

A Two-Stage Precoding Algorithm for Spectrum Access Systems with Different Priorities of Spectrum Utilization

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Abstract. In this paper, a two-stage precoding algorithm, termed as subspace-projection prioritized signal-to-leakage-and-noise ratio (SP-PSLNR) algorithm, is proposed for dynamic spectrum access systems with different priorities of spectrum utilization. In the first stage precoding, the interference from the secondary users (SUs) to the primary users (PUs) is canceled; while in the second stage precoding, we differentiate the interference protection and quality of service (QoS) among SUs with different priorities of spectrum access. For this purpose, we newly introduce a parameter called as “interference leakage weight (ILW)” to be used in the optimization of signal to leakage and noise ratio (SLNR). The simulation results show that the proposed method can increase SUs’ maximum allowed transmit power while maintaining protection to the PUs. Moreover, this method can jointly optimize the transmit power of SUs and minimize the interference among SUs. Furthermore, the QoS of SUs can be differentiated by adjusting the ILWs.

Keywords: 5G · Interference leakage weight · Incumbent user protection · Prioritized dynamic spectrum access · Spectrum access system

1 Introduction

To meet the requirements of the emerging fifth generation (5G) wireless communication systems, such as even higher system capacity and spectrum utilization, cognitive radio (CR) technology has been widely investigated as an important enabling technology. In CR-enabled dynamic spectrum access systems, secondary users (SUs) are allowed to access the spectrum of licensed primary users (PUs) on a non-interference basis. For future wireless networks, a large variety of macrocells, microcells, and femtocells will coexist together with numerous device-to-device (D2D) or machine type communications. Thus, multi-tier or hierarchical wireless systems, in which each tier has different

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quality of service (QoS) requirements, are envisioned [1, 2]. Notably, the U.S. President’s Council of Advisors on Science and Technology (PCAST) recommends a three-tier hierarchy (i.e., Federal primary access, priority secondary access, and general authorized access) for access to Federal spectrum [7]. In this three-tier architecture, the first tier users would be entitled to interference protection to a level such that their communication performance requirements are satisfied. The second tier users would receive short-term priority authorizations. The third tier users would be entitled to use the spectrum on an opportunistic basis and would not be entitled to interference protection. In this study, we consider a two-tier system in which the first tier is the primary user system (PS) and the second tier is the secondary user system (SS). Further, we consider that the SUs in the second tier may have *different* priority levels of spectrum utilization. The high-priority SUs will have better QoS than the low-priority SUs. A new problem which needs to be addressed is how to support *different* QoS requirements of those prioritized SUs while maintaining protection to the PUs.

For most study on cognitive multiple-input multiple-output (MIMO) systems, the SUs have been treated with the same priority. Even though there are many algorithms (e.g., block diagonalization [3], minimum mean square error (MMSE) [3, 4], interference alignment [5]) to mitigate the co-channel interference between the SUs, these algorithms cannot support *different* interference protection and QoS requirements of the prioritized SUs. Ekram Hossain also summaries the challenges of traditional interference management methods (e.g., power control, cell association, etc.) and argues that the existing methods will not be able to address the interference management problem in 5G multi-tier networks because of the more complex interference dynamics (e.g., disparate QoS requirements and priorities at different tiers, huge traffic load imbalance, etc.) [1]. To support the different priorities of interference protection and QoS for spectrum access systems, new interference management algorithms need to be developed.

In this paper, a two-stage precoding algorithm, termed as subspace-projection prioritized signal-to-leakage-and-noise ratio (SP-PSLNR) algorithm, is proposed for spectrum access systems with different priority levels of spectrum utilization. The first stage precoding is based on subspace projection (SP), which mitigates the PU’s interference (PUI) caused by SUs. The second stage precoding is based on maximizing the prioritized signal-to-leakage-and-noise ratio (PSLNR), which suppresses the interference between the SUs and supports different priorities of interference protection. A new parameter called as “*interference leakage weight (ILW)*” is introduced at the second stage precoding to account for the resulting interference leakage from one SU to the other SUs. Simulations are conducted to verify the effectiveness of the proposed algorithm.

The rest of this paper is organized as follows. In Sect. 2, the system model and system parameters are discussed. In Sect. 3, the proposed two-stage precoding algorithm is analyzed in more details. How to assign the appropriate ILWs to SUs with different priority levels is also explained. Simulation results are presented in Sect. 4 followed by the conclusion of this paper.

