Forecasting the Brown Plant Hopper Infection Levels Using Set-Valued Decision Rules

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Abstract. This study introduces a new approach in forecasting the brown plant hopper (BPH) infection levels using the set-valued decision rules. The experiment was conducted in two scenarios, and was supported by the tool SSBPH – the tool is developed by authors. The experimental results help the agricultural managers process information on BPH, forecast the infection of BPH, and give the recommendation to farmers.

Keywords: Brown plant hopper · Set-valued decision rule · Infection level

1 Introduction

Brown plant hoppers (*BPH*) cling to the stem of rice to absorb the sap. In addition, BPH also cause the yellow dwarf disease - an extremely dangerous disease for rice. The life cycle of BPH is 25–28 days. In each month, the peak period that the mature BPH fly into light traps is 5–7 days [3]. BPH not only spread in the local scope, but also migrate from one region to another [2]. Therefore, forecasting the BPH infection levels help the agricultural managers give the recommendation to farmers. The agricultural managers use the number of mature BPH flying into light traps and other influential factors such as the wind direction, the wind speed, the rainfall, the humidity, etc. to conduct the forecast of BPH.

Some studies on warning BPH are presented in [2, 13, 15, 16]. In [15], simulating the BPH density based on the interpolation technique and the multi-agents system supports the managers in observing the distribution of BPH populations on rice fields. In [13], simulating the spread of BPH in the Mekong River delta is based on the life cycle of BPH and the wind direction. In [2], monitoring the cultivated area, the used rice varieties as well as the development stages of the rice are used in order to prevent BPH. In [16], the linear correlation between the areas cultivated by the BHP resistance varieties and the areas infected by BHP, and the linear regression equation was used to delineate the map of the BHP infected areas. However, all these studies are the single-valued approaches.

This paper proposes a new approach to forecast the BPH infection level using the set-valued decision rule. This rule is formed by many factors such as the number of BPH flying into the light traps, the wind speed, the wind direction, the temperature, and

the rainfall. In addition, the peak period must also be selected. This approach will specify the BPH infection level in a region; thereby build the plans for the BPH warning on the immigration levels and the propagation directions in scenarios.

The paper is organized into 5 sections. The first section introduces the context and the approach for solving the problem. The second section focuses on some definitions of set-valued decision information system. The third section proposes the model, forms the set-valued decision rules to warn BPH. The fourth section introduces data and the tool SISBPH used for experiments, and some scenarios for warning BPH. The final section is the conclusion.

2 Set-Valued Decision Rules

2.1 Set-Valued Decision Information System

The set-valued decision information system is defined as as a collection of 4 elements [18] $\langle U, Q_1 \cup \{d\}, V, f \rangle$, where U is a nonempty finite set of N objects $\{x_1, x_2, \ldots, x_N\}$; Q₁ is a finite set of the condition attributes; d is a decision attribute; $Q = Q_1 \cup \{d\}, Q_1 \cap \{d\} = \emptyset$; $V = V_{Q_1} \cup V_d$, where V_{Q_1} is a set of the condition attribute values and V_d is a set of the decision attribute value; f is a mapping from $Ux(Q_1 \cup \{d\})$ to V, that is $f : UxQ_1 \rightarrow 2^{VQ_1}$ is a set-valued mapping and $f : Ux\{d\} \rightarrow V_d$ is a single-valued mapping.

For example, Table 1 presents a set-valued decision information system of 10 objects = $\{x_1, x_2, ..., x_{10}\}$, each object has 1 decision attribute $\{d\}$ and 5 condition attributes $Q_1 = \{q_1, q_2, q_3, q_4, q_5\}$.

U	q_1	q ₂	q ₃	q_4	q ₅	d
x ₁	{1}	{0,1}	{0}	{1,2}	{2}	3
x ₂	{0,1}	{2}	{1,2}	{0}	{0}	1
X ₃	{0}	{1,2}	{1}	{0,1}	{0}	1
x ₄	{0}	{1}	{1}	{1}	{0,2}	2
X5	{2}	{1}	{0,1}	{0}	{1}	2
x ₆	{0,2}	{1}	{0,1}	{0}	{1}	2
X7	{1}	{0,2}	{0,1}	{1}	{2}	3
X ₈	{0}	{2}	{1}	{0}	{0,1}	1
X9	{1}	{0,1}	{0,2}	{1}	{2}	3
x ₁₀	{1}	{1}	{2}	{0,1}	{2}	2

Table 1. A set-valued decision information system of 10 objects.

