Finite Limits and Colimits in Autonomic Systems

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Abstract. Self-* is widely considered as a foundation for autonomic computing. The notion of autonomic systems (ASs) and self-* serves as a basis on which to build our intuition about category of ASs in general. In this paper we will specify ASs and self-* and then move on to consider finite limits and colimits in ASs. All of this material is taken as an investigation of our category, the category of ASs, which we call **AS**.

Keywords: Autonomic computing · Autonomic systems · Coequalizer · Colimit · Equalizer · Limit · Pullback · Pushout · Self-* · Span

1 Introduction

Autonomic computing (AC) imitates and simulates the natural intelligence possessed by the human autonomic nervous system using generic computers. This indicates that the nature of software in AC is the simulation and embodiment of human behaviors, and the extension of human capability, reachability, persistency, memory, and information processing speed. AC was first proposed by IBM in 2001 where it is defined as

> "*Autonomic computing is an approach to self-managed computing systems with a minimum of human interference. The term derives from the body's autonomic nervous system, which controls key functions without conscious awareness or involvement*" [1].

AC in our recent investigations [2–5,7] is generally described as self-*. Formally, let self-* be the set of self-_'s. Each self-_ to be an element in self-* is called a *self-* facet*. That is,

$$\text{self-*} = \{\text{self-\_} \mid \text{self-\_ is a self-* facet}\} \tag{1}$$

We see that self-CHOP is composed of four self-* facets of self-configuration, self-healing, self-optimization and self-protection. Hence, self-CHOP is a subset of self-*. That is, self-CHOP = {self-configuration, self-healing, self-optimization, self-protection} ⊂ self-*. Every self-* facet must satisfy some certain criteria, so-called *self-* properties*.

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In its AC manifesto, IBM proposed eight facets setting forth an AS known as *self-awareness, self-configuration, self-optimization, self-maintenance, self-protection (security and integrity), self-adaptation, self-resource- allocation* and *open-standard-based* [1]. In other words, consciousness (self-awareness) and non-imperative (goal-driven) behaviors are the main features of autonomic systems (ASs).

In this paper we will specify ASs and self-* and then move on to consider finite limits and colimits in ASs. All of this material is taken as an investigation of our category, the category of ASs, which we call **AS**.

2 Outline

In the paper, we attempt to make the presentation as self-contained as possible, although familiarity with the notion of self-* in ASs is assumed. Acquaintance with the associated notion of algebraic language is useful for recognizing the results, but is almost everywhere not strictly necessary.

The rest of this paper is organized as follows: Sect. 3 presents some basic concepts to support consideration of limits and colimits in autonomic systems (ASs). In Sect. 4, we consider some finte limits such as pullbacks of ASs, spans on ASs and equalizers of self-*. In Sect. 5, we consider some finte colimits such as pushouts of ASs and coequalizers of self-*. Finally, a short summary is given in Sect. 6.

3 Basic Concepts

We can think of an AS as a collection of states $x \in AS$, each of which is recognizable as being in AS and such that for each pair of named states $x, y \in AS$ we can tell if $x = y$ or not. The symbol $\oslash$ denotes the AS with no states.

If AS_1 and AS_2 are ASs, we say that AS_1 is a sub-system of AS_2, and write $AS_1 \subseteq AS_2$, if every state of AS_1 is a state of AS_2. Checking the definition, we see that for any system AS, we have sub-systems $\oslash \subseteq AS$ and $AS \subseteq AS$.

We can use system-builder notation to denote sub-systems. For example the autonomic system can be written $\{x \in AS \mid x \text{ is a state of AS}\}$.

The symbol $\exists$ means "there exists". So we can write the autonomic system as $\{x \in AS \mid \exists y \text{ is a final state such that } \textit{self-*action}(x) = y\}$

The symbol $\exists!$ means "there exists a unique". So the statement "$\exists! x \in AS$ is an initial state" means that there is one and only one state to be a start one, that is, the state of the autonomic system before any self-* action is processed.

Finally, the symbol $\forall$ means "for all". So the statement "$\forall x \in AS\ \exists y \in AS$ such that *self-* action*$(x) = y$" means that for every state of autonomic system there is the next one.

