LRS-G^2 Based Non-parametric Spectrum Sensing for Cognitive Radio

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Abstract. In this paper, a novel non-parametric spectrum sensing scheme in cognitive radio (CR) is proposed based on robust Goodness of Fit (GoF) test. The proposed scheme uses likelihood ratio statistics (LRS-G^2), from which goodness of fit test is derived. The test is applied assuming different types of primary user (PU) signals such as static or constant, single frequency sine wave and Gaussian signals, whereas different types of channels such as additive white Gaussian noise (AWGN), block fading and time-varying channels. Considering a real time scenario, uncertainty in noise variance is also assumed. The performance of the proposed scheme is shown using receiver operating characteristics (ROC) and it is compared with energy detection (ED) and prevailing GoF based sensing techniques such as Anderson-Darling (AD) sensing, Order Statistic based sensing and Kolmogrov-Smirnov (KS) sensing. It is shown that the proposed scheme outperforms all these prevailing schemes.

Keywords: Spectrum sensing · Goodness of fit test · Likelihood ratio statistic · Noise uncertainty · Time-varying channel (AR1)

1 Introduction

The opportunistic spectrum access plays an important role to improve spectral efficiency in wireless communications. It becomes achievable using cognitive radio (CR) [1]. One of the most important task in CR is spectrum sensing in which the presence of licensed user, also known as a primary user (PU), is to be detected. If PU is absent in the spectrum, then unlicensed user, also known as a secondary user (SU), can use the spectrum. However, SU has to vacate the spectrum as soon as PU becomes active. Therefore, spectrum sensing technique should take less time with higher detection accuracy. However, the spectrum sensing function is suffered by various factors such as multi-path fading, receiver's uncertainty, interference, etc. Hence, design of a spectrum sensing algorithm is a challenging problem [2,3].

There are two categories for spectrum sensing. First is parametric sensing in which CR uses some known information of PU to sense its presence. In the second

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category of non-parametric sensing, CR does not have any information about PU. A latter approach is realistic in the current scenario of wireless communications as different PU use distinct bandwidth, modulation and coding schemes. In this category, energy detection (ED) based sensing [4] is the simplest one for spectrum sensing due to its low complexity. To improve the performance of ED sensing, antenna diversity [5] or modified ED [6] is used. However, the assumption of having perfect information about distribution of noise at the CR becomes very crucial at the low signal-to-noise ratio (SNR) of the PU signal or time varying nature of wireless channel. In such circumstances, the performance of the ED degrades drastically and results in SNR wall [7]. Therefore, it is of interest to develop a non-parametric sensing algorithm, which provides better performance at low SNR with less number of observations and false alarm probabilities.

Recently, some goodness of fit (GoF) based sensing schemes have been proposed in the category of non-parametric sensing. In this kind of sensing, we determine cumulative distribution function (CDF) of the received observations. This empirical CDF is compared with known CDF of noise, or we test the null hypothesis (H_0), where H_0 denotes absence of PU. Deviation of empirical CDF from the known CDF of noise (F_0) decides presence of PU or hypothesis H_1 [8]. Based on this, [9] has proposed Anderson Darling (AD) sensing, which outperforms ED based sensing at low SNR assuming an additive white Gaussian noise (AWGN) channel. In [10], ordered statistics (OS) based sensing has been proposed. This method outperforms both AD and ED based sensing at low SNR. Furthermore, based on Kolmogorov-Smirnov (KS) GoF test, [11] has proposed KS sensing. The KS sensing outperforms ED based sensing in AWGN channel. In addition to this, based on sequential KS test, [12], has proposed sequential KS sensing scheme in dispersive MIMO channel. In this paper, we propose new GoF based sensing scheme called as likelihood ratio statistics (LRS-G^2), which outperforms OS, AD, KS and ED sensing in AWGN channel with Gaussian noise assumption under H_0.

The above-mentioned papers on GoF have assumed PU as a constant signal in AWGN channel. However, [13] has investigated performance of AD sensing with different PU signals such as independent and identically distributed ($i.i.d$) Gaussian and single frequency sine signals. Under both these PU signals, ED sensing outperforms the AD sensing. We will show that the proposed LRS-G^2 scheme outperforms ED sensing in this condition also.

