A Novel Sequential Phase Difference Detection Method for Spectrum Sensing

Shaojie Liu(✉), Zhiyong Feng, Yifan Zhang, Sai Huang, and Dazhi Bao

Key Laboratory of Universal Wireless Communications, Ministry of Education, Wireless Technology Innovation Institute (WTI), Beijing University of Posts and Telecommunications, Beijing 100876, People's Republic of China
liushaojie@bupt.edu.cn

Abstract. As traditional spectrum sensing approaches in cognitive radio network unable to deal with the contradiction between accuracy and complexity, a novel sequential spectrum detection based on phase difference (SPDD) is proposed in this paper to achieve good performance with less complexity. The variance of phase difference of signal is utilized as the statistics to detect the signal under a realistic Rayleigh fading channel. Moreover, a variable sample size of proposed algorithm is conducted to minimize the complexity while maintained an acceptable performance. Simulation shows that our SPDD method yields about 2 dB gain over the conventional sequential energy detection. In addition, when the cutoff sample number is set to 1000, a substantial efficiency improvement is obtained compared to the fixed sample detection scheme.

Keywords: Cognitive radio · Spectrum sensing · Phase difference · Sequential detection · Gaussian noise

1 Introduction

With the increasing scarcity of spectrum resources, measurement shows the average utilization rate of current spectrum below 3 GHz is merely 5.2 %, which unveils that the spectrum resources are heavily underutilized [1]. Cognitive Radio (CR) is proposed to sense radio environment and utilize vacant spectrum to improve the spectrum utilization [2]. The most important function in CR is to determine whether the primary user (PU) is present or not, which is called spectrum sensing. If CR determines the PU is absent, then the secondary user (SU) can access the licensed bands. The process of detection of spectrum holes is the key enabler for efficient spectrum utilization in CR.

Various spectrum sensing approaches have been proposed in previous literatures, such as energy detection (ED) [3], cyclostationarity based detection [4], matched filter detection [5], multitaper spectrum estimation [6] and sequential energy detection [7]. Matched filtering detection has the optimal performance while detailed information of PU signal such as pulse shaping is required. In practice, it is generally impractical to get priori feature information of PU.

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Cyclostationarity based detection does not require detailed information of PU signal and has robust performance under the low signal to noise ratio (SNR). However, the high computational complexity restricts its widespread usage on energy-constrained devices. For the simplicity of energy detection (ED), it was popularized in the context of IEEE 802.22 cognitive radio networks [8]. However, a longer length of samples of ED is necessary to maintain an acceptable performance at a low SNR. There implements a sequential approach to energy detection to deliver a significant improvement compared with the fixed sample size detection, which is called sequential energy detection (SED) [7]. Since the thresholds setting of SED cannot determined without noise power, we must estimate the noise in advance. Thus it will bring complexity increase and performance deterioration because of the noise uncertainty. Moreover, it also suffers from huge performance degradation at low SNR. In [9] we have proposed a phase difference sensing method, which improves the detection performance compared to energy detection. Simultaneously, it needs a large fixed number samples for detection and thus the complexity increases inevitably.

To solve the contradiction between complexity and accuracy, this paper formulates a novel sequential phase difference detection method (SPDD). In [10,11] R.F. Pawula and F. Adachi have derived the phase difference distribution of the noise-perturbed signal. We have known that there is a obvious difference in the phase difference distribution between noise-perturbed signal and Gaussian noise through plenty of researches [9]. The sequential test sensitivity to the primary signal phase difference addressed in this paper can obviously yield great performance improvement and have a promising future. In addition, compared to the conventional sequential energy detection, the SPDD sensing scheme is immune to the noise uncertainty because the noise power is not required to set the thresholds.

The rest of the paper is organized as follows. In Sect. 2, we formulate the model of spectrum sensing and phase difference distributions of the noise-perturbed signal and Gaussian noise. The algorithm of SPDD is described and the corresponding performance is analyzed in Sect. 3. We provide the simulation results in Sect. 4 and conclude in Sect. 5.