In the paper, we use the $\stackrel{def}{=}$ notation "$AS_1 \stackrel{def}{=} AS_2$" to mean something like "define AS_1 to be AS_2". That is, a $\stackrel{def}{=}$ declaration is not denoting a fact of nature (like $1 + 2 = 3$), but our formal notation. It just so happens that the notation above, such as Self-CHOP $\stackrel{def}{=}$ {self-configuration, self-healing, self-optimization, self-protection}, is a widely-held choice.

If AS and AS' are sets of autonomic system states, then a self-* action *self-*action* from AS to AS', denoted *self-*action*: $AS \rightarrow AS'$, is a mapping that sends each state $x \in AS$ to a state of AS', denoted *self-*action*$(x) \in AS'$. We call AS the domain of *self-*action* and we call AS' the codomain of *self-*action*.

Note that the symbol AS', read "AS-prime", has nothing to do with calculus or derivatives. It is simply notation that we use to name a symbol that is suggested as being somehow like AS. This suggestion of consanguinity between AS and AS' is meant only as an aid for human cognition, and not as part of the mathematics. For every state $x \in AS$, there is exactly one arrow emanating from x, but for a state $y \in AS'$, there can be several arrows pointing to y, or there can be no arrows pointing to y.

Suppose that $AS' \subseteq AS$ is a sub-system. Then we can consider the self-* action $AS' \rightarrow AS$ given by sending every state of AS' to "itself" as a state of AS. For example if $AS = \{a, b, c, d, e, f\}$ and $AS' = \{b, d, e\}$ then $AS' \subseteq AS$ and we turn that into the self-* action $AS' \rightarrow AS$ given by $b \mapsto b, d \mapsto d, e \mapsto e$. This kind of arrow, $\mapsto$, is read aloud as "maps to". A self-* action *self-*action*: $AS \rightarrow AS'$ means a rule for assigning to each state $x \in AS$ a state *self-*action*$(x) \in AS'$. We say that "x maps to *self-*action*(x)" and write $x \mapsto$ *self-*action* (x).

As a matter of notation, we can sometimes say something like the following: Let *self-*action*: $AS' \subseteq AS$ be a sub-system. Here we are making clear that AS' is a sub-system of AS, but that *self-*action* is the name of the associated self-* action.

Given a self-* action *self-*action*: $AS \rightarrow AS'$, the states of AS' that have at least one arrow pointing to them are said to be in the image of *self-*action*; that is we have

$$\mathrm{im}(\textit{self-*action}) \stackrel{def}{=} \{y \in AS' \mid \exists x \in AS \text{ such that } \textit{self-*action}(x) = y\} \quad (2)$$

Given *self-*action*: $AS \rightarrow AS'$ and *self-*action*$'$: $AS' \rightarrow AS''$, where the codomain of *self-*action* is the same set of autonomic system states as the domain of *self-*action*$'$ (namely AS'), we say that *self-*action* and *self-*action*$'$ are composable

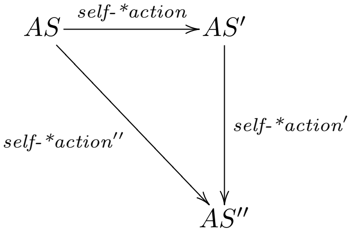
$$AS \xrightarrow{\textit{self-*action}} AS' \xrightarrow{\textit{self-*action}'} AS''$$

The composition of *self-*action* and *self-*action*$'$ is denoted by *self-*action*$' \circ$ *self-*action*: $AS \rightarrow AS''$.

We define the identity *self-*action* on AS, denoted $id_{AS} : AS \rightarrow AS$, to be the self-* action such that for all $x \in AS$ we have $id_{AS}(x) = x$.