The assumption of known variance of noise is very crucial in ED sensing. The change of noise variance deteriorates the performance of ED sensing method. In [14], blind AD sensing scheme has been proposed in block fading channel with constant PU signal. This scheme does not require any information about variance of noise. This blind AD outperforms ED sensing significantly. Our proposed scheme without having knowledge of variance of noise, we call it as Blind LRS-G^2, outperforms Blind AD and ED based sensing methods.

In [14], GoF based sensing has been used assuming a quasi-static channel. However, in a practical scenario, the channel is time-varying. Hence, it is of interest to evaluate the performance of GoF test in a time-varying channel. We have shown performance of the proposed scheme assuming a time-varying channel which is modeled using autoregressive (AR) process. The proposed scheme shows significant improvement in the performance compared to AD and ED sensing.

In [15], authors have proposed likelihood ratio test based sensing scheme under AWGN channel environment. They proposed sensing scheme which outperforms AD and ED based sensing for the system model proposed in [9]. In this paper, we propose a GoF sensing based on a robust normality test [16]. The authors of the paper have used different weighting functions in quadratic equation of the test statistic, called as Zhang's statistic (Z_c), and proposed powerful omnibus GoF test for the Gaussian distribution under H_0. It gives the highest statistical power in comparison with the other GoF test such as AD, KS and Cramer-von-Mises(CvM) tests.

The sampling distribution of the Zhang statistic (Z_c), is mathematically intractable, so it is unattainable to derive the close form expression of the false alarm probability (P_f) and probability of detection (P_d). Hence, we use extensive Monte Carlo Simulations to evaluate the sensing performance of the proposed scheme. We have shown that the proposed LRS-G^2 outperforms all the available GoF based sensing methods and ED sensing method in various scenarios such as different structures of PU, different channel conditions and unknown variance of noise.

The rest of the paper is organized as follows. Section 2 presents the system model and the problem of spectrum sensing as GoF testing using LRS is formulated in Sect. 3. In Sect. 4, the LRS-G^2 sensing algorithm is proposed under known and unknown assumption of noise uncertainty. The simulation results are presented in Sect. 5. Finally, the paper is concluded in Sect. 6.

2 System Model

Let $\mathbf{y} = [y_1, y_2, ..., y_n]^T$ be a vector of n observations of PU, received at CR, where $n \geq 1$. We assume that all the received observations are real as considered in [9,10,14], and each y_i is represented as,

$$y_i = \sqrt{\rho} h_i s_i + w_i, \quad i = 1, 2, 3, \cdots\cdots n, \tag{1}$$

where ρ is the received SNR, h_i represents the channel coefficient. In (1), $w_i \sim \mathcal{N}(0, \sigma^2)$, where $1 \leq i \leq n$, denotes gaussian noise samples and s_i denotes symbol of PU, which can be assumed as constant one or $i.i.d.$ Gaussian as $s_i \sim \mathcal{N}(0, 1)$ or single frequency sine signal as defined in [13]. The CDF of w_i is denoted by $F_0(w)$. The PU signal as a single carrier frequency (f_c) in the discrete version of sine signal can be represented as,

$$s_i = \sqrt{2} sin\left(\frac{2\pi}{k} i + \theta\right), \tag{2}$$

where θ is an initial phase and $k = \frac{f_s}{f_c}$ is the ratio of the sampling frequency (f_s) to the carrier frequency (f_c). The value of k is assumed to be six. Without loss of generality, we assume that all n observations are in ascending order. It means $y_1 \leq y_2 \leq \cdot\cdot \leq y_n$.

We assume three different models for channel coefficient h_i.

