBG). In order to keep a distributed resource allocation, the couple primary interference constraint is embedded into the utility function by mean of pricing. The optimal power allocation strategy for each secondary BS is given by the modified water-filling. We provide a distributed sufficient convergence criterion for the water-filling algorithm. Furthermore, we theoretically demonstrate that our proposed NPBG converges to a Nash-equilibrium (NE) point whenever the convergence criterion is met. In addition to that, we proposed a new distributed algorithm (NDA) to solve the NPBG game. The NDA always converges to a unique NE point. The efficiency of the proposed methods is validated through extensive simulations results.

The rest of this paper is organized as follows: in Sect. 2, we describe the system model together with the problem formulation. In Sect. 3, we introduce a distributed convergence criterion for the water-filling. The new distributed approach that converges to a unique NE point is provided in Sect. 4. Numerical results highlighting some important features of our proposed schemes are given in Sect. 5. Finally, the work is concluded in Sect. 6.

2 System Model and Problem Formulation

Consider a multi-carrier based underlay spectrum sharing CR network that consists of $\mathcal{K}$ active primary users and $\mathcal{S}$ active secondary users. Each active PU and SU is formed by a single transmitter-receiver pair. The total spectrum is divided into L subcarriers. Each subcarrier has a bandwidth B. We consider a downlink transmission where all mobile terminals (MT) and BSs are equipped each with a single antenna. In this configuration, the PUs do not interfere with each other and have a fixed transmission power scheme regardless of the transmission strategy used by the SUs.

The systems that coexist in the network do not cooperate with each other. Moreover, there exists no synchronization neither between any two secondary BSs nor between a secondary BS and a primary BS. Lack of cooperation coupled with asynchronism will create inter-carrier interference which may have detrimental effect on the overall performance of the network. The study of inter-carrier interference was done in [10] where Medjahdi et al. quantified the number of subcarriers affected by interference generated from a given subcarrier. It was demonstrated in [10] that 17 and 3 neighbouring subcarriers suffered from inter-carrier interference in the case of OFDM and FBMC, respectively. The interference weight vector that was derived in [10] can be summarized as

$$\begin{aligned} V^{\mathrm{OFDM}} &= \left[\{705, 89.4, 22.3, 9.95, 5.6, 3.59, 2.5, 1.84, 1.12\} \times 10^{-3}\right] \\ V^{\mathrm{FBMC}} &= \left[8.23 \times 10^{-1}, 8.81 \times 10^{-2}\right] \end{aligned} \tag{1}$$

The interference coefficients will be used throughout this work. In subsequent sections, the interference weight vector is denoted as $V = [V_0, \ldots, V_S]$ where $S = 1$ in the case of FBMC and $S = 8$ for OFDM.

Due to the distributed nature of CR network, all secondary MTs use single user detection i.e., interference caused by other SUs and the PUs are treated as noise. We assume that channel gains which include path loss and shadowing change sufficiently slowly to be considered unchanged during each scheduling interval. Perfect knowledge of channel state information (CSI) is available at each BS. The CSI between secondary BS and primary MT can be periodically measured by a band manager [11]. Also, the MTs can estimate the CSI and feed it back to their respective serving BS.

Denote P_s^l the power that the sth secondary BS allocates on the lth subcarrier. Let $\mathbf{P}_s \triangleq \left(P_s^1, \cdots, P_s^L\right)^\top$ be the power allocation vector of the sth secondary BS and $\mathbf{P}_{-s} \triangleq \{\mathbf{P}_j\}_{j \in \{1, \cdots, s-1, s+1, \cdots, \mathcal{S}\}}$ the set of transmit power of all other secondary BSs. $\mathbf{P} = (\mathbf{P}_1, \cdots, \mathbf{P}_{\mathcal{S}})^\top$ denote all secondary BSs power vector. $G_{s,s}^l$ is the channel gain between secondary BS s and its served MT on subcarrier l. $G_{s,j}^l$ denote the channel gain between BS of SU s and MT of SU j on subcarrier l while $G_{s,k}^l$ is the channel gain between BS of SU s and receiver of the PU k within the lth subcarrier. The achievable data rate of the secondary MT s is given by

$$\mathcal{R}_s(\mathbf{P}_s, \mathbf{P}_{-s}) = \sum_{l=1}^{L} B \log_2 \left(1 + \frac{P_s^l G_{s,s}^l}{\overline{N}_s^l + I_s^l}\right) \tag{2}$$

where

$$I_s^l = \sum_{j \neq s}^{\mathcal{S}} \sum_{l'=1}^{L} P_j^{l'} V_{|l-l'|} G_{j,s}^{l'}, \text{ and } \overline{N}_s^l = N_0 + \sum_{k=1}^{\mathcal{K}} \sum_{l'=1}^{L} P_k^{l'} V_{|l-l'|} G_{k,s}^{l'}$$

N_0 denotes the thermal noise on a subcarrier and $G_{k,s}^l$, the channel gain between the kth primary BS and the mobile terminal of SU s.