2 System Model and Phase Difference

2.1 System Model

The spectrum sensing problem can be formulated as per Eq. (1) for $n = 1, 2...$

$$\begin{aligned} H_1: &\quad r(n) = hs(n) + w(n) \\ H_0: &\quad r(n) = w(n). \end{aligned} \tag{1}$$

where, $r(n)$ is the received signal, h is the instantaneous channel gain, $s(n)$ is the transmitted signal of PU, and $w(n)$ is the Additive White Gaussian Noise (AWGN). H_1 represents that the PU signal is present while H_0 indicates that there is only noise. It is assumed that $s(n)$ is independent identically distributed and $s(n)$ and $w(n)$ are mutually independent.

Instead of the amplitude squares of the received signal as test statistics in conventional sequential energy detection, we focus on the phase difference between two adjacent samples of received signal. We propose a Sequential Probability Ratio Test (SPRT) formulation of the phase difference for detection. Unlike the most sensing method which always have a fixed number of samples to be received before calculating the test statistic, here the samples will be received sequentially, and the likelihood ratio $T(\widetilde{\Delta\theta_n})$ can be calculated as

$$T(\widetilde{\Delta\theta_n}) = \frac{f(\widetilde{\Delta\theta_n}|H_1)}{f(\widetilde{\Delta\theta_n}|H_0)}, \tag{2}$$

where $\Delta\theta_n$ means phase difference between two adjacent samples, $\widetilde{\Delta\theta_n} = [\Delta\theta_1 \Delta\theta_2 ... \Delta\theta_n]$. As $\Delta\theta_i$ is independent identically distributed under H_1 and H_0 hypotheses, we can deduce

$$T(\widetilde{\Delta\theta_n}) = \prod_{i=1}^{n} \frac{f(\Delta\theta_i|H_1)}{f(\Delta\theta_i|H_0)}. \tag{3}$$

The explanation for the independence of the $\Delta\theta_i$ will be given later.

2.2 Phase Difference

In [9] the phase θ_n of the received signal $r(n)$ can be calculated as

$$\theta'_n = \begin{cases} \arctan \frac{\mathrm{Im}(r(n))}{\mathrm{Re}(r(n))}, & \mathrm{Re}(r(n)) \geq 0 \\ \arctan \frac{\mathrm{Im}(r(n))}{\mathrm{Re}(r(n))} + \pi, & \mathrm{Re}(r(n)) < 0, \end{cases} \tag{4}$$

$$\theta_n = \theta'_n \bmod 2\pi. \tag{5}$$

Where $\mathrm{Re}(r(n))$ and $\mathrm{Im}(r(n))$ mean the real and imaginary part of $r(n)$ respectively, and $(\bullet) \bmod 2\pi$ can make the phase θ fall between 0 and 2π. Then, the phase difference $\Delta\theta$ mentioned in Eq. (2) can be obtained as

$$\Delta\theta_n = (\theta_{n+1} - \theta_n) \bmod 2\pi. \tag{6}$$

2.3 Phase Difference Distribution

We know that the phase θ_n of Gaussian noise follows a uniform distribution, which means $\theta_n \sim U(0, 2\pi)$. According to our assumption, the instantaneous phases of Gaussian noise are all mutually independent and also identically distributed. Then $\Delta\theta'_n = \theta_{n+1} - \theta_n$ follows a triangular distribution from -2π to 2π, which can be shown that

$$f_{\Delta\theta'_n}(\Delta\theta'_n) = \begin{cases} \frac{1}{2\pi} + \frac{\Delta\theta'_n}{4\pi^2}, & -2\pi \leq \Delta\theta'_n < 0 \\ \frac{1}{2\pi} - \frac{\Delta\theta'_n}{4\pi^2}, & 0 \leq \Delta\theta'_n \leq 2\pi. \end{cases} \tag{7}$$