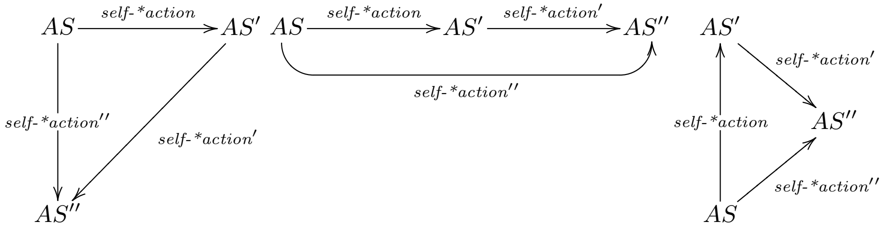
A *self-*action*: $AS \rightarrow AS'$ is called an *isomorphism*, denoted *self-*action*: $AS \xrightarrow{\cong} AS'$, if there exists a self-* action *self-*action*$'$: $AS' \rightarrow AS$ such that *self-*action*$' \circ$ *self-*action*$= id_{AS}$ and *self-*action* $\circ$ *self-*action*$' = id_{AS'}$. We also say that *self-*action* is *invertible* and we say that *self-*action*$'$ is the *inverse* of *self-*action*. If there exists an isomorphism $AS \overset{\cong}{\to} AS'$ we say that AS and AS' are isomorphic autonomic systems and may write $AS \cong AS'$.

Consider the following diagram:



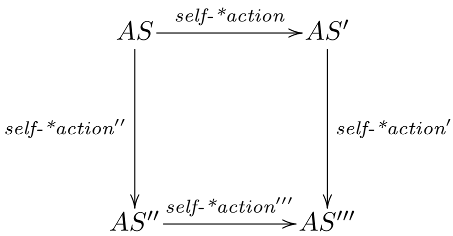
(3)

We say this is a diagram of autonomic systems if each of AS, AS', AS'' is an autonomic system and each of *self-*action*, *self-*action′*, *self-*action″* is a self-* action. We say this diagram commutes if $\textit{self-*action}' \circ \textit{self-*action} = \textit{self-*action}''$. In this case we refer to it as a commutative triangle of autonomic systems. Diagram (3) is considered to be the same diagram as each of the following:



(4)

Consider the following picture:



(5)

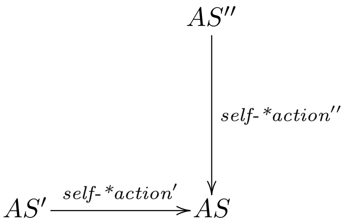
We say this is a diagram of autonomic systems if each of AS, AS', AS'', AS''' is an autonomic system and each of *self-*action*, *self-*action′*, *self-*action″*, *self-*action‴* is a self-* action. We say this diagram commutes if $\textit{self-*action}' \circ \textit{self-*action} = \textit{self-*action}''' \circ \textit{self-*action}''$. In this case we refer to it as a commutative square of autonomic systems.

4 Finite Limits in Autonomic Systems

In this section, we consider what are called limits of variously-shaped diagrams of ASs.

4.1 Pullbacks of Autonomic Systems

Suppose given the diagram of ASs and self-*actions below.

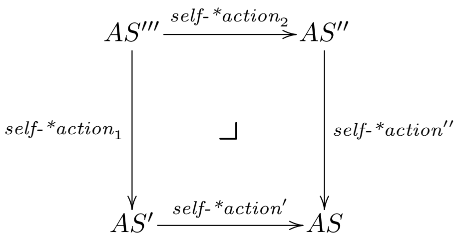


(6)

Its fiber product is the AS

$$AS' \times_{AS} AS'' \stackrel{def}{=} \{(x, w, y) | \textit{self-*action}'(x) = w = \textit{self-*action}''(y)\}$$

There are obvious projections $\textit{self-*action}_1 : AS' \times_{AS} AS'' \to AS'$ and $\textit{self-*action}_2 : AS' \times_{AS} AS'' \to AS''$. Note that if $AS''' = AS' \times_{AS} AS''$ then the following diagram commutes

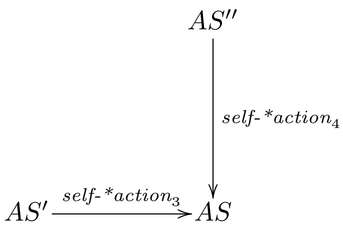


(7)

Given the setup of diagram (7) we come to the pullback of AS' and AS'' over AS to be any AS''' for which we have an isomorphism $AS''' \stackrel{\cong}{\to} AS' \times_{AS} AS''$. The corner symbol "⌟" in diagram (7) indicates that AS''' is the pullback.

Some may prefer to denote this fiber product by $\textit{self-*action}' \times_{AS} \textit{self-*action}''$ rather than $AS' \times_{AS} AS''$. The former is mathematically better notation, but human-readability is often enhanced by the latter, which is also more common in the literature. We use whichever is more convenient.