For underlay CR networks, secondary users can simultaneously with the PUs communicate on the same frequency band provided that the degradation induced on the QoS of the primary users is tolerable. This is captured by preventing the per subcarrier total interference caused by SUs activity to the kth PU from exceeding a predefined threshold.

$$\sum_{s=1}^{\mathcal{S}} \sum_{l'=1}^{L} P_s^{l'} G_{s,k}^{l'} V_{|l-l'|} \leq I_k^{l,\max}, \forall k, \forall l \tag{3}$$

The main bottleneck of (3) is that the interference constraint does include the power of all secondary users. Circumvent this coupling constraint is of great important since we ought to provide distributed solutions.

In this work, we formulate the transmission strategy of the SUs as a noncooperative game. Let $\mathcal{P}_s$ be the feasible set of the transmission strategy of secondary BS s.

$$\mathcal{P}_s(\mathbf{P}_{-s}) \triangleq \left\{\mathbf{P}_s : \mathcal{R}_s(\mathbf{P}_s, \mathbf{P}_{-s}) \geq \widehat{\mathcal{R}}_s,\ P_s^l \geq 0, \forall l\right\}$$

where $\widehat{\mathcal{R}}_s$ is the rate constraint of the SU s. To deal with the coupling PUs interference constraint that is not incorporated in the feasible set, we introduce a pricing in the secondary utility function which is given by

$$\mathcal{U}(\mathbf{P}_s;\boldsymbol{\mu}) \triangleq \sum_{l=1}^{L} P_k^l + \sum_{k=1}^{\mathcal{K}}\sum_{l=1}^{L} \mu_k^l \sum_{l'=1}^{L} P_s^{l'} G_{s,k}^{l'} V_{|l-l'|} \tag{4}$$

by rearranging the terms on the left hand side, we have

$$\mathcal{U}(\mathbf{P}_s;\boldsymbol{\mu}) \triangleq \sum_{l=1}^{L} P_s^l + \sum_{l=1}^{L} P_s^l \left(\sum_{k=1}^{\mathcal{K}} G_{s,k}^l \left(\sum_{l' \in \mathcal{I}_{s,k}^l}^{L} \mu_k^{l'} V_{|l-l'|}\right)\right)$$

where $\mathcal{I}_{s,k}^l$ represents the set of subcarrier of kth primary BS that suffers from interferences generated by the lth subcarrier of the s-th secondary BS. $\boldsymbol{\mu} = (\boldsymbol{\mu}_1, \cdots, \boldsymbol{\mu}_{\mathcal{K}})^\top$ where $\boldsymbol{\mu}_k = (\mu_k^1, \cdots, \mu_k^L)^\top$ represents the vector prices set by the kth PU due to the activity of SUs. The prices are chosen such that the complementary conditions are met, i.e.,

$$\begin{aligned} &\boldsymbol{\mu} \geq 0 \\ &\mu_k^l \left(\sum_{s=1}^{\mathcal{S}}\sum_{l'=1}^{L} P_s^{l'} G_{s,k}^{l'} V_{|l-l'|} - I_k^{l,\max}\right) = 0,\ \forall k, l \end{aligned} \tag{5}$$

Clearly, the prices set by the PUs aim to control the interference generated by the secondary users. It is straightforward to see that the vector of prices will be null whenever the interference generated by the SU is less than the interference threshold.

Denote $\mathcal{G} = \{S, \{\mathcal{P}_s\}, \{\mathcal{U}_s\}\}$, the non-cooperative pricing-based game (NPBG). $S = \{1, 2, \cdots, \mathcal{S}\}$ represents the index set of the secondary BSs. $\mathcal{U}_s$ and $\mathcal{P}_s$ denote the utility function and the strategy space for secondary BS s, respectively.

