On Convergence of a Distributed Cooperative Spectrum Sensing Procedure in Cognitive Radio Networks

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Abstract. In this work, we analyze a distributed cooperative spectrum sensing scheme where N secondary users (SUs) of a cognitive wireless network try to agree about the primary user presence (absence) by iterative interchanging individual opinions (states) over an unreliable wireless propagation medium. It is assumed that the SUs update their personal states based on the "K-out-of-N" rule, and the interchange session fails, if the consensus has not been reached within a fixed number of iterations. The problem of forming a joint opinion becomes challenging because a SU makes its personal decision based on local observations distorted by a wireless propagation medium. This fact may cause a disorder. In this paper, we formulate sufficient conditions of reaching the agreement on the basis of local observations.

Keywords: Cognitive radio networks · Distributed spectrum sensing · Social wireless networks · Wireless propagation

1 Introduction

Spectrum sensing (SS) is a crucial function of cognitive radio since it provides secondary users (SUs) information about spectrum availability and preserves primary users (PUs) from interference coming from unlicensed spectrum users. In order to improve the SS quality in the wireless medium characterized by fading, interference, and path-loss effects, cooperative SS (CSS) schemes employing SU spatial diversity have been proposed [1,2]. A large amount of research has been devoted to analyzing and designing CSS algorithms, and the most works on the topic considered centralized schemes where a fusion center makes a joint decision on the basis of local decisions or/and measurements [3,4]. In [5], a distributed CSS algorithm was analyzed where the SUs attempted to reach a joint decision on the PU presence via interchange of their individual measurements, which were received undistorted at each node.

In contrast to the absolute majority of previous works on CSS, in this paper, we consider a distributed CSS scheme where the SUs try to reach the agreement on the PU presence by interchanging their personal binary opinions (yes/no) via an unreliable propagation medium. Such scenarios are typical in wireless

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networks where the nodes have also social ties, and a dedicated control channel is used for opinion interchange. Trying to reach the agreement, the SUs update their personal opinions (states) based on the "K-out-of-N" rule. But each SU changes its opinion based on only the local observations of the network state, which are different for different users since the wireless medium distorts the transmitted binary signals in a random manner. Therefore the above distributed procedure may result in a disorder (divergence).

In this paper, we obtain sufficient conditions assuring the stochastic convergence of the presented distributed algorithm.

2 System Model

2.1 Model of Opinion Interchange

We consider a secondary network comprising N nodes operating in a finite area. The SUs cooperate to form a joint decision on the PU presence. We denote the network state vector

$$\mathbf{x}(t) = \{x_1(t), x_2(t), \ldots, x_N(t)\} = \{\underbrace{+1, \ldots +1}_{\mathbf{S}^+(t)}, \underbrace{-1, \ldots -1}_{\mathbf{S}^-(t)}\} \tag{1}$$

where the random variate (RV) $x_i(t)$ corresponds to the opinion of the i th SU (yes/no) on the PU presence at the t th iteration, $0 \leq t \leq T$, where T is a fixed integer. The initial state $\mathbf{x}(0)$ is formed based on individual spectrum sensing, after which the SUs start to update their opinions following the "K-out-of-N" rule as

$$x_i(t+1) = \text{Sign}\left[x_i(t) + \sum_{j \neq i} x_{i,j}(t) + N - 2K\right] \tag{2}$$

where $x_{i,j}(t)$ is the state of node j observed at node i. Taking into account that the binary opinion $x_j(t)$ can be interpreted either correctly or incorrectly, $x_{i,j}(t)$ can be represented as

$$x_{i,j}(t) = w_{i,j}(t) x_j(t) \tag{3}$$

where $w_{i,j}(t)$ is a two-point RV taking on the value $+1$ with the probability of correct bit detection $P_{\text{cd}_{i,j}}$ and taking on the value -1 with the probability $(1 - P_{\text{cd}_{i,j}})$. Obviously, $w_{i,j}$ follows a Bernoulli distribution [6] with the success probability equal to $P_{\text{cd}_{i,j}}$, which can be defined as the probability of correct bit detection of binary phase shift keying as [7]

$$P_{\text{cd}_{i,j}} = 1 - Q\left(\sqrt{2\gamma_{i,j}}\right) \tag{4}$$

where $Q(.)$ is the Gaussian Q function, and $\gamma_{i,j}$ is the signal-to-noise ratio (SNR) characterizing the transmission path between the nodes j and i.