Notations: we use \mathbf{A}^H , $E\{\mathbf{A}\}$ to denote the conjugate transpose and the statistical expectation of matrix \mathbf{A} , respectively. $Tr\{\mathbf{A}\}$ denotes the trace of matrix \mathbf{A} . \mathbf{I}_N denotes an $N \times N$ identity matrix.

2 System Model

Figure 1 shows the system model of spectrum access systems with different priority levels of spectrum utilization. For the system model discussed in this paper, one PS coexists with k SSs. These SSs have different priority levels of spectrum utilization. All SUs are assumed to operate in the same spectrum used by PU while the interference to the PU should be kept below the predefined threshold. In this system model, a pair of active transmitter and receiver equipped with multiple antennas is considered in the PS or SS. As shown in Fig. 1, N_{T_p} and N_{T_s} represents the number of transmitting antennas at PU and SU, respectively. N_{R_p} , N_{R_s} represents the number of receiving antennas at PU and SU, respectively. In Fig. 1, the solid arrow lines represent the desired signals, while the dashed arrow lines stand for the interference. A database is employed to store/retrieve the priority information, geolocation information as well as the channel state information of the PUs and the SUs.

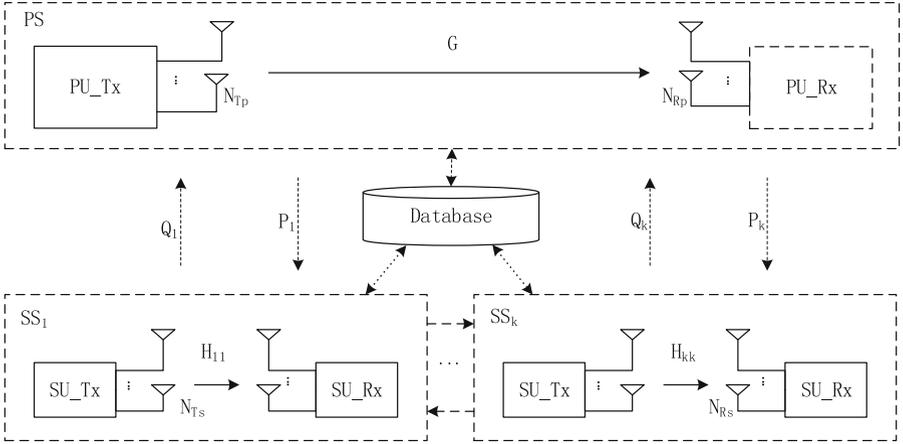


Fig. 1. System model of spectrum access systems with different priority levels of spectrum utilization

The received signal vectors y_p and y_{si} at the PU receiver and the i -th SU receiver are expressed as follows:

$$y_p = Gs + \sum_{i=1}^k Q_i F_i x_i + n_p, \quad (1)$$

$$y_{si} = H_{ii} F_i x_i + \sum_{r=1, r \neq i}^k H_{ir} F_r x_r + P_i s + n_{si}, \quad (2)$$

where s is the transmitted symbol vector from the PU transmitter; x_i is the transmitted symbol vector from the i -th SU transmitter; G is the channel matrix between the PU

transmitter and receiver; \mathbf{Q}_i is the channel matrix between the i -th SU transmitter and PU receiver; the term \mathbf{F}_i is the precoding matrix of the i -th SU; \mathbf{H}_{ii} is the channel matrix between the i -th SU transmitter and receiver; \mathbf{H}_{ir} is the channel matrix between the r -th SU transmitter and the i -th SU receiver; \mathbf{P}_i is the channel matrix between the PU transmitter and i -th SU receiver; \mathbf{n}_p and \mathbf{n}_{si} are the additive white Gaussian noise (AWGN) vectors with zero mean and unit variance.

The first term in (1) and (2) represents the desired signal at the receiver of PU and i -th SU, respectively. The second term in (1) represents the aggregated PUI caused by SUs. The second and third term in (2) represents the SU's interference (SUI) caused by the other SUs and the PU, respectively. The last term in (1) and (2) is the AWGN noise. In this paper, unless stated otherwise, it is assumed that perfect channel state information (CSI) is known at the transmitters and receivers of SUs.