- AWGN channel: In this case, h_i is assumed to be one and noise distribution is Gaussian with mean zero and variance σ^2.
- Block fading channel: In this case, $h_i \sim \mathcal{N}(0,1)$, however it remains constant during a block of n symbols.
- Time-varying channel: In this case, $h_i \sim \mathcal{N}(0,1)$, however it varies with time in a block of n symbols. This channel is generated using first ordered autoregressive ($AR1$) process,

$$h_i = ah_{i-1} + \sqrt{1-a^2}v_i, \quad 0 \leq a \leq 1 \tag{3}$$

where v_i denotes $i.i.d$ as Gaussian with mean zero and variance one. In (3), a indicates correlation coefficient between consecutive symbols i.e. $a = E[h^*_{i-1}h_i]$, where $E[\cdot]$ represents expectation operator. Here, $a = 1$ and $a = 0$ denote a constant (block fading) channel and an independent channel respectively. The value of a is determined using Jake's autocorrelation function [17] as $a = J_0(2\pi f_d T_s)$, where f_d and T_s denote doppler frequency in Hz and symbol time in seconds respectively.

3 Goodness of Fit Based Sensing Using Likelihood Ratio Statistics (LRS-G^2)

In GoF based sensing, we test the received observations whether they are drawn from null hypothesis (H_0) or not. We assume that the CDF of Gaussian noise under H_0 is known and denoted by $F_0(t)$. In literature, null hypothesis testing algorithms are classified in two ways, Pearson's Chi-squared test and empirical distribution function (EDF) test. The AD, KS and CvM tests are under the category of EDF tests. In [18], authors have proposed a new hypothesis test based on power divergence statistics for null-hypothesis testing as,

$$\begin{aligned} 2nI^\lambda = \frac{2n}{\lambda(\lambda+1)} \Bigg\{ & F_n(t)\left[\frac{F_n(t)}{F_0(t)}\right]^\lambda \\ & + [1-F_n(t)]\left[\frac{1-F_n(t)}{1-F_0(t)}\right]^\lambda - 1 \Bigg\} \end{aligned} \tag{4}$$

where, λ represents a parameter for selection of goodness of fit test, n and $F_n(t)$ denote number of received observations and empirical CDF respectively.

By selecting $\lambda = 1$, (4) represents Pearson's Chi-squared test statistics ($\mathbb{X}^2$) as,

$$\mathbb{X}^2 = \frac{n[F_n(t) - F_0(t)]^2}{F_0(t)[1-F_0(t)]} \tag{5}$$

and $\lambda = 0$, (4) represents Likelihood Ratio Statistics (LRS-$\mathbb{G}^2$) as,

$$\mathbb{G}^2 = 2n\left\{F_n(t)log\frac{F_n(t)}{F_0(t)} + [1 - F_n(t)]log\frac{1 - F_n(t)}{1 - F_0(t)}\right\} \tag{6}$$

In [16], authors have proposed a parametrization approach to construct a generalized omnibus GoF tests for a specified distribution (F_0) under hypothesis H_0 as normal distribution using different weight functions. They have proposed general test statistics called as Z statistics using,

$$Z = \int_{-\infty}^{\infty} z_t \; w(t) \; dt, \tag{7}$$

where z_t indicates a type of goodness of fit test statistics and $w(t)$ denotes weighting function. The power of any goodness of fit test depends on these two parameters z_t and $w(t)$.

Let $z_t = \mathbb{X}^2$ as shown in (5). Then, (7) can be expressed as

$$Z = \int_{-\infty}^{\infty} \frac{n[F_n(t) - F_0(t)]^2}{F_0(t)[1 - F_0(t)]} \; w(t) \; dt \tag{8}$$

Substituting the distinct weighting functions $w(t) = F_0(t)$, $w(t) = n^{-1}F_0(t)[1 - F_0(t)]$ and $w(t) = F_0(t)[1 - F_0(t)]$ in (8), the Z statistics represent AD, KS and CvM statistics respectively as discussed in [8]. Using these AD, KS and CvM statistics, different spectrum sensing schemes have been proposed in [9,11,12,19].

The authors of [16] have proposed powerful omnibus tests. To derive such test, they used LRS-$\mathbb{G}^2$ by substituting (6) into (7) in place of z_t,

$$\begin{aligned} Z &= \int_{-\infty}^{\infty} \mathbb{G}^2 \; w(t) \; dt \\ &= \int_{-\infty}^{\infty} 2n\left\{F_n(t)log\frac{F_n(t)}{F_0(t)} + [1 - F_n(t)]log\frac{1 - F_n(t)}{1 - F_0(t)}\right\}w(t)dt \end{aligned} \tag{9}$$

By using different weight functions ($w(t)$) in (9) as mentioned below, Z produces Z_k, Z_a and Z_c statistics called as Zhang's omnibus statistics.