Suppose given the diagram of ASs and self-actions as in (8).



(8)

For any AS''' and commutative solid arrow diagram as in (9). In other words, $\textit{self-*action}_1 : AS''' \to AS'$ and $\textit{self-*action}_2 : AS''' \to AS''$ such that *self-*action*$_3$ ∘ *self-*action*$_1$ = *self-*action*$_4$ ∘ *self-*action*$_2$ there exists a unique arrow

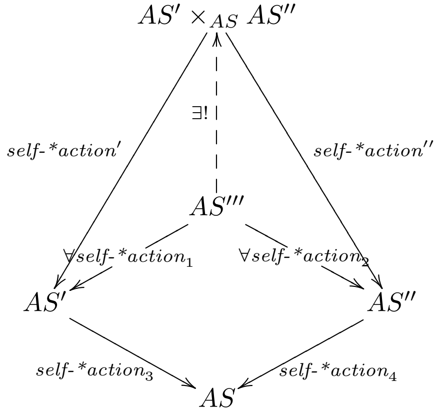
$$< \textit{self-*action}_1, \textit{self-*action}_1 >_{AS}: AS''' \rightarrow AS' \times_{AS} AS''$$

making everything commute. In other words,

$$\textit{self-*action}_1 = \textit{self-*action}' \circ < \textit{self-*action}_1, \textit{self-*action}_1 >_{AS}$$

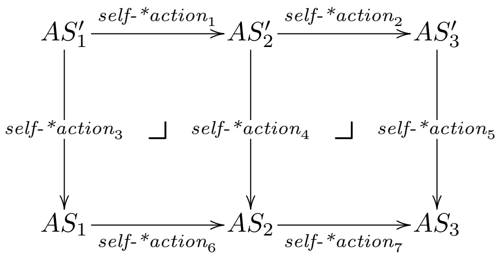
and

$$\textit{self-*action}_2 = \textit{self-*action}'' \circ < \textit{self-*action}_1, \textit{self-*action}_1 >_{AS}$$



(9)

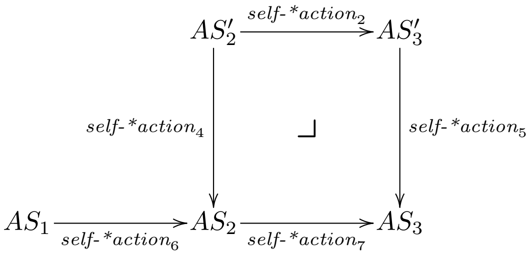
Consider the diagram drawn in (10), which includes a left-hand square, a right-hand square, and a big rectangle



(10)

If $AS'_2 \cong AS_2 \times_{AS_3} AS'_3$ then the right-hand square is a pullback. The right-hand square has a corner symbol indicating that $AS'_2 \cong AS_2 \times_{AS_3} AS'_3$ is a pullback. But the corner symbol on the left might be indicating that the left-hand square is a pullback, or the big rectangle is a pullback. Thus, If $AS'_2 \cong AS_2 \times_{AS_3} AS'_3$ then the left-hand square is a pullback if and only if the big rectangle is.

Consider the diagram drawn in (11)



(11)

where $AS'_2 \cong AS_2 \times_{AS_3} AS'_3$ is a pullback. Then there is an isomorphism

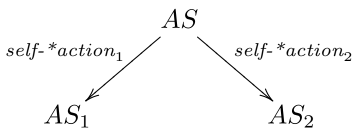
$$AS_1 \times_{AS_2} AS'_2 \cong AS_1 \times_{AS_3} AS'_3$$

In other words,

$$AS_1 \times_{AS_2} (AS_2 \times_{AS_3} AS'_3) \cong AS_1 \times_{AS_3} AS'_3$$

4.2 Spans on Autonomic Systems

Consider AS_1 and AS_2, a span on AS_1 and AS_2 is an AS together with self-* actions *self-*action*$_1 : AS \to AS_1$ and *self-*action*$_2 : AS \to AS_2$.