3 Two-Stage Precoding Algorithm (SP-PSLNR)

In this section, the proposed two-stage precoding algorithm "SP-PSLNR" is discussed in a stage-by-stage approach in the first two subsections. Then the combination of two stages, i.e., the SP-PSLNR algorithm, is presented in the third subsection. In the last subsection, how to assign the appropriate ILWs to SUs of different priority levels is further discussed.

3.1 First Stage Precoding: SP Algorithm

The first stage precoding is based on SP, which eliminates the PUI caused by the SUs operating in the same spectrum. In a CR-enabled dynamic spectrum access system, the SUs are allowed to access the same spectrum used by the PU only if the resulting interference to the PU remains below the predefined PUI threshold. Therefore, the SUs' maximum allowed transmit power has to be limited by the predefined interference threshold at the PU. Consequently, the SUs' transmit power might be too low to meet the communication quality requirements. To increase the allowed transmit power of SUs, more effective interference suppression algorithm is quite needed. The SP algorithm can help SUs to transmit signals in the null space of the interference channel (i.e., the channel between SUs transmitter and PU receiver), thus eliminating the PUI caused by SUs. In this way, the maximum allowed transmit power of SUs with the SP-based precoding can be significantly higher than that when using the traditional power control method.

Based on the geolocation database approach such as the advanced geolocation engine (AGE) database (please refer to [6] for more details about AGE database), the i -th SU first finds out the PU within its interference range, and then the channel matrix \mathbf{Q}_i is retrieved when evaluating the PUI caused by the i -th SU transmitter. By applying the singular value decomposition (SVD) of \mathbf{Q}_i , the null of \mathbf{Q}_i , i.e., $\mathbf{V}^{(0)}$, which is the first stage precoding vector $\mathbf{F}_i^{(1)}$ for the i -th SU transmitter, can be obtained as follows.

$$Q_i = U \sum [V^{(1)}V^{(0)}]^H, \quad (3)$$

$$F_i^{(1)} = V^{(0)}, \quad (4)$$

where $V^{(1)}$ is the matrix composed of the right singular vectors corresponding to non-zero singular values of Q_i , and $V^{(0)}$ is the matrix composed of the zero singular values of Q_i . To obtain the null space of non-zero elements, the number of the SU's transmitting antennas should be no less than the number of the PU's receiving antennas.

3.2 Second Stage Precoding: PSLNR Algorithm

To support different priority levels of QoS requirements for SUs, the PSLNR algorithm is proposed by introducing a new parameter called as "ILW" into the traditional SLNR algorithm. The PSLNR measured by the i -th SU receiver is expressed as follows:

$$\begin{aligned} PSLNR_i &= \frac{E\left\{ |H_{ii}F_i x_i|^2 \right\}}{E\left\{ \left| \alpha_i \sum_{r=1, r \neq i}^k H_{ri}F_i x_i + n_{si} \right|^2 \right\}} \\ &= \frac{\text{Tr}(F_i^H H_{ii}^H H_{ii} F_i)}{\text{Tr}(\alpha_i^2 \sum_{r=1, r \neq i}^k F_i^H H_{ri}^H H_{ri} F_i) + I_{N_{Rs}}}, \end{aligned} \quad (5)$$

where α_i represents the i -th SU's ILW. The higher the α_i value of i -th SU, the stronger constraint is forced to the investigated SU on its interference leakage to the other SUs. Therefore, a smaller ILW will be assigned to the high-priority SU. That is to say, the high-priority SU is endowed with looser constraint on its interference leakage to other lower-priority SUs.

The second stage precoding is based on maximizing the PSLNR, which suppresses the interference among the SUs and supports different priorities of interference protection and QoS.

The optimization problem is formulated as follows:

$$\begin{cases} \max_{F_i^{(2)}} PSLNR_i \\ s.t. \|F_i\|^2 \leq p_{Si} \\ F_i = F_i^{(1)} F_i^{(2)}, \end{cases} \quad (6)$$

where $F_i^{(2)}$ represents the second stage precoding matrix of the i -th SU and p_{Si} represents the transmit power of i -th SU.