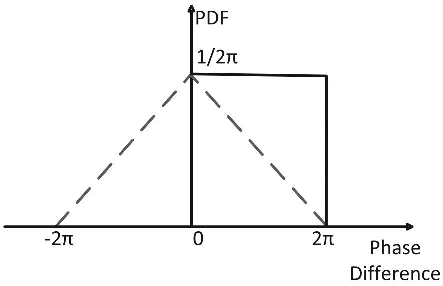

Fig. 1. Distribution of phase difference for Gaussian noise

Since $\varDelta\theta_n = (\varDelta\theta'_n) mod 2\pi$, we can express the distribution of phase difference of Gaussian noise $f^n_{\varDelta\theta_n}(\varDelta\theta_n)$ as

$$f^n_{\varDelta\theta_n}(\varDelta\theta_n) = f_{\varDelta\theta'_n}(\varDelta\theta_n) + f_{\varDelta\theta'_n}(\varDelta\theta_n - 2\pi) = \frac{1}{2\pi}. \tag{8}$$

As shown in Fig. 1, it indicates the phase difference $\varDelta\theta \sim U(0, 2\pi)$. It seems that $\varDelta\theta_{n+1}$ and $\varDelta\theta_n$ may be correlated because both of them refer to θ_{n+1}, but the correlation between them can be eliminated after $(\bullet)mod$. So we assume $\varDelta\theta_i$ is independent identically distributed, and this assumption matches very well with our simulation results. The phase difference cumulative distribution of the signal perturbed by Gaussian noise has been given as formula (17) in [11], the formula can be written as

$$F_{\varDelta\theta_n}(\varDelta\theta_n) = \frac{1}{4\pi}\int_{-\pi/2}^{\pi/2} e^{-E}\left[\frac{W\sin\alpha}{E} + Q\right]dt, \tag{9}$$

where,

$$E = U - V\sin t - W\cos\beta\cos t$$

$$U = \frac{1}{2}(SNR_{n+1} - SNR_n)$$

$$V = \frac{1}{2}(SNR_{n+1} + SNR_n)$$

$$W = \sqrt{SNR_{n+1}SNR_n} = \sqrt{U^2 - V^2}$$

$$Q = \frac{\rho\sin\varDelta\theta_n - \lambda\cos\varDelta\theta_n}{1 - (\rho\cos\varDelta\theta_n + \lambda\sin\varDelta\theta_n)\cos t}$$

$$\alpha = (\varDelta\phi_n - \varDelta\theta_n) mod 2\pi$$

$\varDelta\phi_n$ represents the phase difference between nth and $(n+1)$th sampling point, $\rho + \lambda i$ means complex correlation of Rayleigh fading signal and noise. It should be noted that SNR_n is the instantaneous SNR of the nth received sampling signal, and SNR_n will not change for phase modulation through amplitude and QAM modulation. The received sample signal is considered to be constant, thus we can assume $SNR_{n+1} = SNR_n = \gamma$.

In addition, the phase difference can be assumed to be independent with each other just as noise. For received continuous wave, if the Gaussian noise is without fading, $Q = 0$. Here we discuss the case of Rayleigh fading, thus $\rho + j\lambda = \sqrt{\rho^2 + \lambda^2} e^{j\Delta\phi_n} = \frac{\gamma e^{j\Delta\phi_n}}{\gamma+1}$. The performance of SPDD algorithm shows significant improvement in comparison to the conventional algorithm at extremely low SNR, then we derive $e^{-E} \approx 1$. Therefore, the form of $F_{\Delta\theta_n}(\Delta\theta_n)$ can be rewritten as

$$\begin{aligned} & F_{\Delta\theta_n}(\Delta\theta_n) \\ &= \frac{1}{4\pi}\int_{-\pi/2}^{\pi/2}\left[\frac{\sin\alpha}{1-\cos\alpha\cos t} + \frac{\gamma\sin\alpha}{\gamma(1-\cos\alpha\cos t)+1}\right]dt \\ &= \frac{\sin\alpha}{\pi|\sin\alpha|}\arctan\left|\cot\frac{\alpha}{2}\right| \\ &\quad + \frac{\sin\alpha}{\pi\sqrt{\left(1+\frac{1}{\gamma}\right)^2 - cos^2\alpha}}\arctan\sqrt{\frac{(\gamma+1)+\gamma\cos\alpha}{(\gamma+1)-\gamma\cos\alpha}}. \end{aligned} \tag{10}$$