In this paper, each SU selfishly minimizes its utility function while satisfying its rate constraint. More specifically, the game is formulated as

$$\text{NPBG}: \quad \min_{\mathbf{P}_s \in \mathcal{P}_s(\mathbf{P}_{-s})} \mathcal{U}(\mathbf{P}_s;\boldsymbol{\mu}), \quad \forall s \in S \tag{6}$$

For a fixed prices $\boldsymbol{\mu}^\star$, a strategy profile $\mathbf{P}^\star$ is said to be a pure-strategy Nash equilibrium (NE) if no single SU has the incentive to unilaterally change its own transmission power to achieve a lower utility function.

3 Convergence Criterion

For any fixed and non-negative pricing $\boldsymbol{\mu}$ and power allocation $\mathbf{P}_{-s}$, the optimal solution of problem (6) is given by the modified water-filling, i.e.,

$$P_s^l = \left[\frac{\nu_s \frac{B}{\ln 2}}{1 + \sum_{k=1}^{\mathcal{K}} G_{s,k}^l \left(\sum_{l' \in \mathcal{I}_{s,k}^l}^{L} \mu_k^{l'} V_{|l-l'|}\right)} - \frac{\overline{N}_s^l + I_s^l}{G_{s,s}^l}\right]^+ \tag{7}$$

where $[x]^+ \triangleq \max(x,0)$ and ν_s is the Lagrangian multiplier associated to the rate constraint. Let $\boldsymbol{\nu} = (\nu_1, \cdots, \nu_{\mathcal{S}})$, (7) can be compactly written as

$$\mathbf{P} = \varXi(\boldsymbol{\nu}, \boldsymbol{\mu}) - \mathbf{G}^{-1}\overline{\mathbf{N}} - \mathbf{G}^{-1}\overline{\mathbf{G}}\mathbf{P} \tag{8}$$

where $\varXi(\boldsymbol{\nu}, \boldsymbol{\mu})$, $\mathbf{G}^{-1}$, $\overline{\mathbf{N}}$, $\overline{\mathbf{G}}$ are defined in (9). More specifically, $\mathbf{0}_s$ denotes a $L \times L$ zero entry matrix and $\overline{\mathbf{G}}$ is the interference matrix of the entire secondary system.

$$\begin{aligned}
\mathbf{G} &= \operatorname{diag}\left(G^1_{1,1}, \cdots, G^L_{1,1}, \cdots, G^1_{\mathcal{S},\mathcal{S}}, \cdots, G^L_{\mathcal{S},\mathcal{S}}\right),\\
\overline{\mathbf{N}} &= (\overline{N}^l_1, \cdots, \overline{N}^L_1, \cdots, \overline{N}^1_{\mathcal{S}}, \cdots, \overline{N}^L_{\mathcal{S}})^\top,\ \varXi(\boldsymbol{\nu}, \boldsymbol{\mu}) \triangleq (\xi(\nu_1, \boldsymbol{\mu}), \cdots, \xi(\nu_{\mathcal{S}}, \boldsymbol{\mu}))^\top\\
\xi(\nu_s, \boldsymbol{\mu}) &\triangleq \left(\frac{\nu_s \frac{B}{\ln 2}}{1 + \sum_{k=1}^{\mathcal{K}} G^l_{s,k}\left(\sum_{l' \in \mathcal{I}^l_{s,k}}^{L} \mu^{l'}_k V_{|l-l'|}\right)}, \cdots, \right.\\
&\qquad\qquad \left. \frac{\nu_s \frac{B}{\ln 2}}{1 + \sum_{k=1}^{\mathcal{K}} G^L_{s,k}\left(\sum_{l' \in \mathcal{I}^L_{s,k}}^{L} \mu^{l'}_k V_{|L-l'|}\right)}\right)^\top\\
\overline{\mathbf{G}} &\triangleq \begin{pmatrix} \mathbf{0}_1 & \overline{\mathbf{G}}_{12} & \ldots & \overline{\mathbf{G}}_{1\mathcal{S}} \\ \overline{\mathbf{G}}_{21} & \mathbf{0}_2 & \ldots & \overline{\mathbf{G}}_{2\mathcal{S}} \\ \vdots & \vdots & \ddots & \vdots \\ \overline{\mathbf{G}}_{\mathcal{S}1} & \overline{\mathbf{G}}_{\mathcal{S}2} & \ldots & \mathbf{0}_{\mathcal{S}} \end{pmatrix},\\
\overline{\mathbf{G}}_{j,s} &= \begin{pmatrix} G^1_{j,s}V_0 & G^2_{j,s}V_1 & \ldots & G^L_{j,s}V_{|L-1|} \\ G^1_{j,s}V_1 & G^2_{j,s}V_0 & \ldots & G^L_{j,s}V_{|L-2|} \\ \vdots & \vdots & \ddots & \vdots \\ G^1_{j,s}V_{|L-1|} & G^2_{j,s}V_{|L-2|} & \ldots & G^L_{j,s}V_0 \end{pmatrix}
\end{aligned} \tag{9}$$