By solving the above optimization problem according to the solution of traditional SLNR algorithm, the second stage precoding matrix for the i -th SU can be expressed as follows:

$$F_i^{(2)} = \left[f_i^{(2)} \dots f_i^{(2)} \right]_{N_{T_i} \times N_{R_s}}, \quad (7)$$

where $f_i^{(2)} = \Phi \left[\left(\alpha_i^2 \sum_{r=1, r \neq i}^k F_i^{(1)H} H_{ri}^H H_{ri} F_i^{(1)} + I_{N_{R_s}} \right)^{-1} F_i^{(1)H} H_{ii}^H H_{ii} F_i^{(1)} \right]$, and $\Phi[A]$

represents the eigenvector corresponding to the largest eigenvalue of A .

Supposing there are two SSs of different priorities, the ratio of two SUs' signal to interference plus noise ratio ($SINR$) can be written as:

$$\frac{SINR_1}{SINR_2} = \frac{Tr(F_1^H H_{11}^H H_{11} F_1)}{Tr(F_2^H H_{22}^H H_{22} F_2)} \times \frac{Tr(F_1^H H_{21}^H H_{21} F_1) + Tr(P_2^H P_2) + N_0}{Tr(F_2^H H_{12}^H H_{12} F_2) + Tr(P_1^H P_1) + N_0}, \quad (8)$$

where $F_1 = F_1^{(1)} F_1^{(2)}$, $F_2 = F_2^{(1)} F_2^{(2)}$, and N_0 represents the noise power. As a special case, when these two pairs of SU transmitters and receivers are symmetrically distributed with regarding to the PU transmitter (as shown in Fig. 3 in the next Section), the ratio of two SUs' $SINR$ is only affected by the second precoding matrix. As a result, the ratio of two SUs' $SINR$ will be mainly determined by the ILWs assigned to these two SUs.

3.3 Two-Stage Precoding Algorithm (SP-PSLNR)

The proposed scheme, termed as SP-PSLNR, is the combination of SP precoding and PSLNR precoding. By using the SP-PSLNR algorithm, the different priorities of interference protection and QoS requirements can be supported for spectrum access systems with different priority levels of spectrum utilization. The two-stage precoding scheme at the i -th SU transmitter is carried out by the following 4 steps:

- (1) Identify the PU within the SU's interference range. We get the channel matrix \mathbf{Q}_i which is the interference channel matrix between the i -th SU transmitter and the PU receiver.
- (2) Obtain the null of \mathbf{Q}_i , (i.e., the first stage precoding matrix) by applying the SVD of \mathbf{Q}_i .
- (3) Assign the appropriate ILW to the i -th SU transmitter according to the procedures detailed in the next subsection.
- (4) Obtain the second stage precoding matrix by using the PSLNR criterion.

The proposed two-stage precoding algorithm is employed at the SU transmitter. The flow chart of the desired signal from the i -th SU transmitter to its intended receiver is depicted in Fig. 2.

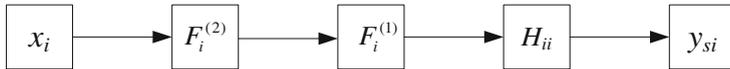


Fig. 2. Flow chart of the desired signal from the i -th SU transmitter to the i -th SU receiver

3.4 ILW Assignment

As mentioned in the Subsect. 3.2, ILW is introduced to account for the interference leakage power in the objective function of SU. In this way, the high-priority SU has a looser constraint on its interference leakage to the other low-priority SUs, contrarily the low-priority SU has to obey a stronger constraint on its interference leakage to the other high-priority SUs. Therefore, the high-priority SU can obtain better interference protection and QoS guarantee.

When adopting the proposed SP-PSLNR algorithm, how to assign the appropriate ILWs to SUs of different priority levels is an important issue. According to the PSLNR expressed by (5) and the SINR ratio expressed by (8), the priority level, required QoS (e.g., SINR) and the transmit power of SUs are the key factors to be considered when making the ILW assignment. Based on simulations or field tests, the ratio of ILWs can be pre-determined according to the required QoS (say, SINR) difference and the transmit power at different SUs.

As shown from the simulation results presented in the next section, the ratio of ILWs has significant impact on the differentiation of the SINR at different prioritized SUs. Therefore, a proportional ratio method is proposed to adjust ILWs for SUs with different priorities. The sum of ILWs assigned to all active SUs is normalized to 1.