In this work, we assume that $x_i(t+1) = x_i(t)$ if the sum in the brackets in (2) is zero. The transform $\mathbf{x}(0) \to \mathbf{x}(1) \to \ldots \to \mathbf{x}(T)$ represents the stochastic dynamics of the considered system. We obtain below sufficient conditions assuring the convergence of $\mathbf{x}(0)$ to a consensus in a probabilistic sense.

2.2 Model of Wireless Propagation

In this work, we model wireless propagation by taking into account fading and path-loss (PL) effects. We apply a bounded PL model that can be represented as [8]

$$\gamma_{\mathrm{pl}} = \begin{cases} 1, R < R_0 \\ \left(\frac{R}{R_0}\right)^{-\kappa}, R \geq R_0 \end{cases} \tag{5}$$

where R is the transmitter-receiver distance, κ is the path-loss exponent, and R_0 is a path-loss constant.

A gamma distribution models fading effects as

$$f_{\gamma_{\mathrm{f}}}(x) = \frac{x^{m-1}}{\Gamma(m)\theta^m} \exp\left(-\frac{x}{\theta}\right) \tag{6}$$

where m and θ are the respective shape and scale parameters, and $\Gamma(.)$ is the gamma function. In fading channels, m is inversely proportional to the amount of fading. This model represents the channel power gains in Nakagami-m small-scale fading, as well as it is used as a substitute for composite Nakagami-m-log-normal shadowing fading [9].

3 Convergence to Consensus

In view of (1)–(2), the agreement in the considered network means that the network state is either $\mathbf{x}^+ = \{\underbrace{1, 1, \ldots, 1}_{N}\}$ or $\mathbf{x}^- = \{\underbrace{-1, -1, \ldots, -1}_{N}\}$. From the point of view of convergence analysis, $\mathbf{x}^+$ and $\mathbf{x}^-$ are equivalent, and we will concentrate on the convergence to $\mathbf{x}^+$, and the convergence will be understood in the sense of the ϵ-convergence given by Definition 1.

Definition 1. A state $\mathbf{x}(0) = \{\underbrace{1, 1, \ldots, 1}_{n} \underbrace{-1, -1, \ldots, -1}_{m=N-n}\}$ converges to $\mathbf{x}^+$ if the probability $Pr\left\{\cap_{i=1}^{N}\{x_i(t) = 1\}\right\} \geq 1 - \epsilon$, where ϵ is a predetermined number.

From the state update Eq. (2), it is seen that there are two reasons that may affect the convergence to the consensus: the initial state $\mathbf{x}(0)$ and statistics of $w_{i,j}$. We give below more details about them.

3.1 Distribution of Network Initial State

The network initial state $\mathbf{x}(0)$ is defined by results of individual SS. For example, in the case of energy detection, the probabilities of correct detection P_{d_i} and false alarm P_{f} at node i can be defined as [10]

$$P_{\mathrm{d}_i} = Q_u\left(\sqrt{2\gamma_i}, \sqrt{2\lambda}\right), \tag{7}$$

and

$$P_{\mathrm{f}} = \frac{\Gamma\left(u, \lambda/2\right)}{\Gamma\left(u\right)} \tag{8}$$

where u is the product of the observation time and signal bandwidth, $\Gamma(a, x) = \int_x^{\infty} t^{a-1} \exp(-t)\, dt$ is the upper incomplete gamma function, $Q_u\left(\sqrt{2\gamma_i}, \sqrt{2\lambda}\right)$ is the generalized Marcum Q function [11], γ_i denotes the signal-to-noise ratio (SNR) at the node i, and λ is the detector threshold.

In the scenario considered in this work, we assume that u and λ are the same for all SUs, and thus the false probability is the same for all SUs, while the received SNR γ_i is obviously defined by the channel gain and distance between the PU and node i. Then the probability of obtaining less than M indications (I) of PU presence $P_I(M) = Pr\{I \leq M\}$ is the probability of less than M successes in N independent and non-identical (i.n.d.) trials where the success probability of i th trial is P_{d_i}. This probability is defined by the cumulative distribution function (CDF) $F_{\mathcal{B}_P}(N, \mathbf{p})$ of the Poisson binomial distribution $\mathcal{B}_P$ [12], where $\mathbf{p} = \{P_{\mathrm{d}_1}, \ldots, P_{\mathrm{d}_N}\}$. In each interchange epoch, the CDF $F_{\mathcal{B}_P}(N, \mathbf{p})$ is random, and it is defined by a concrete realization of γ_i, $i = 1, \ldots, N$. $P_I(M)$ can be averaged over the channel and node location statistics.