2.2 Maximal Tolerance Class

Given a set-valued decision information system $\langle U, Q_1 \cup \{d\}, V, f \rangle$, a tolerance class of $x \in U$ based on set of the condition attributes $B \subseteq Q_1$ is defined as the following [18]: $T_B(x) = \{y/y \in U, \forall b \in B : b(x) \cap b(y) \neq \emptyset\} = \bigcap_{b \in B} T_b(x)$.

For example, using Table 1 and if $B = Q_1 = \{q_1, q_2, q_3, q_4, q_5\}$, we have $T_B(x_1) = T_B(x_7) = \{x_1, x_7, x_9\}$, $T_B(x_2) = T_B(x_8) = \{x_2, x_3, x_8\}$, $T_B(x_3) = \{x_2, x_3, x_4, x_8\}$, $T_B(x_4) = \{x_3, x_4\}$, $T_B(x_5) = T_B(x_6) = \{x_5, x_6\}$, $T_B(x_9) = \{x_1, x_7, x_9, x_{10}\}$, $T_B(x_{10}) = \{x_9, x_{10}\}$.

Classifying the set of objects U using the concept of the tolerance class can lead to two cases: (1) the objects of $T_Q(x)$ may not possess any common attribute values; (2) with $x \neq y$; x, $y \in U$, there exist the inclusion relation between $T_Q(x)$ and $T_Q(y)$. In the above example, the inclusion relation is $T_B(x_2) \in T_B(x_3)$.

The maximal tolerance class is the maximal set of objects which are tolerant with each other. For example, using Table 1, the maximal tolerance classes are presented in Table 2.

Maximal tolerance class	Tolerance value
$\{x_{1,}, x_{7}, x_{9}\}$	(1, 0, 0, 1, 2)
$\{x_2, x_3, x_8\}$	(0, 2, 1, 0, 0)
$\{x_3, x_4\}$	(0, 1, 1, 1, 0)
$\{x_5, x_6\}$	$(2, 1, \{0, 1\}, 0, 1)$
$\{x_9, x_{10}\}$	(1, 1, 2, 1, 2)

Table 2. The maximal tolerance classes.

2.3 Set-Valued Decision Rules

Given a set-valued decision information system $\langle U, Q_1 \cup \{d\}, V, f \rangle$, and $B \subseteq Q_1$, K^B (K^P) is called the set of maximal tolerance classes on B (P). des K^B is the set of tolerance values of K^B . The decision rule defined by K^B is [18]:

$$desK^B \rightarrow \bigvee_{i \in d(K^B)}(d,i)$$
 where $d(K^B) = \{i | \exists x \in K^B, d(x) = i\}$

- (i) if $d(K^B) = \{i\}, des(K^B) \rightarrow (d, i)$ is called the finite set-valued rule;
- (ii) if $i \in d(K^B)$ and $d(K^B) \{i\} \neq \emptyset$, $des(K^B) \to (d, i)$ is called the infinite set-valued rule.

Table 3. An example of the decision rules (* : the finite decision rule; ** : the infinite decision rule).

K_i^B	$des(K_i^B)$	The decision rule
$K_1^B = \{ \mathbf{x}_{1,} \mathbf{x}_{7}, \mathbf{x}_{9} \}$	(1, 0, 0, 1, 2)	$(1, 0, 0, 1, 2) \rightarrow (d,3)^*$
$K_2^B = \{\mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_8\}$	(0, 2, 1, 0, 0)	$(0, 2, 1, 0, 0) \rightarrow (d, 1)^*$
$K_3^B = \{x_3, x_4\}$	(0, 1, 1, 1, 0)	$(0, 1, 1, 1, 0) \rightarrow (d,1) \lor (d,2)^{**}$
$K_4^B = \{x_5, x_6\}$	$(2, 1, \{0, 1\}, 0, 1)$	$(2, 1, \{0, 1\}, 0, 1) \rightarrow (d, 2)^*$
$K_5^B = \{\mathbf{x}_9, \mathbf{x}_{10}\}$	(1, 1, 2, 1, 2)	$(1, 1, 2, 1, 2) \rightarrow (d,2) \lor (d,3)^{**}$

For example, using Table 1 and if $B = Q_1 = \{q_1, q_2, q_3, q_4, q_5\}$, we have the results as follows (Table 3).