At the nth iteration, for any fixed, $\boldsymbol{\mu}$, the modified water-filling function denoted as $\boldsymbol{\psi}$ can be expressed as

$$\mathbf{P}^{(n)} = \boldsymbol{\psi}\left(\mathbf{P}^{(n-1)}, \boldsymbol{\mu}^{(n-1)}\right) = \varXi(\boldsymbol{\nu}, \boldsymbol{\mu}^{(n-1)}) - \mathbf{G}^{-1}\overline{\mathbf{G}} - \mathbf{G}^{-1}\overline{\mathbf{G}}\mathbf{P}^{(n-1)} \tag{10}$$

Now, we proceed to define the price update.

Definition 1. *[9] Let $\{n^{l,t(k,l)}_k\}^{\infty}_{t(k,l)=1}$ of $\{n\}^{\infty}_{n=1}$ be a unique subsequence associated with μ^l_k, the asynchronous price update is defined as the price update approach in which each is updated only at time instances $\{n^{l,t(k,l)}_k\}^{\infty}_{t(k,l)=1}$.*

By using Definition 1, problem (6) can be solved by iteratively solving (10). The proposed approach is summarized as follow.

Algorithm 1. Iterative modified water-filling algorithm for solving (6)

1: **Input** A solution accuracy $\epsilon > 0$ and a feasible $\mathbf{P}^0$.
2: Initialize $\boldsymbol{\mu}^{(0)}, \boldsymbol{\nu}^{(0)}$, set $n = 0$ and let $t(k,l) = 1$;
3: **repeat**
4: $n = n + 1$;
5: Find $\mathbf{P}^n$ by using (10);
6: For each secondary BS s, update $\nu_s^{(n)}$ by using bisection method.
7: Asynchronous update of the interferences prices μ_k^l, $\forall k, l$

$$\mu_k^{l,(n)} = \begin{cases} \left[\mu_k^{l,(n-1)} + \delta \left(I_k^{l,(n)} - I_k^{l,\max}\right)\right]^+, & \text{if } n+1 = n_k^{l,t(k,l)} \\ \mu_k^{l,(n-1)}, & \text{otherwise} \end{cases}$$

If $n + 1 = n_k^{l,t(k,l)}$ then $t(k,l) = t(k,l) + 1$;
8: **until** $n > \mathcal{N}$ or $\left| \left(\mathcal{U}(\mathbf{P}_s^{(n)}; \boldsymbol{\mu}) - \mathcal{U}(\mathbf{P}_s^{(n-1)}; \boldsymbol{\mu})\right) / \mathcal{U}(\mathbf{P}_s^{(n-1)}; \boldsymbol{\mu}) \right| \leq \epsilon$, $\forall s$
9: **Output** $\mathbf{P}^n$.

where $\sum_{s=1}^{\mathcal{S}} \sum_{l'}^{L} P_s^{l',n} G_{s,k}^{l'} V_{|l-l'|} = I_k^{l,n}$ and $\delta \in (0,1)$ is a coefficient to control the convergence speed of the price update. The following lemma captures the convergence of the interference prices. See [9] for the proof.

Lemma 1. *The sequence of interferences prices $\{\boldsymbol{\mu}^{(n)}\}_{n=1}^{\infty}$ generated by Algorithm 1 converges, i.e.,*

$$\lim_{n\to\infty} \boldsymbol{\mu}^{(n)} = \boldsymbol{\mu}^\star$$

Now, we provide a sufficient criterion for convergence of the proposed Algorithm 1 to a unique NE point of the game $\mathcal{G}$. This is done in the following theorem.