In Fig. 1, we show simulation results for the complementary CDFs $Pr\{I > N/2\}$ and $Pr\{I > 2N/3\}$ for the SUs uniformly distributed over a circle of the radius $R_{\max}$ and PU located at the origin. The network and propagation parameters are: $m = 1.7$ and $m = 3.5$, $\kappa = 2.6$, $R_0 = 0.1 R_{\max}$. We assume that the probability of false detection $Q_f = 0.1$, and the product of the observation time and signal bandwidth $u = 2$.

Actually, the estimates in Fig. 1 characterize the average (over the channel statistics and operating area) CSS performance for either centralized or distributed scenarios where a decision is made via "K-out-of N" rule on the basis of perfect (undistorted) SU decisions that, however, are made by taking into

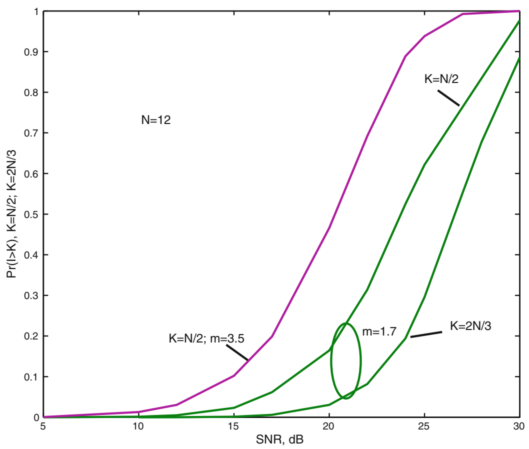

Fig. 1. Complementary CDF, $Pr\{I > K\}$ ($N = 12$, $K = N/2$ and $K = 2N/3$), versus the PU SNR. SUs are uniformly distributed over a circle, and the PU is located at the origin.

account imperfections imposed by the wireless propagation medium on individual SS. Both scenarios correspond to cases of a very high SNR of the control channel: $\mathrm{SNR_c} \to \infty$. This condition assures that for $\forall\ i,\ j,\ P_{\mathrm{cd}_{i,j}} \approx 1$, and even under a distributed scenario, all nodes make decisions on the basis of the same information. We focus below on distributed scenarios where the wireless propagation medium distorts the individual opinions.

3.2 Statistical Properties of Node Weights

The node weights $w_{i,j}$ are two-point RVs, and thus their statistical properties are defined by Bernoulli distribution [6]. We assume that $w_{i,j}$ may differ in different interchange epochs, but they are constants at a fixed interchange epoch. It is seen that generally $w_{i,j}$ differ for different i and j due to different (random) propagation conditions between different node pairs caused by random fading and node locations. Thus, any sum of $w_{i,j}$ follows the Poisson binomial distribution. Moreover, we note that

$$D_i = \left(\sum_{j \in J} w_{i,j} - \sum_{k \in K} w_{i,k} \right) \stackrel{d}{=}$$
$$\Sigma_i = \left(\sum_{j \in J} w_{i,j} + \sum_{k \in K} w'_{i,k} \right) \tag{9}$$

where $\stackrel{d}{=}$ means equal in distribution, and $w'_{i,j}$ is a two-point RV: $w'_{i,j} = +1$ with the probability $(1 - P_{\mathrm{cd}_{i,j}})$, and $w'_{i,j} = -1$ with the probability $P_{\mathrm{cd}_{i,j}}$. Thus D_i also follows the Poisson binomial distribution with the average success probability

$$\bar{p}_i = \frac{1}{|J| + |K|} \left(\sum_{j \in J} P_{\mathrm{cd}_{i,j}} + \sum_{k \in K} (1 - P_{\mathrm{cd}_{i,j}}) \right) \tag{10}$$

3.3 Sufficient Conditions of Convergence

We introduce node subsets $\mathbf{S}^+(0) = \{k : x_k(0) = 1\}$ ($|\mathbf{S}^+(0)| = n$) and $\mathbf{S}^-(0) = \{k : x_k(0) = -1\}$ ($|\mathbf{S}^-(0)| = m = N - n$). For the sake of simplicity, we omit below the iteration index t and assume that each SU makes its decision following the opinion of the majority, that is $K = N/2$. Also for the sake of simplicity and without loss of generality, we assume that $(N-1)$ is even (otherwise we had just to use the corresponding integer parts). We suppose that the control channel is designed in such a way that $P_{\mathrm{cd}_{i,j}} > 0.5$ for $\forall i, j$ since otherwise the probability of incorrect opinion reception is larger than that of correct reception.