3 Forecasting the BPH Infection Level Using the Set-Valued Decision Rules

3.1 The Set-Valued Decision Information System of the BPH Forecast

The set-valued decision information system is defined as a collection of 4 elements $\langle U, Q, V, f \rangle$. U is a nonempty finite set of N objects which are the light traps $U = \{b_1, b_2, ..., b_N\}$. Q is the set of 5 condition attributes which are the number of BPH, the wind direction, the wind speed, the temperature, and the rainfall $Q = \{q_1, q_2, q_3, q_4, q_5\}$; and 1 decision attribute. To solve the problem using the set-valued decision information system, some conventions are proposed as follows.

The number of BPH (q1): this attribute is the number of BPH flying into a light trap, and is described in Table 4.

The wind direction (q_2) : Vietnam has the tropical monsoon climate with two seasons annually (the dry season and the rainy season); and two main wind directions: southwest from January to July and northeast from late July to December. Therefore, in

The number of BPH (r)	Value of q_1 (the migration level of BPH)
0	0
$0 < r \le 200$	1
$200 < r \le 600$	2
$600 < r \le 900$	3
$900 < r \le 1200$	4
1200 < r	5

Table 4. The convention on the number of BPH flying into a light trap (unit: individual).

 Table 5. The description of the set-valued decision rules.

d	Explanation			
0	At present, the place of a light trap and its neighbours do not have BPH.			
1.1	At present, the migration level and the propagation direction at the place of a light trap			
	and its neighbours is 1 and 1 respectively.			
1.2	The migration level and the propagation direction are 1 and 2 respectively.			
2.1	The migration level and the propagation direction are 2 and 1 respectively.			
2.2	The migration level and the propagation direction are 2 and 2 respectively.			
3.1	The migration level and the propagation direction are 3 and 1 respectively.			
3.2	The migration level and the propagation direction are 3 and 2 respectively.			
4.1	The migration level and the propagation direction are 4 and 1 respectively.			
4.2	The migration level and the propagation direction are 4 and 2 respectively.			
5.1	The migration level and the propagation direction is 5 and 1 respectively.			
5.2	The migration level and the propagation direction is 5 and 2 respectively.			

the proposed model, information on the wind direction at a light trap is assigned one of two values: 1 (southwest) and 2 (northeast).

The wind speed, the temperature, and the rainfall (q_3, q_4, q_5) : are assigned the measured values in the reality (unit of wind speed is m/s, unit of temperature is °C, and unit of rainfall is mm).

The decision attribute (d): is defined as Table 5.

For example, data collected at 3 light traps and on 5 constitutive days is shown in Table 6. The value f of the light trap b_i and the attribute q_j is a set of 5 values corresponding to 5 days.

Table 6. A set-valued decision information system collected at 3 light traps and on 5 constitutive days.

U	q ₁	q ₂	q ₃	q ₄	q ₅	d
b_1	{1,1,1,1,1}	{1,1,1,1,1}	{2.3,3,4,4,2,3.6}	{27,28,28,28,29}	{4,3,0,0,0}	1.1
b_2	{1,1,2,1,1}	{1,1,1,1,1}	{2.3,3,4,4,2,3.5}	{27,28,28,28,29}	{3,1,0,0,0}	2.1
b_3	{3,1,1,1,1}	{1,1,1,1,1}	{2.3,3,4,4,2,3.6}	{27,28,28,28,29}	{4,3,0,0,0}	3.2

3.2 Forecasting the Migration of BPH – Scenario 1

At the nymphal period of the life cycle (7–10 days), BPH can move from this rice field to other rice fields. The migration of BPH will outbreak when the density is greater than 10000 individual/m² or the food is depleted (rice at the pre-harvest stage). When BPH migrate to a particular area, after 5 to 7 days, they will spawn. In the suitable environmental conditions, the proportion of female: male is 3: 1; each female can lay 150–250 eggs, and the hatching time is about 1 week, therefore there are the overlap generations [14].

The objective of the scenario is to identify locations where BPH migrate to and the propagation direction of BPH.