The procedures of the ILW assignment are detailed as follows:

- (1) Estimate the interference range of the SU based on the maximal transmit power, the height of transmitting antennas and other related information;
- (2) Find all the active SUs (i.e., the SUs in active communications) in this interference range, and then sort their priorities;
- (3) Determine the ratio of ILWs for the SUs in a proportional manner according to the required SINR difference and the transmit power at different SUs;
- (4) Assign an appropriate ILW to each active SU such that the sum of ILWs assigned to all active SUs is equal to 1.

4 Performance Simulation

Simulations are conducted to further evaluate the performance of the proposed two-stage precoding algorithm. This section presents the simulation results, which demonstrate the effectiveness of the proposed SP-PSLNR algorithm.

The simulation model is depicted in Fig. 3, in which one PS and two prioritized SSs share the same spectrum. Assuming that at a given time instance, there is only one pair of active communication users in the PS and SS. SS₁ (serving the SU₁) has higher priority than SS₂ (serving the SU₂). The cell radius of the PS and SS is set as 30 m. The transmitter is located at the cell center. As shown in Fig. 3, the propagation distances of

the desired signal are designated as d_{11} and d_{22} ; the propagation distances of the interference from the neighboring SS are designated as d_{12} and d_{21} ; and the propagation distance of interference from PS is designated as d_{01} and d_{02} . Rayleigh fading channel model and free-space path loss model are assumed in the simulation. The modulation type is binary phase shift keying (BPSK). Moreover, it is assumed that the complete CSI is known at the transmitters of SS₁ and SS₂. Some other system parameters are listed in Table 1.

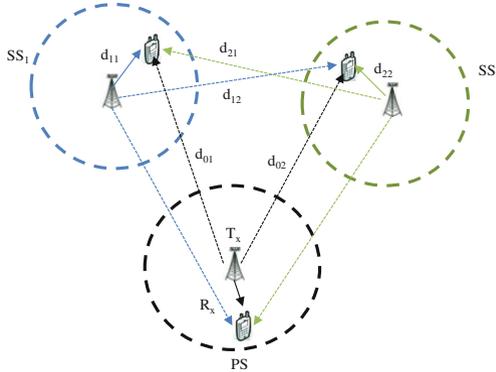


Fig. 3. Simulation model of spectrum access of SUs with different priority levels

Table 1. System parameters used in the simulation (unless stated otherwise)

Parameters	Value
Operating frequency	2 GHz
Channel bandwidth	10 MHz
Noise figure at the PU/SU receiver	5 dB
PU transmit power	5 dBm
SU transmit power	5 dBm
Number of transmitting antennas at PU	2
Number of receiving antennas at PU	2
Number of transmitting antennas at SU	2
Number of receiving antennas at SU	2

Without loss of generality, the SU receiver is randomly distributed in the small cell of SS, and thus the desired signal distance or the interference distance may not be equal. The simulation results are shown in Figs. 4, 5, 6 and 7. Then, we introduce a special case to show the differentiated SUs' performance due to different ILW assignments, in which we set the two pairs of SU transmitters and receivers symmetrically distributed with regarding to the PU transmitter. For this special case, both the desired signal distances and the interference distances are equal for the receivers of two SUs, i.e., $d_{11} = d_{22}$ and $d_{01} = d_{02}$, the corresponding simulation results are shown in Figs. 8, 9 and 10.