It is seen from (2) that starting from $t = 1$ the components of $\mathbf{x}(t)$ become dependent RVs. Thus, a question is, which values of n, m, and the success probabilities $P_{\mathrm{cd}_{i,j}}$ can guarantee the ϵ-convergence? Sufficient conditions of ϵ-convergence can be formulated via Proposition 1.

Proposition 1. *The network is ϵ-convergent if*

$$n > m, \tag{11}$$

and for each node $x_i \in \mathbf{S}^+$,

$$\bar{p}^{(+)} \geq \left[\frac{(N-1)\left(1 - I^{-1}_{\epsilon/N}\left[(N-1)/2+2, (N-1)/2-1\right]\right)}{n-1} - \frac{(N-n)(1-\bar{p}_i^{(-)})}{n-1} \right], \tag{12}$$

while for each node $x_i \in \mathbf{S}^-$,

$$\bar{p}_i^{(+)} \geq \max\left\{ \left[0.5 + \frac{2}{N-1}\right]; \left[\frac{(N-1)\left(1 - I^{-1}_{\epsilon/N}\left[(N-1)/2-1, (N-1)/2+2\right]\right)}{n} - \frac{(N-n-1)(1-\bar{p}_i^{(-)})}{n} \right] \right\} \tag{13}$$

where $\bar{p}_i^{(+)}$ is the average success probability for the neighborhood of node $i \in \mathbf{S}^+$, $\bar{p}_i^{(-)}$ is the average success probability for the neighborhood of node $i \in \mathbf{S}^-$, and $I_r^{-1}()$ is the inverse regularized beta function [11].

Proof. Let E_i be the event of $x_i = 1$. Then $Pr\left\{\cap_{i=1}^N E_i\right\}=1 - Pr\left\{\cup_{i=1}^N \bar{E}_i\right\}$, where $Pr\left\{\cup_{i=1}^N \bar{E}_i\right\}$ is the probability that at least one of E_i is not true. By Boole's inequality,

$$Pr\left\{\cap_{i=1}^N E_i\right\} \geq 1 - \sum_{i=1}^N Pr\left\{\bar{E}_i\right\}. \tag{14}$$

Thus, conditions assuring $Pr\left\{\bar{E}_i\right\} \leq \epsilon/N$ for $\forall\ i$ guarantee the ϵ-convergence.

If $x_i \in \mathbf{S}^+$, then the probability that it will change the opinion is

$$P^+ = Pr\left\{ \underbrace{\left(\sum_{j \in \mathbf{S}_i^+, j\neq i} w_{i,j} - \sum_{k \in \mathbf{S}_i^-} w_{i,k} \right)}_{\Sigma_i, i\in\mathbf{S}^+} < -1 \right\}, \tag{15}$$

and the probability that a node $x_i \in \mathbf{S}^-$ will not change the opinion is

$$P^{-} = Pr\left\{\underbrace{\left(\sum_{j\in \mathbf{S}_i^{+}} w_{i,j} - \sum_{k\in \mathbf{S}_i^{-},k\neq i} w_{i,k}\right)}_{\Sigma_i, i\in \mathbf{S}^{-}} \leq +1\right\}. \tag{16}$$

The RV Σ_i in (15)–(16) follow the Poisson binomial distribution. Bounds on the CDF $U \sim \mathcal{B}_P$ can be obtained due to Hoeffding as [12]

$$Pr\left\{U \leq M\right\} \leq \sum_{k=0}^{M} \binom{N-1}{k} \bar{p}^{k} (1-\bar{p})^{N-1-k} \tag{17}$$

iff $\bar{p} \geq (M+1)/(N-1)$, where $\bar{p}$ is the average success probability. On the right-hand side of (17) we observe the CDF $F(N-1,\bar{p})$ of ordinary binomial distribution with the parameters $(N-1)$ and $\bar{p}$ defined as [6]

$$F_{\mathrm{B}}(N-1,\bar{p}) = I_{1-\bar{p}}(N-1-M, M+1) \tag{18}$$

where $I_r(a,b)$ is the regularized beta function [11].

Then using (15) and (17)–(18) as well as taking into account that $P_{\mathrm{cd}_{i,j}} > 0.5$ and $\bar{p} = (n-1)\bar{p}^{(+)} + (N-n)(1-\bar{p}^{(-)})$, we conclude that $P^{+} \leq F_{\Sigma_i}\left((N-1)/2-2\right) \leq \epsilon/N$ if (11)–(12) hold.