For $w(t) = 1$, Z approaches Z_k statistic, which is expressed as

$$Z_k = \max_{1 \le i \le n}\left(\left(i - \frac{1}{2}\right) log\left\{\frac{i - \frac{1}{2}}{nF_0(y_{(i)})}\right\} + \left(n - i + \frac{1}{2}\right) log\left\{\frac{n - i + \frac{1}{2}}{n\left\{1 - F_0(y_{(i)})\right\}}\right\}\right) \tag{10}$$

For $w(t) = F_n(t)^{-1}\left\{1 - F_n(t)\right\}^{-1}$, Z approaches Z_a statistic, which is expressed as

$$Z_a = -\sum_{i=1}^{n}\left[\frac{log\left\{F_0(y_{(i)})\right\}}{n - i + \frac{1}{2}} + \frac{log\left\{1 - F_0(y_{(i)})\right\}}{i - \frac{1}{2}}\right] \tag{11}$$

For $w(t) = F_0(t)^{-1}\{1 - F_0(t)\}^{-1}$, Z approaches Z_c statistic, which is expressed as

$$Z_c = \sum_{i=1}^{n} \left[log \left\{ \frac{F_0(y_{(i)})^{-1} - 1}{(n - \frac{1}{2})/(i - \frac{3}{4}) - 1)} \right\} \right]^2 \tag{12}$$

We choose above mentioned statistics and use it for hypothesis testing considering different conditions for channels and PU. The effect of the different Zhang statistics [16] on the detection performance of SU are discussed in Sect. 5.

4 LRS-G^2 Spectrum Sensing Algorithm

The problem of spectrum sensing as a null-hypothesis testing problem is defined as [9],

$$\begin{aligned} H_0 &: F_Y(y) = F_0(y) \\ H_1 &: F_Y(y) \neq F_0(y) \end{aligned} \tag{13}$$

For LRS-G^2 sensing, we use statistics defined in (12) to measure distance between $F_Y(y)$ and $F_0(y)$. Let $F_n(y)$ be the empirical cumulative distribution function (ECDF) of the received observations which can be expressed as,

$$F_n(y) = \frac{|\{i - \frac{1}{2} : y_i \leq y, 1 \leq i \leq n\}|}{n} \tag{14}$$

where $|.|$ indicates cardinality.

4.1 LRS-G^2 Sensing Without Noise Uncertainty

We assume that the noise power is known a priori. The noise under H_0 is $w_i \sim \mathcal{N}(0, \sigma^2)$. Here, we assume that $\sigma^2 = 1$.

First, for the detection of PU at the CR, the value of threshold (ξ) is selected so that the false alarm probability (P_f) is at a desired level (α) as,

$$\alpha = \mathbb{P}\{\ Z_c > \xi | H_0\} \tag{15}$$

To find ξ, it is worth mentioning that the distribution of Z_c under H_0 is independent of the $F_0(y)$. Hence, after applying the probability integration transform (PIT) for available observations,

$$\begin{aligned} Z_c = \int_0^1 2n \Bigg\{ F_Z(z) log \frac{F_Z(z)}{z} + [1 - F_Z(z)] \\ \times \log \frac{1 - F_Z(z)}{1 - z} \Bigg\} z^{-1} \{1 - z\}^{-1} dz, \end{aligned} \tag{16}$$

where $z = F_0(y)$ and $F_Z(z_i)$ denotes ECDF of the transformed observations z_i, where $z_i = F_0(y_i)$ for $1 \leq i \leq n$. All statistics of observations are independent and uniformly distributed over $[0,1]$. As shown in [9] for AD sensing, the distribution of A^2 is independent of the $F_0(y)$. The same is also true for the distribution of Z_c. As given in [16], the value of ξ is determined for a specific value of P_f. For example, when $P_f = 10^{-3}$ and $n = 50$, then the value of ξ is 31.707.