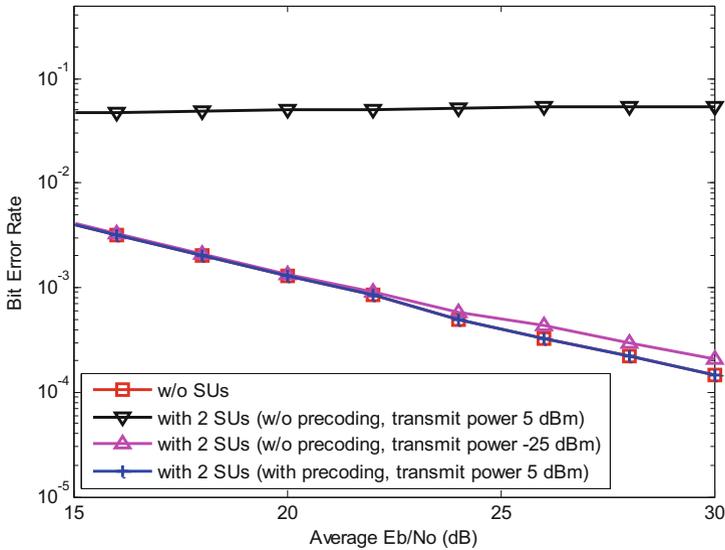


Fig. 4. BER performance of PU under different settings

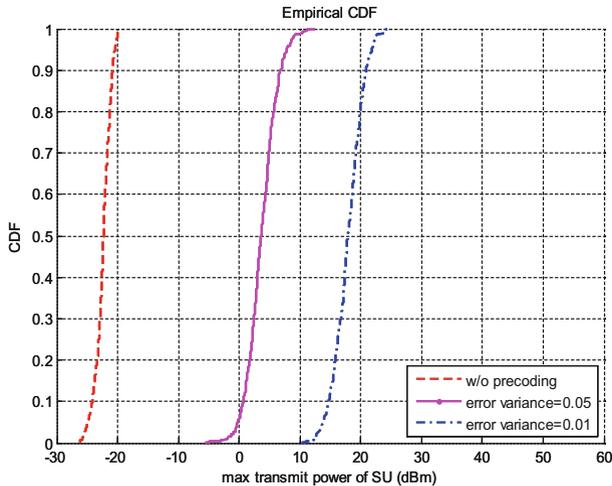


Fig. 5. CDF of SUs' maximum allowed transmit power

Figure 4 shows the BER performance of PU under different settings. Especially, the performance of SP-based precoding algorithm is compared against that of the traditional power control method (i.e., without precoding). The PU receiver is located at the cell edge of PS. The simulation results show that if all the SUs employ the SP-based precoding with the perfect CSI, the PUI can be completely eliminated. More importantly, the maximum allowed transmit power at SUs can be increased significantly by

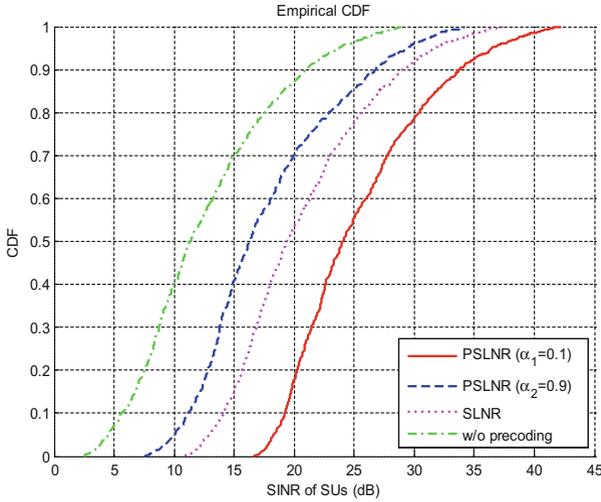


Fig. 6. CDF of SUs’ SINR

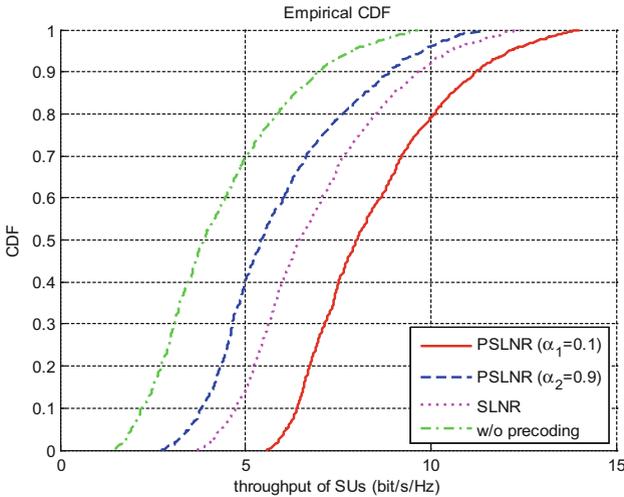


Fig. 7. CDF of SUs’ throughput

about 30 dB as compared to the traditional power control method (i.e., without precoding). The reason is that SP precoding enables SUs to transmit signal in the null space of interference channel to the PU. Therefore, the proposed SP-PSLNR algorithm can protect the PU’s QoS while significantly relaxing the limitation on SUs’ maximum allowed transmit power.