Theorem 1. *The sequence $\{\mathbf{P}^{(n)}\}_{n=1}^{\infty}$ generated by the proposed Algorithm 1 converges to a unique NE regardless of the initial power allocation value if*

$$\sum_{j=1, j\neq s}^{\mathcal{S}} \frac{\sum_{l'=1}^{L} V_{|l-l'|} G_{j,s}^{l'}}{G_{s,s}^l} \leq 1, \ \forall s, l \tag{11}$$

Proof: The proof of Theorem 1 follows the step of the proof of [9, Theorem 4]. Due to limited space, we leave the details for future publication. ■

4 New Distributed Scheme

From Theorem 1, we notice that our proposed Algorithm 1 converges to a unique NE point only if the sufficient convergence condition is met. In this section, we propose a distributed algorithm that always convergences to a unique NE point of $\mathcal{G}$. This is done by providing a new distributed convergence criterion that can be embedded into problem (6).

The power allocation to solve (6) can be done by

$$P_s^l = \gamma_s^l \left(\frac{\overline{N}_s^l}{G_{s,s}^l} + \frac{\sum_{j\neq s}^{\mathcal{S}} \sum_{l'=1}^{L} P_j^{l'} V_{|l-l'|} G_{j,s}^{l'}}{G_{s,s}^l} \right) \tag{12}$$

where γ_s^l is the signal-to-interference-plus-noise ratio (SINR) for secondary user s on subcarrier l. (12) can be compactly written as

$$\mathbf{P} = \mathbf{G}^{-1}\mathbf{\Gamma}\overline{\mathbf{G}}\mathbf{P} + \mathbf{G}^{-1}\mathbf{\Gamma}\overline{\mathbf{N}} \tag{13}$$

where $\mathbf{\Gamma} = \mathrm{diag}\left(\gamma_1^1, \cdots, \gamma_1^L, \cdots, \gamma_{\mathcal{S}}^1, \cdots, \gamma_{\mathcal{S}}^L\right)$. For a fixed SINR $\mathbf{\Gamma}$, at the nth iteration, the power allocation function ϕ is expressed as

$$\mathbf{P}^{(n)} = \phi\left(\mathbf{P}^{(n-1)}, \mathbf{\Gamma}\right) = \mathbf{G}^{-1}\mathbf{\Gamma}\overline{\mathbf{G}}\mathbf{P}^{(n-1)} + \mathbf{G}^{-1}\mathbf{\Gamma}\overline{\mathbf{N}} \tag{14}$$

Theorem 2. *The power allocation scheme* (14) *converges to a unique fixed point for any arbitrary starting point if*

$$\frac{\gamma_s^l\left(\sum_{j\neq s}^{\mathcal{S}}\sum_{l'=1}^{L} V_{|l-l'|}G_{j,s}^{l'}\right)}{G_{s,s}^l} < 1, \;\; \forall s, l \tag{15}$$

Proof: Given an arbitrary initial power $\mathbf{P}^{(0)}$, we have

$$\left\|\mathbf{P}^{(n+1)} - \mathbf{P}^{(n)}\right\| = \left\|\mathbf{G}^{-1}\mathbf{\Gamma}\overline{\mathbf{G}}\left(\mathbf{P}^{(n)} - \mathbf{P}^{(n-1)}\right)\right\| \leq \zeta^{n+1}\left\|\mathbf{P}^{(1)} - \mathbf{P}^{(0)}\right\| \tag{16}$$

where $\kappa = \max_{\substack{1\leq s\leq \mathcal{S}\\ 1\leq l\leq L}} \frac{\gamma_s^l\left(\sum_{j\neq s}^{\mathcal{S}}\sum_{l'=1}^{L} V_{|l-l'|}G_{j,s}^{l'}\right)}{G_{s,s}^l}$. It follows that for $\forall n, M \geq 0$,

$$\left\|\mathbf{P}^{(n+M)} - \mathbf{P}^{(n)}\right\| = \sum_{m=1}^{M}\left\|\mathbf{P}^{(n+m)} - \mathbf{P}^{(n+m-1)}\right\| \overset{(b)}{\leq} \frac{\zeta^n}{1-\zeta}\left\|\mathbf{P}^{(1)} - \mathbf{P}^{(0)}\right\| \tag{17}$$

(b) is verified if $\frac{\gamma_s^l\left(\sum_{j\neq s}^{\mathcal{S}}\sum_{l'=1}^{L} V_{|l-l'|}G_{j,s}^{l'}\right)}{G_{s,s}^l} < 1, \forall l, \forall s$. Hence, we obtain a Cauchy sequence which is a convergent sequence. Moreover, it is straightforward to demonstrate that $\phi(\cdot)$ is a contraction function. Therefore, the power allocation scheme converges to a unique fixed point [12] $\mathbf{P}^\star = \left(\mathbf{I} - \mathbf{G}^{-1}\mathbf{\Gamma}\overline{\mathbf{G}}\right)^{-1}\mathbf{G}^{-1}\mathbf{\Gamma}\overline{\mathbf{N}}$. ■

Remark 1. First, the criterion in Theorem 2 is a convergence condition per subcarrier. Secondly, we notice that when $l = 1$, our proposed sufficient condition (15) coincides with the convergence criterion given in [13] for the water-filling.