The decision rules

 $f(q_{1i})$ is called the value from day 1 to day i, *n.k* is called the decision value. If $\exists f(q_{1i}) = n \text{ and } n = \max(f(q_{1i})) \text{ then } d = n.k$. Suppose that the value of wind direction q_2 is 1, and attributes q_3 , q_4 , q_5 are suitable for the development and the migration of BPH, we define the decision rules as follows.

$$\begin{split} &\{q_{1i}=1,q_2=1,q_3,q_4,q_5\} \rightarrow (d,1.1); \\ &\{q_{1i}=2,q_2=1,q_3,q_4,q_5\} \rightarrow (d,2.1); \\ &\{q_{1i}=3,q_2=1,q_3,q_4,q_5\} \rightarrow (d,3.1); \\ &\{q_{1i}=4,q_2=1,q_3,q_4,q_5\} \rightarrow (d,4.1); \\ &\{q_{1i}=5,q_2=1,q_3,q_4,q_5\} \rightarrow (d,1.1). \end{split}$$

The propagation direction of BPH

If BPH migrate, they mainly move by jumping from this rice leaf to other leaf. Only individual with the long wing can move far. The migration of BPH is based on wind, therefore determining the wind direction and the wind speed is very important [11].

Suppose that the condition attributes q_3 , q_4 , q_5 are suitable for the development and the migration of BPH, and the value of q_1 is 3, the decision rules corresponding to the wind directions are:

$$\{q_1 = 3, q_2 = 1, q_3, q_4, q_5\} \to (d, 3.1); \{q_1 = 3, q_2 = 2, q_3, q_4, q_5\} \to (d, 3.2).$$

We call R to be the propagation radius $R = V_{wind}xT_{wind}(1)$ where V_{wind} is the wind speed, T_{wind} is the moving time. The spread area is the circular sector with an angle β . Its area is calculated by $S = \frac{1}{2}R^2\beta(2)$. BPH can migrate to regions of this area.

3.3 Forecasting the Migration of BPH – Scenario 2

The objective of the scenario is to identify locations where BPH do not migrate. The mature BPH tend to fly into the light traps strongly [3]. Therefore,

- If a light trap does not collect any BPH, there are three cases: (1) at present, the migration of BPH does not occur at the place of that light trap and its neighbors;
 (2) at present, BPH is not yet mature; (3) both of the above cases. The decision rule will be {q1 = 0, q2, q3, q4, q5} → (d, 0).
- If a light trap has BPH, the food (rice) is copious, and the BPH density in the rice field <10000 individual/m²; then the decision rule will be
- $\{q_1 \neq 0, q_2, q_3, q_4, q_5\} \to (d, 0).$

4 Experiment

4.1 Experimental Data

Experimental data is collected at 7 places locating 7 light traps $\{b_1, b_2, ..., b_7\}$ on 5 constitutive days from 02/10/2014 to 02/14/2014. For each place, data is stored in the table as the follow (Table 7).

	The number	Wind direction	Wind speed	Temperature	Rainfall
	of BPH			_	
Day 1	0	Southwest	2.3	27	30
Day 2	0	Southwest	3.4	28	3
Day 3	0	Southwest	4	28	0
Day 4	0	Southwest	2	28	0
Day 5	0	Southwest	3.4	29	0

Table 7. The collected data of the first place (b_1) on 5 constitutive days.

4.2 Tool SSBPH

Tool SSBPH (Set valued decision Information System - warning Brown Plant Hopper) is implemented by using Smalltalk [10], and integrated into Netgen framework [9]. This tool support users the functions: (i) read and store data into a template; (ii) standardize the single-valued data to store into a set-valued decision information systems; (iii) build the tolerance classes; (iv) define the maximal tolerance classes; (v) create the set-valued decision rules; (vi) display the migration level and the propagation direction of BPH on the map; (vii) produce the reports, the charts, and the support plans on warning BPH (Fig. 1).



Fig. 1. The tool SISBPH: (I) is the input data – information collected from the places locating the light traps are the number of BPH, the wind direction, the wind speed, the temperature, the rainfall; (II) is the list of functions of this tool; and (III) is the output such as the chart, the report, and the support plans.

4.3 Identifying the Migration of BPH – Scenario 1

The objective of the scenario is to identify locations where BPH migrate to and the propagation direction of BPH for 7 locations $\{b_1, b_2, b_3, b_4, b_5, b_6, b_7\}$.

Using the tool SISBPH and empirical data collected from 7 light traps, the obtained results are the follows.