As mentioned above, theoretically, the SP-based precoding can completely eliminate the PUI with perfect CSI. However, in practice, the estimated channel matrix

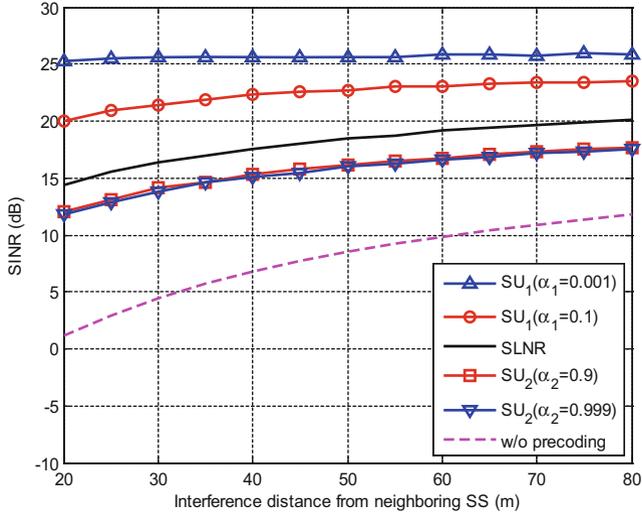


Fig. 8. SINR of SUs with different algorithms or ILW values

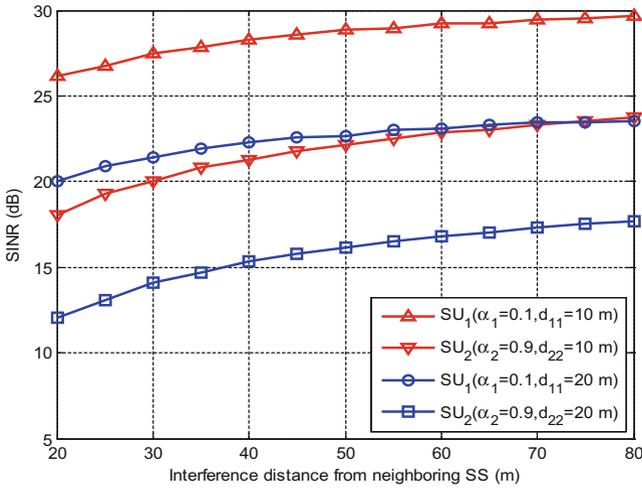


Fig. 9. SINR of SUs with different desired signal distances

always contains some errors. In Fig. 5, the element of error matrix is considered to be a random variable, e_{ij} , following the complex Gaussian distribution with zero mean and σ^2 variance, i.e., $CN(0, \sigma^2)$. In this simulation, the interference to noise ratio (INR) threshold at PU is set to 0 dB. Note that, for the simulation shown in Fig. 5, the number of SUs' transmitting antennas is set to 8. The maximum allowed transmit power of SU refers to the transmit power of SU when the PUI caused by this SU is equal to the interference tolerance threshold of PU. And the maximum allowed transmit

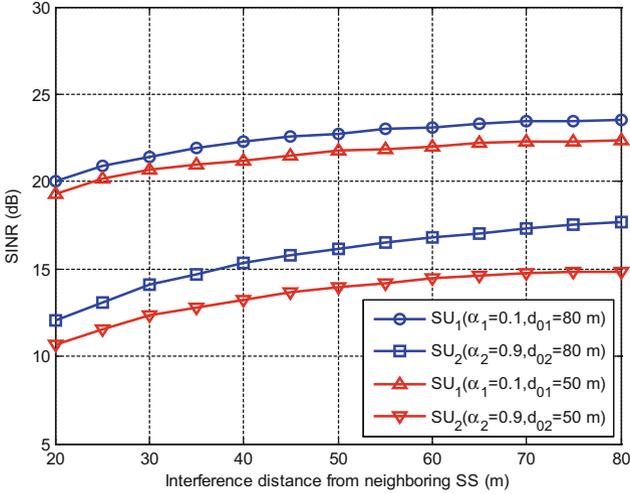


Fig. 10. SINR of SUs under different interference distance from the PS transmitter

power is related with the INR threshold of PU, the distance between SU and PU, and the adopted interference suppression algorithm of SU. As can be seen in Fig. 5, when the error variance is set to 0.05 and 0.01, the maximum allowed transmit power can be increased by about 23 ~ 30 dB, 38 ~ 45 dB, respectively. Larger error variance results in larger PUI caused by SUs. As expected, the smaller the error variance, the higher the maximum allowed transmit power.