To be able to use (12) as a solution to the NPBG, the value of γ_s^l, $\forall s, l$ is required. From (12), we see there exists a one-to-one mapping from P_s^l to γ_s^l, $\forall s, l$. This one-to-one mapping is defined by $P_s^l = \gamma_s^l \widehat{I}_s^l$ where

$$\widehat{I}_s^l \triangleq \frac{\left(\overline{N}_s^l + \sum_{j\neq s}^{\mathcal{S}}\sum_{l'=1}^{L} P_j^{l'} V_{|l-l'|}G_{j,s}^{l'}\right)}{G_{s,s}^l}$$

Define the following variable

$$C_s^l \triangleq \frac{G_{s,s}^l}{\sum_{j\neq s}^{\mathcal{S}}\sum_{l'=1}^{L} V_{|l-l'|}G_{j,s}^{l'}}$$

Let $\mathbf{\Gamma}_s = (\gamma_s^1, \cdots, \gamma_s^L)^\top$ be the SINR vector for secondary user s. At the nth round, $\mathbf{\Gamma}_s^{(n)}$ can be found by solving the following convex optimization problem

$$\begin{aligned} \max_{\boldsymbol{\Gamma}_s \geq 0} \ & \sum_{l=1}^{L} \gamma_s^l \widehat{I}_s^{l,(n-1)} \left(1 + \sum_{k=1}^{\mathcal{K}} G_{s,k}^l \left(\sum_{l' \in \mathcal{I}_{s,k}^l}^{L} \mu_k^{l'} V_{|l-l'|} \right)\right) \\ \text{s.t. } & \widehat{\mathcal{R}}_s \leq \sum_{l=1}^{L} B \log_2 \left(1 + \gamma_s^l\right) \\ & \gamma_s^l \leq C_s^l - \delta_1, \ \forall l \end{aligned} \tag{18}$$

The optimal solution of problem (18) is given by

$$\gamma_s^{l\star} = \left[\frac{\lambda_s \frac{B}{\ln 2}}{\widehat{I}_s^{l,(n-1)} \left(1 + \left(\sum_{k=1}^{\mathcal{K}} G_{s,k}^l \left(\sum_{l' \in \mathcal{I}_{s,k}^l}^{L} \mu_k^{l'} V_{|l-l'|}\right)\right)\right)} - 1 \right]_0^{C_s^l - \delta_1} \tag{19}$$

Where λ_s is the dual associated with the rate constraint. In fact, it is important to notice that without the second constraint, problem (18) is equivalent to problem (6). The criterion in (15) is embedded into the optimization problem (18) as a constraint in order to assure the convergence of the algorithm to a fixed point. Notice that an infinitesimal positive constant δ_1 is deducted from the convergence criterion to relax the constraint. The game $\mathcal{G}$ given in (6) is solved by alternately solving problem (18) and substituting each γ_s^l, $\forall s, l$ in (12). The proposed new algorithm is summarized as

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Algorithm 2. New distributed algorithm to solve (6)

1: **Input** A solution accuracy $\epsilon > 0$ and a feasible $\mathbf{P}^0$.
2: Initialize $\boldsymbol{\mu}^0$, set $n = 0$ and let $t(k, l) = 1$;
3: **repeat**
4: $\quad n = n + 1$;
5: $\quad$ Obtain $\boldsymbol{\Gamma}_s^{(n)}$, $\forall s$ by solving problem (18);
6: $\quad$ Calculate $P_s^{l,(n)}$, $\forall s, l$ by using (12).
7: $\quad$ Assychronous update of the interferences prices μ_k^l, $\forall k, l$

$$\mu_k^{l,n} = \begin{cases} \left[\mu_k^{l,n-1} + \delta\left(I_k^{l,n} - I_k^{l,\max}\right)\right]^+, & \text{if } n+1 = n_k^{l,t(k,l)} \\ \mu_k^{l,n-1}, & \text{otherwise} \end{cases}$$

$\quad$ If $n + 1 = n_k^{l,t(k,l)}$ then $t(k, l) = t(k, l) + 1$;
8: **until** $n > \mathcal{N}$ or $\left| \left(\mathcal{U}(\mathbf{P}_s^n; \boldsymbol{\mu}) - \mathcal{U}(\mathbf{P}_s^{n-1}; \boldsymbol{\mu})\right) / \mathcal{U}(\mathbf{P}_s^{n-1}; \boldsymbol{\mu}) \right| \leq \epsilon$, $\forall s$
9: **Output** $\mathbf{P}^n$.