After simplifying, we have

$$F_\alpha(\alpha) = \frac{1}{2} + \frac{\alpha}{2\pi} + \frac{\sin\alpha G(\alpha)}{2\pi H(\alpha)}, \tag{11}$$

in which,

$$G(\alpha) = \frac{\pi}{2} + \arcsin\frac{\gamma\cos\alpha}{\gamma+1}$$

$$H(\alpha) = \sqrt{\left(1+\frac{1}{\gamma}\right)^2 - \cos^2\alpha}$$

Since $\alpha = (\Delta\phi_n - \Delta\theta_n) \bmod 2\pi$, we can use the formula Eq. (11) to get the derivation. Finally, the distribution of phase difference of the received signal perturbed by Gaussian noise can be obtained as

$$\begin{aligned} f^s_{\Delta\theta_n}(\Delta\theta_n) = & \frac{1}{2\pi} + \frac{\cos\alpha G(\alpha)}{2\pi H(\alpha)} - \frac{\cos\alpha sin^2\alpha G(\alpha)}{2H^3(\alpha)} \\ & - \frac{\gamma sin^2\alpha}{2\pi(\gamma+1)H(\alpha)\sqrt{1-\frac{\gamma^2\cos^2\alpha}{(\gamma+1)^2}}}. \end{aligned} \tag{12}$$

Here $\Delta\phi_{\mathrm{n}} = \frac{\pi}{2}$, which means that sampling rate is set to four times of the residual carrier frequency. It is obvious that the phase difference distribution of the received signal perturbed by Gaussian noise is quite different with that of noise, which can be utilized to detect PU signal sequentially. As shown in Fig. 2, the curves represent the phase difference distribution when SNR is 0 dB, −5 dB, −10 dB. With the decrease of SNR, the distribution of phase difference of pure signal converges to linear distribution $\frac{1}{2\pi}$, which is the distribution of phase difference for Gaussian noise. This indicates the proposed SPDD is not only reasonable in theory but also feasible in practice.

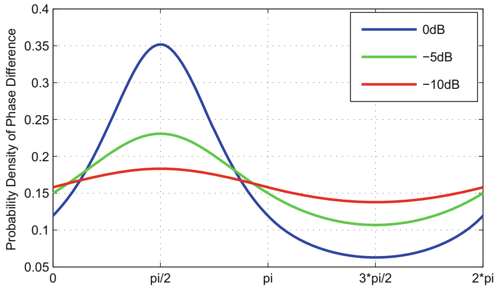

Fig. 2. Phase difference distribution (Color figure online)

3 SPDD Algorithm and Analysis

3.1 SPDD Algorithm

Through the description above, the distribution of phase difference between signal perturbed by noise and Gaussian noise is completely different. Therefore, we propose a novel SPDD algorithm to improve detection efficiency. It is assumed that the received signal is perturbed by noise and signal Rayleigh fading is slow, then using formula (3), (8), (12) we rewrite the likelihood ratio $T(\widetilde{\Delta\theta_n})$ as

$$T(\widetilde{\Delta\theta_n}) = \prod_{i=1}^{n} \frac{f^s_{\Delta\theta_i}(\Delta\theta_i)}{f^n_{\Delta\theta_i}(\Delta\theta_i)} = (2\pi)^n \prod_{i=1}^{n} f^s_{\Delta\theta_i}(\Delta\theta_i). \tag{13}$$

where, $f^s_{\Delta\theta_i}(\Delta\theta_i)$ denotes the distribution of phase difference when the PU signal is present while $f^n_{\Delta\theta_i}(\Delta\theta_i)$ indicates that of only noise. The proposed SPDD calculates the likelihood ratio $T(\widetilde{\Delta\theta_n})$ sequentially, and thus the statistic test $T(\widetilde{\Delta\theta_n})$ is compared with two thresholds, the upper threshold B and the lower threshold A. If the likelihood radio is above B then PU is present, if it is below A then the PU is absent, else accept new samples. The decision rule can be given as

$$D = \begin{cases} H_1, & T(\widetilde{\Delta\theta_n}) \geq B \\ Accept\ New\ Sample & A < T(\widetilde{\Delta\theta_n}) < B \\ H_0, & T(\widetilde{\Delta\theta_n}) \leq A. \end{cases} \tag{14}$$

where D means the sensing decision, and the SPDD algorithm is described as the following **Algorithm** 1.