Figures 6 and 7 present the cumulative distribution function (CDF) of the SUs’ SINR and throughput, respectively. It shows that when using the proposed two-stage precoding algorithm, the high-priority SU always has better SINR and higher throughput as compared to that using the traditional SLNR algorithm. This is because the high-priority SU (with lower ILW) has looser constraint on its interference leakage than the SU with traditional SLNR algorithm. However, the low-priority SU always has poorer SINR and lower throughput as compared to that using the traditional SLNR algorithm. This observation implies that the performance improvement of the high-priority SU is obtained at the cost of performance degradation of the low-priority SU.

Figure 8 shows the SINR of different prioritized SUs. In this simulation, $d_{11} = d_{22} = 20$ m and $d_{01} = d_{02} = 80$ m. The ILWs assigned to the SUs with different priority levels of spectrum utilization are shown in the legend.

Figure 8 shows that, firstly, PSLNR can reduce the interference between SSs, and significant benefits can be obtained as compared to that without precoding; secondly, according to the comparison of PSLNR and SLNR, the higher prioritized user has obtained better SINR as compared to that with SLNR algorithm, whereas the lower prioritized user has even lower SINR than that with SLNR algorithm; thirdly, as expected, the SU with smaller ILW results in higher SINR than the SU with larger ILW. When the ratio of the ILWs between the high-priority SU and the low-priority SU gets smaller, the SINR difference of between these SUs becomes even larger. For example, when the ratio of the SUs’ ILWs (i.e., α_1/α_2 in this simulation) is reduced to

1/999 from 1/9, the SINR difference of these SUs is increased to 13.5 dB from about 8 dB (when the interference distance from the neighboring SS transmitter is 20 m). In sum, the smaller the SUs' ILW ratio, the larger the SUs' SINR difference. Fourthly, for a given SUs' ILW ratio, the difference of SUs' SINR decreases as the distance from the neighboring SS transmitter increases. As the distance from the neighboring SS transmitter increases, the interference between prioritized SUs becomes even smaller. As a result, the SUs' SINR difference narrows down.

Figure 9 shows the impact of the desired signal distance on the SUs' SINR difference when the ILWs (i.e., 0.1 and 0.9) to the two SUs remain the same. It shows that when the desired signal distance is reduced from 20 m to 10 m, the SINR of SUs increases about 6 dB on average, while the SUs' SINR difference changes little. This is because the SUI is independent with the desired signal distance.

Figure 10 shows when the interference distance from the PS transmitter decreases from 80 m to 50 m, the SINR of SUs decreases in general and the SINR difference of two SUs may increase about 2 dB. When the distance from the PS transmitter decreases, the SUI becomes more serious. In general, the stronger the SUI, the larger the SUs' SINR difference. Hence, the SINR difference increases when the interference distance from the PS transmitter decreases.

As we expected, simulation results shown in Figs. 8, 9 and 10 demonstrate that the SINR difference of SUs mainly depends on the ratio of SUs' ILWs (i.e., α_1/α_2 in this paper).

5 Conclusion

To support the coexistence of PU and prioritized SUs in spectrum access systems with different priority levels of spectrum utilization, a two-stage precoding algorithm (termed as "SP-PSLNR algorithm") is proposed in this paper. With the help of the first stage precoding based on SP, the SUs can access the spectrum without degrading the QoS of PU. In the meanwhile, SUs' maximum allowed transmit power can be significantly increased as compared to that when using the traditional power control method. By maximizing the PSLNR with the newly introduced parameter "ILW", the second stage precoding can differentiate the priorities of interference protection and QoS (e.g., the received SINR) for prioritized SUs. In this way, the high-priority SUs can obtain better SINR than the low-priority SUs, and the SUs' SINR difference can be adjusted by the ratio of ILWs. The stronger the SUI, the larger SUs' SINR difference can be obtained. Performances of the proposed algorithm are verified through simulations. For the future work, in the second stage precoding, the SU's interference caused by the PU will also be taken into account. The proposed algorithm could also be further investigated and exploited to deal with the challenging interference issues in 5G mobile communication systems.

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