From the structure of the proposed Algorithm 2, we see that it always converges to a unique and fixed NE point of the game $\mathcal{G}$, the solution is given by $\mathbf{P}^\star = \left(\mathbf{I} - \mathbf{G}^{-1}\boldsymbol{\Gamma}\overline{\mathbf{G}}\right)^{-1} \mathbf{G}^{-1}\boldsymbol{\Gamma}\overline{\mathbf{N}}$. The SINR vector will also converge. Due to space constraint, we leave the detailed proof of the SINR convergence for future publication. In Sect. 5, numerical analysis of the convergence of the SINR together with the convergence of the interference prices will be provided.

To implement our proposed distributed Algorithms 1 and 2, the secondary MTs need to measure the noise-plus-interference on each subcarrier at each iteration. This value is then feeding back to the respective secondary BS. This operation is repeated until convergence or stopping criterion of both algorithms is reached. Clearly, in terms of signalling overhead, our proposed algorithms by using only local information need little signalling overhead.

5 Numerical Results

In this section, the performance of our proposed algorithm is evaluated via numerical results. All results are conducted using Monte Carlo simulation by averaging over 300 channel realizations. We consider an underlay CR network with 2 PUs and 5 SUs. The secondary BSs are randomly located at a distance varying from 0.1 km to 0.5 km away from the primary BSs. Each MT is uniformly located within a 0.5 km radius circle from its serving BS. There are $L = 32$ subcarriers having each a bandwidth of $B = 15$ KHz.

The path loss model for the channel is LdB(d) $= 128.1 + 37.6 \times \log_{10}(d)$, where d is the distance between a BS and a MT. The shadowing's standard deviation is 6 dB and $N_0 = -174$ dBm/Hz. The primary BS has a uniform power transmission $P_p^l = \frac{P_{\max}}{L}, \forall l$ with $P_{\max} = 33$ dBm. The interference threshold $I_k^{l,\max}$ is computed by assuming only 10 % of the PU k interference-free achievable rate degradation is permitted on subcarrier l, $\forall l$. Unless otherwise stated, $\widehat{\mathcal{R}}_s = 30$ Kbits/s. The maximum number of iterations is $\mathcal{N} = 40$ while $\delta_1 = 10^{-5}$ and $\epsilon = 10^{-4}$.

To evaluate the proposed Algorithms 1 and 2, we also compare with the perfect synchronization case denoted as PS. In this case, the interference weight is $V^{\text{PS}} = \{1\}$ We will clearly see that asynchronism lead to a loss of performance. Our Algorithms are initialized by assuming uniform power on each subcarrier mainly $P_{\max}/L$.

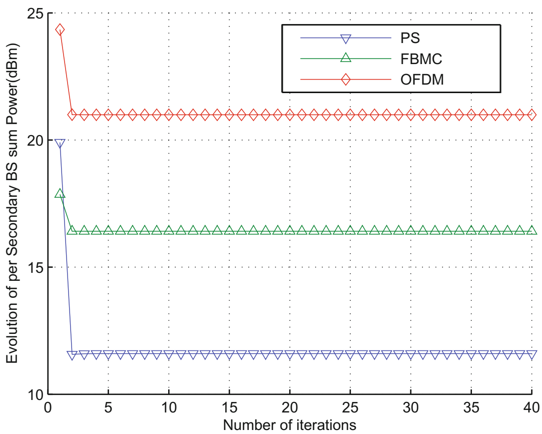

Fig. 1. Average sum secondary power versus rate constraint (Color figure online)