We can iteratively update the $T(\widetilde{\Delta\theta_n})$ as

$$\begin{aligned} \ln T(\widetilde{\Delta\theta_{n+1}}) &= (n+1)\ln(2\pi) + \sum_{i=1}^{n+1} \ln f^s_{\Delta\theta_i}(\Delta\theta_i) \\ &= n\ln(2\pi) + \sum_{i=1}^{n} \ln f^s_{\Delta\theta_i}(\Delta\theta_i) + \ln(2\pi) + \ln f^s_{\Delta\theta_{n+1}}(\Delta\theta_{n+1}) \\ &= \ln T(\widetilde{\Delta\theta_n}) + \ln(2\pi) + \ln f^s_{\Delta\theta_{n+1}}(\Delta\theta_{n+1}). \end{aligned} \tag{15}$$

Algorithm 1. Sequential Phase Difference Detection Algorithm

Require: The decision threshold A and B
Ensure: $D \in H_0, H_1$
1: Calculate the distribution of phase difference of the received sample signal using the formula(12);
2: Calculate $T(\widetilde{\Delta\theta_n})$ to get the decision D by the formula (13), (14);
3: **if** $T(\widetilde{\Delta\theta_n}) \geq B$ **then**
4: $H_1 \leftarrow D$,declaring PU is present;
5: **else**
6: **if** $T(\widetilde{\Delta\theta_n}) \leq A$ **then**
7: $H_0 \leftarrow D$,declaring PU is absent;break
8: **else**
9: Accept new sample, go to step1 and update $T(\widetilde{\Delta\theta_n})$;
10: **end if**
11: **end if**

The formula (15) provides a natural simplification for SPDD implementation and avoids approximations caused by the threshold setting and others. The computational complexity to update $T(\widetilde{\Delta\theta_n})$ can be reduced substantially by using this iterative method as well.

3.2 Threshold Setting

These thresholds A and B are calculated by using the Wald approximations. [12] has proved that the SPRT has the minimum average expected sample size amongst the class of all sequential and fixed size likelihood ratio for a given fixed P_d and P_f, which provides a theoretical basis for our algorithm.

$$A = \frac{1 - P_d}{1 - P_f} \quad \& \quad B = \frac{P_d}{P_f}. \tag{16}$$

The P_d and P_f used to set the thresholds mean the detection probability and false alarm probability are called Design Values, which are set to fixed values before detection. The thresholds and the likelihood radio $T(\widetilde{\Delta\theta_{n+1}})$ are determined without the demand of noise power because the distribution of phase difference of noise is uniformly distributed despite of noise power. Therefore, the proposed SPDD is completely immune to the noise uncertainty problem, which means considerable superiority over the sequential energy detection.

3.3 Performance Analysis

The stopping sample size of the sequential phase difference detection is a random variable, and the relative performance gained by SPDD can be characterized by comparing the detection duration with the conventional fixed sample size detection. In the case of the hypotheses H_1, we can derive from [12] the following lower bounds for the expected values of the termination time as (18) where $\beta = 1 - P_d$,

$$\begin{aligned} P[\ln T(\widetilde{\Delta\theta_n}\,|H_1\,) \leq \ln A] = \beta, \\ P[\ln T(\widetilde{\Delta\theta_n}\,|H_1\,) \geq \ln B] = 1 - \beta. \end{aligned} \tag{17}$$

Let $\xi = \frac{f^s_{\Delta\theta}(\Delta\theta)}{f^n_{\Delta\theta}(\Delta\theta)}$, with the assumption of negligible overshoot of the test statistic, then the expectation of the average sample number can be approximatively given as

$$E\,[N|H_1] = \frac{1}{E[\ln\xi|H_1]}\left(\frac{A(B-1)}{B-A}\ln A + \frac{B(1-A)}{B-A}\ln B\right). \tag{18}$$