The maximal tolerance classes are $K_1 = \{b_1, b_3\}; K_2 = \{b_2, b_6\}; K_3 = \{b_4, b_5, b_7\}$ *The decision rules* are $desK_1 = (0, 1, \{2.3, 3.4, 4\}, \{27, 28, 29\}, \{30, 3, 0\}) \rightarrow (d, 0)$ $desK_2 = (\{1, 2\}, 1, \{2.3, 4, 2\}, \{27, 28, 29\}, \{3, 0\}) \rightarrow (d, 1) \lor (d, 2)$ $desK_3 = (\{2, 3\}, 1, 2, \{28\}, \{3, 1, 0\}) \rightarrow (d, 2) \lor (d, 3)$

Because of $(\{q_1 > 0\}, q_2, q_3, q_4, q_5) \rightarrow (d, a)$ and a > 0, the decision rules created by K₂ and K₃ are used for identifying the migration of BPH. For K₂, we choose $d_2 = \max(d_{2i}) = 2(2.1)$; For K₃, we choose $d_3 = \max(d_{3i}) = 3(3.1)$. Therefore, the migration level of BPH at locations $K_2 = \{b_2, b_6\}$ is level 2, and at locations $K_3 = \{b_4, b_5, b_7\}$ is level 3.

The propagation direction. At locations $(\{b_2, b_6\}, \{b_4, b_5, b_7\})$, given the moving time $T_{wind} = 2$ h and $\beta = 30^{\circ}$, using formulae (1) and (2), the propagation radius R and the area of the circular sector S are R = 19.92 km and S = 597.6 km². The result is shown in Fig. 2.

At that time, the mature BPH flying into the light traps is in level 2 (200–600 individuals) and level 3 (600–900 individuals), we propose the following plans.

Plan 1: at locations K_2 and K_3 , rice planted at the propagation regions is very young (seedlings).



Fig. 2. The map displaying the propagation of BPH at locations K_2 and K_3 with the propagation radius R = 19.92 km and the propagation area S = 597.6 km².

The farmers should let the rice sprout to be flooded in water at night (from 5 pm to 7 am the next morning), and outcropped the water at daytime; this work is maintained in 3–4 days; until the farmers do not see the mature BPH flying the light traps, the water will be managed under the normal method.

Plan 2: at locations K_2 and K_3 , rice planted at the propagation regions is less than 20 days old.

Plan 1 can be applied. However, if the density of migration of BPH is high, the farmers should spray the pesticide to keep the natural enemies system on the rice fields. The best time to spray is the time that the number of BPH flying into the light traps is the largest.

Plan 3: at locations K_2 and K_3 , rice planted at the propagation regions is greater than 20 days old.

If on the rice field, the BPH is at the age of 1-3, or the number of mature BPH is equal or greater than 3 individuals/strand, the farmers should spray the pesticide.

Plan 4: at locations K₂ and K₃, the propagation regions are prepared for sowing.

The number of BPH flying into the light traps is still collected in next days, if this number decreases, the farmers should prepare seeds and sow them.

4.4 Identifying the Migration of BPH – Scenario 2

The objective of the scenario is to identify locations where BPH do not migrate for 7 locations $\{b_1, b_2, b_3, b_4, b_5, b_6, b_7\}$.

Similar to scenario 1, the tool SISBPH and empirical data collected from 7 light traps are also used. The maximal tolerance classes K_1 , K_2 , K_3 and the decision rules of both scenarios are the same. Because of $desK_1 \rightarrow (d, 0)$, there is no BPH at locations $K_1 = \{b_1, b_3\}$ and its neighbors. In this case, we do not calculate the radius R and identify the propagation direction.

Plan 1: b_1 , b_3 are the propagation regions.

The farmers should visit the rice fields regularly; specify the age of BPH; continue to monitor the light traps every day. When there is the migration of BPH, scenario 1 will be used.

Plan 2: b_1 , b_3 are not the propagation regions.

The farmers should visit the rice fields regularly; specify the density of BPH; follow the recommendations of experts on using rice varieties, the pesticide, etc.

5 Conclusion

The paper presents the set-valued decision information system and the basis for establishing the set-valued decision rules. Based on the presented theory, this paper proposes a model to warn the migration of BPH.

In this study, the tool SISBPH is developed to support the experts on warning BPH. This tool process the collected data, established the set-valued decision rules, identify the locations where BPH migrate to as well as the infection levels. The experiment is conducted at seven locations in the Mekong Delta; from this experiment the plans are proposed.

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