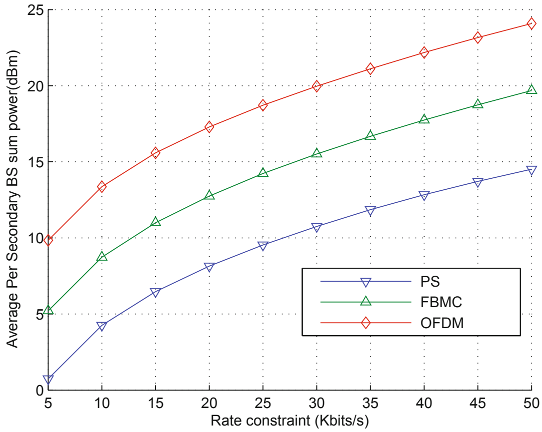

Fig. 2. Average sum secondary power versus rate constraint (Color figure online)

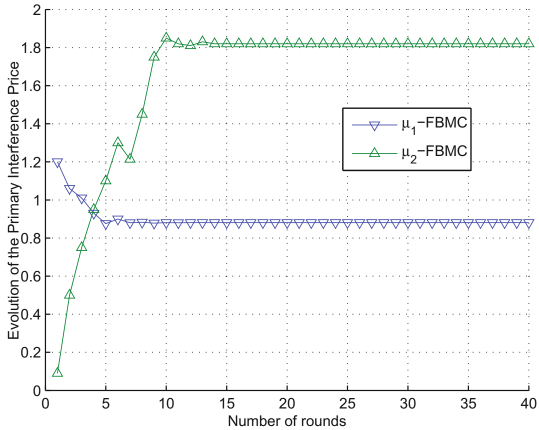

Fig. 3. Convergence behaviour of the interference price. (Color figure online)

Figures 1 and 2 portray the convergence properties and the performance of our proposed Algorithm 1 for different multi-carrier modulation scheme. Figure 1 depicts the evolution of the per secondary BS sum power. From Fig. 1, it can be clearly inferred that the proposed Algorithm 1 converges irrespective of the modulation method. It is important to observe the gap between the performance of PS and the one achieved by OFDM and FBMC. This is the consequence of inter-carrier interference induced by asynchronism and lack of cooperation.

Figure 2 depicts the performance of our proposed Algorithm 1 in terms of average sum power versus per BS power rate constraint. We can see that the sum power achieved by the proposed Algorithm 1 tends to increase as the rate constraint increases. From Fig. 2, we also observe a gain varying from 21.98 % to 22.70 % between the sum power with FBMC compared with the sum power achieved with OFDM.

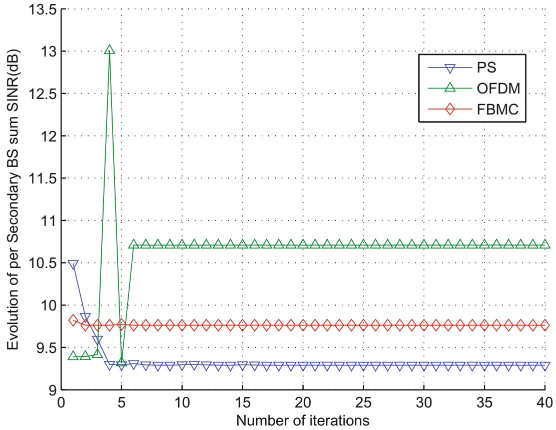

Fig. 4. Convergence behaviour of the SINR. (Color figure online)

Figure 3 demonstrates the convergence behaviour of the interference update of Algorithm 2. The convergence of interference prices is only given only for the case of FBMC. We observe that the interference prices of both primary BSs converge.

In Sect. 4, we stated that the SINR vector sequence $\{\mathbf{\Gamma}_s^{(n)}\}_{n=1}^{\infty}$, $\forall s$ converges. We prove our assertion by means of simulations. Indeed, Fig. 4 depicts the performance of our proposed Algorithm 2. It shows the convergence behaviour of the SINR vector. From Fig. 4, we clearly observe that the sequence of the SINR vector converges regardless of the multi-carrier modulation scheme.

6 Conclusion

In this work, we proposed two distributed algorithms to solve the problem of secondary sum power minimization for an underlay downlink asynchronous CR network with OFDM/FBMC. The problem was reformulated as a pricing-based non-cooperative game. We provide a sufficient convergence criterion to a NE point of the NPBG. Moreover, we provide a new algorithm that solves alternately power vector and SINR vector. The new algorithm always converges to a unique fixed NE point. Furthermore, we have through numerical results validated the efficiency of the proposed schemes. The simulation results highlighted the advantages of using FBMC over OFDM for asynchronous network.

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