(12)

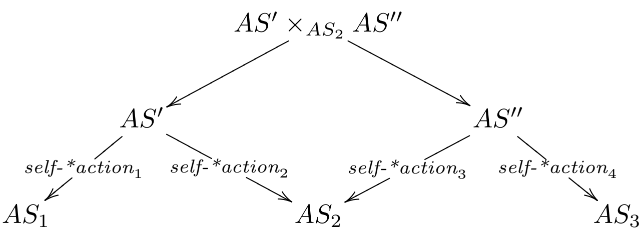
Let AS_1, AS_2, and AS_3 be autonomic systems, and let

$$AS_1 \overset{\textit{self-*action}_1}{\longleftarrow} AS' \overset{\textit{self-*action}_2}{\longrightarrow} AS_2$$

and

$$AS_2 \overset{\textit{self-*action}_3}{\longleftarrow} AS'' \overset{\textit{self-*action}_4}{\longrightarrow} AS_3$$

be spans. Their composite span is given by the fiber product $AS' \times_{AS_2} AS''$ as in the diagram (13):



(13)

If there is a span as $AS_1 \leftarrow AS \rightarrow AS_2$ then by the universal property of products [6], we have a unique map $AS \overset{\exists!}{\rightarrow} AS_1 \times AS_2$.

If there are two spans as $AS_1 \leftarrow AS' \rightarrow AS_2$ and $AS_1 \leftarrow AS'' \rightarrow AS_2$. We can take the disjoint union $AS' \sqcup AS''$ and by the universal property of coproducts, we have a unique span $AS_1 \leftarrow AS' \sqcup AS'' \rightarrow AS_2$ making the diagram (14) commute.

$$\begin{array}{ccccc} & \swarrow & AS' & \searrow & \\ & & \downarrow & & \\ AS_1 & \longleftarrow & AS' \sqcup AS'' & \longrightarrow & AS_2 \\ & & \uparrow & & \\ & \nwarrow & AS'' & \nearrow & \end{array} \tag{14}$$

Given a span $AS_1 \overset{\textit{self-*action}_1}{\longleftarrow} AS \overset{\textit{self-*action}_2}{\longrightarrow} AS_2$, we can draw a bipartite graph with each state of AS_1 drawn as a dot on the left, each state of AS_2 drawn as a dot on the right, and each state a in AS drawn as an arrow connecting vertex $\textit{self-*action}_1(a)$ on the left to vertex $\textit{self-*action}_2(a)$ on the right.

4.3 Equalizers of Self-*

Suppose given two parallel self-* actions

$$AS_1 \underset{\textit{self-*action}_2}{\overset{\textit{self-*action}_1}{\rightrightarrows}} AS_2$$

The equalizer of $\textit{self-*action}_1$ and $\textit{self-*action}_2$ is the commutative diagram in (15),

$$Eq(\textit{self-*action}_1, \textit{self-*action}_2) \xrightarrow{p} AS_1 \underset{\textit{self-*action}_2}{\overset{\textit{self-*action}_1}{\rightrightarrows}} AS_2 \tag{15}$$

where we define

$$Eq(\textit{self-*action}_1, \textit{self-*action}_2) \overset{def}{=} \{a \in AS_1 \mid \textit{self-*action}_1(a) = \textit{self-*action}_2(a)\}$$

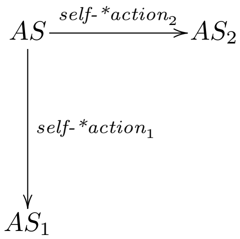
and where p is the canonical inclusion

5 Finite Colimits in Autonomic Systems

We consider several types of finite colimits to obtain some intuition about them, without formally defining them yet.

5.1 Pushouts of Autonomic Systems

Suppose given the diagram (16) of ASs and self-* actions below:



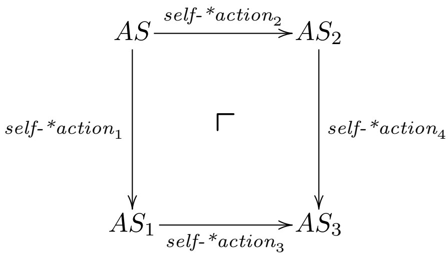
(16)