Second, sort all the received observations in ascending order. Then, we get

$$y_{(1)} \leq y_{(2)} \leq \cdot\cdot \leq y_{(n)}. \tag{17}$$

Third, calculate the test statistics (Z_c) using (12) as,

$$Z_c = \sum_{i=1}^{n} \left[log \left\{ \frac{u_i^{-1} - 1}{(n - \frac{1}{2})/(i - \frac{3}{4}) - 1)} \right\} \right]^2 \tag{18}$$

where $u_i = F_0(y_{(i)})$.

At last, compare the value of (18) with ξ. If $Z_c > \xi$, then reject the null hypothesis H_0 in favor of the presence of PU signal. Otherwise, declare that the PU is absent. Compute performance metric as Probability of Detection (P_d) with a given value of P_f. Furthermore, the detection probability (P_d) is computed theoretically as,

$$\begin{aligned} P_d &= \mathbb{P}\{\ Z_c > \xi | H_1\} \\ &= 1 - F_{Z_c, H_1}(\xi) \end{aligned} \tag{19}$$

4.2 LRS-G^2 Sensing with Noise Uncertainty

In this case, LRS-G^2 sensing method is used considering an uncertainty in the variance of noise, we call it Blind LRS-G^2 sensing.

Recently, [14] has proposed the Blind AD sensing method, where noise uncertainty was considered. Authors of the papers have considered the spectrum sensing problem as Student's t-distribution testing problem. We have used the same approach by replacing AD test with the proposed Zhang test in LRS-G^2 sensing. The summary of the algorithm is as follows:

Step:1 Select an integer m, where $m > 1$ and it is a factor of n. Divide all the samples $Y = \{y_i\}_{i=1}^{n}$ into $g = \frac{n}{m}$ groups, where m number of received observations are there in one group [14].
Step:2 For the j^{th} group ($j = 1, 2, 3 \cdots\cdots g$), calculate T_j,

$$T_j = \frac{\overline{Y_j}}{S_j/\sqrt{m}}, j = 1, 2 \cdots, g \tag{20}$$

where $\overline{Y_j}$ is mean and S_j^2 is variance of the received observations in the j^{th} group,

$$\overline{Y_j} = \sum_{k=0}^{m-1} \frac{Y_{mj-k}}{m} \ and \ S_j^2 = \sum_{k=0}^{m-1} \frac{(Y_{mj-k} - \overline{Y_j})^2}{m} \tag{21}$$

Step:3 Find the threshold ξ for a given probability of false alarm P_f using (15).
Step:4 Sort T_j in ascending order. Hence, we get

$$T_{(1)} \leq T_{(2)} \leq \cdot\cdot \leq T_{(g)}$$

Step:5 Calculate the required test statistic Z_c for each group as shown in (18) by replacing $y_{(i)}$ by $T_{(j)}$.
Step:6 If $Z_c < \xi$, then reject null hypothesis H_0 i.e. If $T_j \sim N(0, \sigma^2)$, then T_j is Student's t-distributed variable with $m-1$ degrees. It shows the absence of PU. Compute P_d for the fixed value of P_f. Repeat the above-mentioned steps for other values of P_f.

5 Simulation Results

In this section, we have presented receiver operating characteristics (ROC) i.e. plot of P_d versus P_f for different values of SNR for the proposed LRS-G^2 sensing method using simulations. We have also presented P_d versus SNR for lower values of P_f. We have considered three types of channels such as AWGN, block fading and time-varying channels using auto regressive process ($AR1$). model. We have also considered three types of PU such as constant, single frequency sine wave and $i.i.d$ Gaussian with mean zero.

In AWGN channel environment, Z_c, Z_k and Z_a provide similar detection performance. So, we choose the Z_c statistic for taking decision at secondary user (SU). However, in fading channel, Z_k statistic provides better performance Therefore, we choose Z_k statistic for block fading and time varying channel. Furthermore, we have considered the noise uncertainty and shown its effect on detection performance by varying SNR.

Finally, we have compared all our results with prevailing GoF sensing such as AD, KS, OS and ED schemes.