Similarly, in the case of the hypotheses H_0,

$$E[N|H_0] = \frac{1}{E_0[\ln\xi|H_0]}\left(\frac{B-1}{A-1}\ln A + \frac{1-A}{B-A}\ln B\right). \tag{19}$$

Using (8) and (12),

$$\begin{aligned} E[\ln\xi\,|H_1\,] = 2\pi E[\ln f^s_{\Delta\theta}(\Delta\theta)\,|H_1\,], \\ E[\ln\xi\,|H_0\,] = 2\pi E[\ln f^s_{\Delta\theta}(\Delta\theta)\,|H_0\,]. \end{aligned} \tag{20}$$

Unfortunately, as the complicated formulation of $f^s_{\Delta\theta}(\Delta\theta)$, an exhaustive mathematical formulation of the average sample size of SPDD is intractable. But we can get that the distribution of the stopping sample size depends on the P_d, P_f and the SNR of our sensing system. In practical detection, it may occur the situation that the sample size is too excessive to degrade the performance at extremely low probability. In order to avoid this situation, we set a cut-off number M, which is much larger than the average sample number. The result of SPDD will be decided as H_0 when the system sample number comes to M and the decision has not been made. Otherwise, SPDD only needs to store N samples. Thus the computational complexity is $O(N)$, which is a great advantage compared to other more sophisticated schemes such as cyclostationary detection.

4 Simulation Analysis

In this section, Monte Carlo Simulation is conducted to analyze the performance (2000 experiments for each point) of the proposed scheme. The influence of SNR, different kinds of modulated signals and Design Values on the performance of SPDD are analyzed carefully and rigorously here.

Figure 3 shows the relationship of detection probability P_d of SPDD with SNR for several basic modulation signals, when the Design Values $P_d = 0.9$, $P_f = 0.01$ and the cut-off sample number is set to 1000. It can be observed that the SNR of Sine Wave, BPSK, 16QAM and 2FSK are −9.3 dB, −9.2 dB, −9.2 dB and −8.5 dB correspondingly when the actual detection probability is 0.9. The P_d curves for kinds of modulated signals are similar, which demonstrates that SPDD

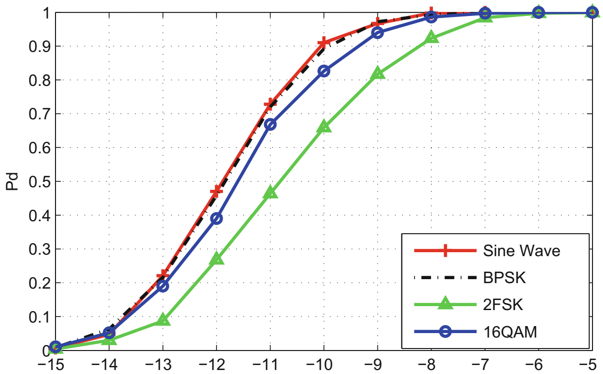

Fig. 3. Detection probability P_d for Sine Wave, BPSK, 16QAM and 2FSK (Color figure online)

is robust with respect to modulation mode. Generally, the sample symbols in one modulation are independently distributed and occur with an equal probability. Therefore, the opposite influence on phase difference of the sequences $r(i)r(j)$ and $r(j)r(i)$ can be offset by each other.