Its fiber sum, denoted $AS_1 \sqcup_{AS} AS_2$, is defined as the quotient of $AS_1 \sqcup AS \sqcup AS_2$ by the equivalence relation $\sim$ generated by $a \sim \textit{self-*action}_1(a)$ and $a \sim \textit{self-*action}_2(a)$ for all states a in AS. In other words,

$$AS_1 \sqcup_{AS} AS_2 \overset{def}{=} (AS_1 \sqcup AS \sqcup AS_2)/\sim$$

where $\forall a \in AS, a \sim \textit{self-*action}_1(a)$ and $a \sim \textit{self-*action}_2(a)$

There are obvious inclusions $\textit{self-*action}_3 : AS_1 \rightarrow AS_1 \sqcup_{AS} AS_2$ and $\textit{self-*action}_4 : AS_2 \rightarrow AS_1 \sqcup_{AS} AS_2$. Note that if $AS_3 = AS_1 \sqcup_{AS} AS_2$ then the diagram (17) commutes.



(17)

Given the setup of diagram (17), we define the pushout of AS_1 and AS_2 over AS to be any autonomic system AS_3 for which we have an isomorphism $AS_3 \xrightarrow{\cong} AS_1 \sqcup_{AS} AS_2$. The corner symbol "$\ulcorner$" in diagram (17) indicates that AS_3 is the pushout.

For diagram (16), For any autonomic system AS_3 and commutative solid arrow diagram in (18). In other words, self-* actions $\textit{self-*action}_3 : AS_1 \rightarrow AS_3$ and $\textit{self-*action}_4 : AS_2 \rightarrow AS_3$ such that $\textit{self-*action}_3 \circ \textit{self-*action}_1 = \textit{self-*action}_4 \circ \textit{self-*action}_2$, there exists a unique arrow

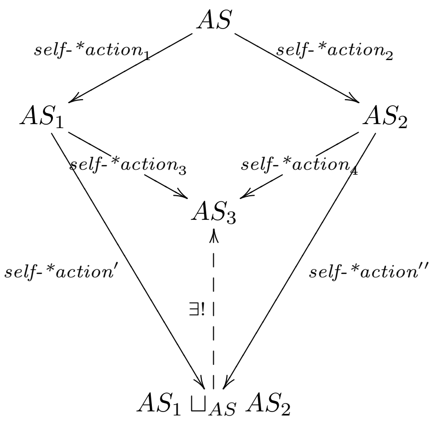
$$\ll \textit{self-*action}_3, \textit{self-*action}_4 \gg : AS_1 \sqcup_{AS} AS_2 \rightarrow AS_3$$

making everything commute. In other words,

$$\textit{self-*action}_3 = \ll \textit{self-*action}_3, \textit{self-*action}_4 \gg \circ \textit{self-*action}'$$

and

$$\textit{self-*action}_4 = \ll \textit{self-*action}_3, \textit{self-*action}_4 \gg \circ \textit{self-*action}''$$



(18)

5.2 Coequalizers of Self-*

Suppose given two parallel self-* actions

$$AS_1 \underset{\textit{self-*action}_2}{\overset{\textit{self-*action}_1}{\rightrightarrows}} AS_2$$

The coequalizer of $\textit{self-*action}_1$ and $\textit{self-*action}_2$ is the commutative diagram in (19),

$$AS_1 \underset{\textit{self-*action}_2}{\overset{\textit{self-*action}_1}{\rightrightarrows}} AS_2 \xrightarrow{q} \textit{Coeq}(\textit{self-*action}_1, \textit{self-*action}_2) \quad (19)$$

where we define the coequalizer of $\textit{self-*action}_1$ and $\textit{self-*action}_2$ is the quotient of AS_2 by the equivalence relation generated by

$$\{(\textit{self-*action}_1(a), \textit{self-*action}_2(a)) | a \in AS_1\} \subseteq AS_2 \times AS_2$$

In other words,

$$\textit{Coeq}(\textit{self-*action}_1, \textit{self-*action}_2) \overset{def}{=} AS_2 / \textit{self-*action}_1(a) \sim \textit{self-*action}_2(a)$$

6 Conclusions

The paper is a reference material for readers who already have a basic understanding of self-* in ASs and are now ready to consider finite limits and colimits in ASs using algebraic language. Algebraic specification is presented in a straightforward fashion by discussing in detail the necessary components and briefly touching on the more advanced components.

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