Figure 1 shows the ROC for the proposed LRS-G^2 method in comparison with prevailing GOF sensing schemes at SNR $= -4$ dB, $n = 30$ and constant PU signal. It can be seen that the proposed technique outperforms all under AWGN channel. To observe the performance of the proposed scheme at lower value of P_f such as 0.01, we have shown P_d versus SNR with $n = 30$ under AWGN channel in Fig. 2. At SNR $= -8dB$, the detection probabilities of 0.7293, 0.5505, 0.4026, 0.3206 and 0.0195 are achieved for LRS-G^2, KS, OS, AD and ED sensing respectively.

Considering the PU signal as a discrete sinusoidal signal or independent and identically distributed Gaussian signal [13], Fig. 3 shows ROC for the proposed scheme along with AD and ED sensing at an SNR of -5dB and $n = 30$. It can be seen that the proposed scheme outperforms both the AD and ED sensing in both the PU signals. Furthermore, it can be seen that the ED sensing outperforms GoF based AD sensing, however proposed GoF based LRS-G^2 scheme outperforms ED sensing. It proves that the LRS-G^2 scheme is robust against the nature of PU signal.

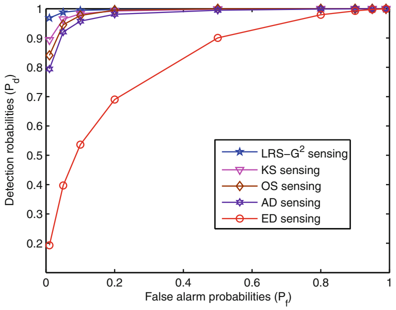

Fig. 1. ROC for different sensing schemes in AWGN channel for constant PU signal at SNR $= -4$dB and $n = 30$.

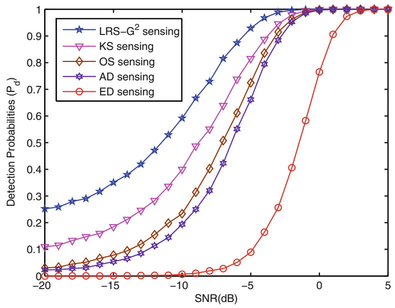

Fig. 2. P_d versus SNR for different sensing schemes in AWGN channel for constant PU signal at $P_f = 0.01$ and $n = 30$.

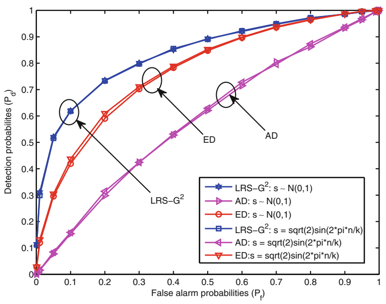

Fig. 3. ROC with different types of PU signals at SNR $= -5$dB and $n = 30$ in AWGN channel.

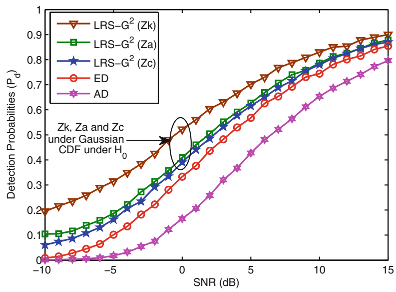

Fig. 4. P_d versus SNR in block fading channels with PU Signal as single frequency sine signal at $P_f = 10^{-3}$.

So far, we have shown performance of the proposed scheme in AWGN channel with different PU signals. Further, in Fig. 4, the detection performance of LRS-G^2 is shown under block fading channel with PU signal as single frequency sine signal with $n = 30$. We have also presented performance for LRS-G^2 sensing taking all Zhang test statistics as derived in [16]. The ED outperforms AD sensing. Interestingly, we can observe that the LRS-G^2 with Z_k, Z_a and Z_c outperform ED and AD sensing under fading environment.