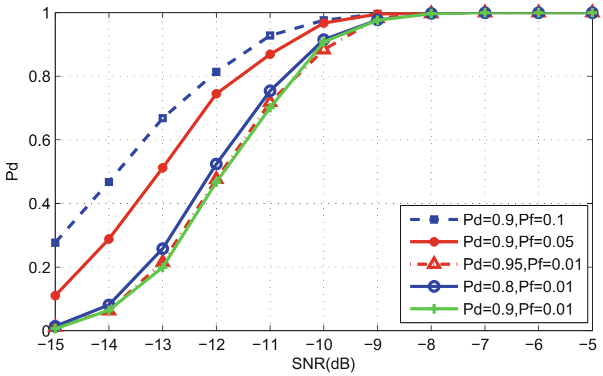

Fig. 4. Detection probability P_d for different design values (Color figure online)

Figure 4 illustrates the receiver operating characteristic (ROC) curves of sine wave for different design values, where shows the effect of SNR on the actual P_d obtained via Monte Carlo simulation for increasing design values P_d and P_f. It can be seen that the proposed SPDD matches its design specifications upto a SNR of -10 dB, below which the actual detection probability P_d precipitously declines. Also, the actual P_d is influenced by thresholds setting, which are determined by design values P_d and P_f.

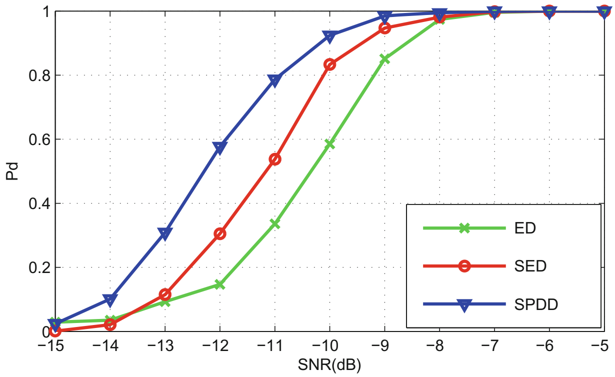

Fig. 5. Detection probability P_d for SPDD, SED and ED (Color figure online)

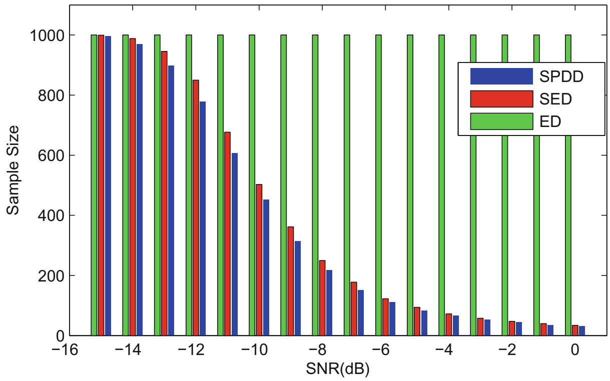

Fig. 6. Sample size for SPDD, SED and ED (Color figure online)

Figure 5 compares the ROC curves of SPDD, conventional sequential energy detection(SED) and energy detection (ED), where the design values of SPDD, SED $P_d = 0.9, P_f = 0.01$ and the signal used is continuous carrier. Moreover, the corresponding stopping sample number are compared in Fig. 6, where the cut-off sample number of SPDD, SED is set to 1000. It shows substantial performance improvement in the SPDD by analyzing the distribution of phase difference. The actual P_d equals 1 till $SNR = -7$ dB and the P_d is still > 0.9 upto -9 dB. Meanwhile, SPDD can achieve 2 dB gain compared with conventional SED and ED, which demonstrates that SPDD has a significant performance improvement. The ratio of the average sample size of SPDD test to the sample size of fixed sample ED test reaches 0.5 while the ratio between SPDD and SED reaches about 0.95 when the actual P_d are both 0.9. It is obvious that the proposed SPDD is more efficient and flexible.

5 Conclusions

In this paper, the novel SPDD spectrum sensing scheme based on phase difference has been shown to deliver substantial efficiency gain over the conventional detection methods based on fixed sample size and amplitude statistics, which are susceptive to the noise uncertainty and inefficient. Through careful analysis, the distribution of phase difference between two adjacent received signal samples of signal perturbed by noise and Gaussian noise can be utilized to sense signal. Moreover, the Iterative Probabilistic Update method proposed above has been developed to robustly evaluate the likelihood ratio and thus bring significant performance improvement. Meanwhile, the detection thresholds setting of SPDD are free from noise power, therefore the proposed scheme is immune to noise uncertainty.

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