Similarly, one can show that $P^{-} \leq \epsilon/N$ if (11), (13) hold. In this case, $M = (N-1)/2+1$ in (17), and $\bar{p} = n\bar{p}^{(+)} + (N-n-1)(1-\bar{p}^{(-)})$.

It is possible to formulate stricter sufficient conditions of ϵ-convergence for both $x_i \in \mathbf{S}^{+}$ and $x_i \in \mathbf{S}^{-}$.

Corollary 1. *The network is ϵ-convergent if for $\forall\ x_i$, $i = 1,\ldots,N$, (11) holds and*

$$\bar{p}_i^{(+)} > \max\left\{\left\lceil\frac{(N-n)\bar{p}_i^{(-)} + n - \frac{N+1}{2} + 2}{n-1}\right\rceil; \right.$$
$$\left.\left\lceil\frac{(N-1)\left(1 - I_{\epsilon/N}^{-1}\left[(N-1)/2-1,(N-1)/2+2\right]\right)}{n-1} - \frac{(N-n)(1-\bar{p}_i^{(-)})}{n-1}.\right\rceil\right\} \tag{19}$$

Proof. Aiming at formulating joint convergence conditions for all nodes, we note that Σ_i for $i \in \mathbf{S}^{+}$ defined by (15) is the sum of $(n-1)$ i.n.d. Bernoulli RVs, each with the success probability larger than 0.5 and m i.n.d. Bernoulli RVs, each with the success probability less than 0.5. In Σ_i for $i \in \mathbf{S}^{-}$ specified by (16), the number of the i.n.d. Bernoulli RVs with the success probability larger than 0.5 is n, and the number of the i.n.d. Bernoulli RVs with the success probability less than 0.5 is $(m-1)$. This is due to (9)–(10) and $P_{\text{cd}_{i,j}} > 0.5$. Then it is easy to show that under other equal conditions (that is under equal $P_{\text{cd}_{i,j}}$), conditions assuring $Pr\{\Sigma_i, i \in \mathbf{S}^+ \leq +1\} \leq \epsilon/N$ guarantee also that $P^+ < \epsilon/N$ and $P^- \leq \epsilon/N$.

The validity of (11)–(13) and (19) is defined by many factors such as the cardinality N of the node set, shape and size of the operating area, node distribution in the area, control channel reliability (that is the SNR and coding used). In Fig. 2, we show graphs presenting the probability $Pr(P_\epsilon)$ that (19) holds at the first iteration. The operating area and parameters of wireless propagation medium are described in Subsect. 3.1. The results presented in Fig. 2 show that the wireless propagation medium affects significantly the validity of sufficient conditions. At the same time, we emphasize that (11)–(13) represent sufficient conditions of ϵ-convergence, and (19) (valid for $\forall\ x_i$) represents rather strict conditions.

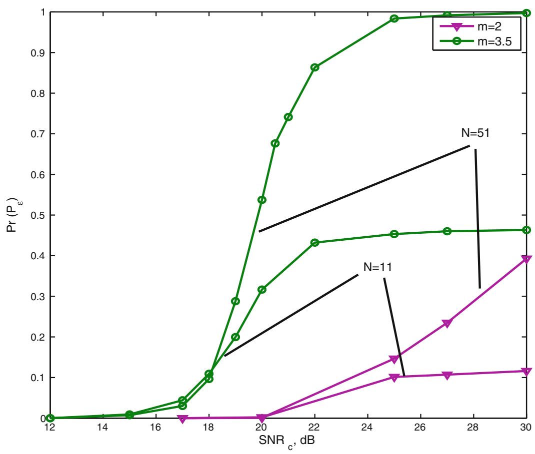

Fig. 2. Graphs of probability of validity of (19) under considered scenario versus the SNR_c of the control channel. The PU SNR $= 30$ dB.

4 Conclusion

In this work, we analyzed a distributed cooperative spectrum sensing algorithm where the SUs tried to reach an agreement about the PU presence/absence by interchanging their personal opinions via an untrustworthy propagation medium. Such scenarios are typical in scenarios where the SUs have also social ties implemented via a dedicated control channel. Under conditions that the SUs can make their decision only on the basis of local observations that can be misinterpreted, we obtained sufficient conditions of convergence to the consensus. Our numerical results showed that propagation conditions affect significantly the validity of the derived sufficient conditions.

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