Now, we consider blind LRS-G^2 with uncertainty in noise, i.e. the noise variance (σ^2) is unknown. We assume that the channel (h) is block fading and PU signal is constant [14]. In Fig. 5, we have shown P_d versus SNR for $P_f = 0.05$ with $m = 4$ and $n = 32$. It can be seen that uncertainty in noise degrades the performance as expected. We have also presented performance of AD sensing and blind AD sensing (for $m = 4$ and $m = 2$) along with performance of ED sensing with known variance of noise. It can be seen that the blind LRS-G^2 outperforms AD and ED sensing with known variance also. In Fig. 6, we have shown ROC for the proposed scheme assuming PU signal as a single frequency sine signal and channel is time-varying modeled by $AR1$ process. The ROC for

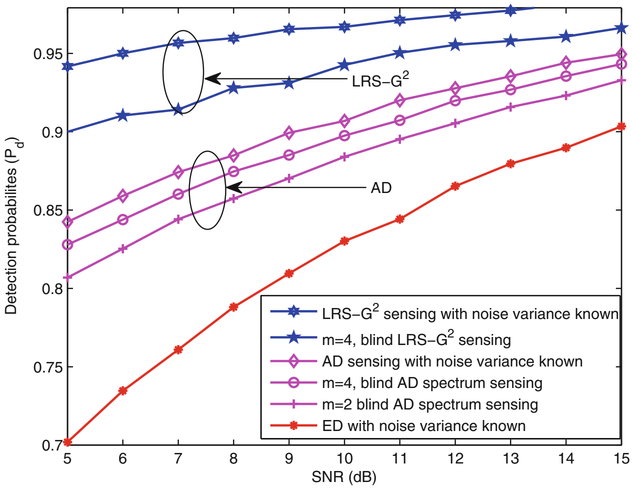

Fig. 5. P_d versus SNR in block fading channels with noise uncertainty for constant PU signal at $P_f = 0.05$ and $n = 32$.

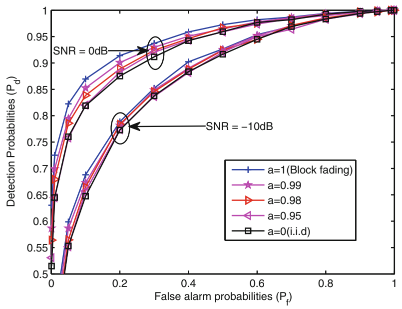

Fig. 6. ROC for LRS-G^2 sensing with different correlation coefficient (a) at $n = 30$ in time-varying channel.

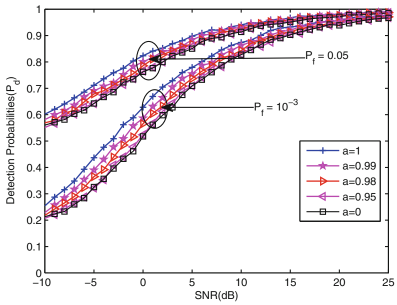

Fig. 7. P_d versus SNR for LRS-G^2 sensing with different values of correlation coefficient (a) at $n = 30$.

LRS-G^2 sensing is presented for different values of correlation coefficient (a) such as 1, 0.99, 0.98, 0.95, 0 at $n = 30$ and SNR of 0dB and -10dB. It can be seen that performance improves as the value of a increases towards unity. In Fig. 7, we have shown P_d versus SNR for $P_f = 0.05$, 0.001 for the same values of n and a. From the results, shown in Figs. 6 and 7, we can say that LRS-G^2 sensing improves P_d when the channel is block faded ($a = 1$). However, as the value of a decreases, the performance degrades as the channel becomes time-varying.

6 Conclusion

In this paper, a novel non-parametric spectrum sensing scheme based on likelihood ratio statistics using goodness of fit test has been proposed. The detection performance is presented using ROC assuming various types of primary user signals as well as different channel conditions. Furthermore, the adverse effect of noise uncertainty is also shown on the performance. The ROC for ED and prevailing GoF based sensing schemes such as AD, OS and KS are compared with the proposed one. The ED based sensing usually outperforms traditional GoF based sensing schemes when PU signal is not static. However, the proposed GoF based scheme outperforms ED as well as all these GoF based sensing. In case of time-varying channel, the performance of the proposed scheme degrades as the channel changes from slow time varying to fast time varying.

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