Multimodality in the Rainfall Drop Size Distribution in Southern England

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Abstract. Mutimodality appears in the modelling of rainfall drop size distributions (DSDs), and the understanding of the distribution in general is important as it helps in the predicting and mitigation of attenuation due to rain of satellite signals in frequencies above 10 GHz. This work looks at the occurrence of multimodality in the rainfall DSDs in southern England, with data captured at the Chilbolton Observatory for a seven year period (2003 to 2009). The investigation looks at the variation in the number of modes against different rain rates and seasons. It shows that multimodality is a relatively common occurrence, and hence there is a need to model this phenomenon when attempting to predict rain attenuation of satellite signals.

Keywords: Probabilistic forecasting · Theoretical modelling · Space and satellite communications · Estimation and forecasting · Probability distributions · Rainfall drop size distribution (DSD) · Multimodality

1 Introduction

Understanding the rainfall drop size distribution (DSD) is important in the understanding of transmissions using frequencies above 10 GHz if optimal use is to be made of bandwidth. There is presently enormous demand for telecommunication services leading to an increasing need to utilize more transmission bandwidth. Lower frequency bands are now congested, and providers presently utilise higher frequencies. The transmission of signals at frequencies above 10 GHz is however much more susceptible to attenuation due to precipitation. The precipitation leads to degradation in the desired quality of service and link availability. Raindrops, in particular, absorb and scatter radio wave energy.

There is the need to properly estimate the attenuation due to rainfall, as over-estimating is wasteful of resources, while under-estimating may lead to system outages. In order for engineers to design dependable systems, there is a need to reliably predict how precipitation, and rain in particular, attenuates transmitted signals.

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Modelling the rainfall drop size distribution (DSD) is a key ingredient in the prediction of rain attenuation thereby allowing providers to design mitigation techniques to counter attenuation due to these rain events.

This work describes a study of rainfall drop size distributions, particularly the occurrence of multimodal distributions, based on data collected between 2003 and 2009 from a disdrometer located at the Chilbolton Observatory in southern England.

2 DSD Modelling

2.1 Standard Statistical Models

Rainfall drop size distribution, $N(D)$ (in m^{-3} mm^{-1}), is defined as the number of raindrops per unit volume per unit diameter, centred on D (in mm). $N(D)dD$, expressed in m^{-3}, is the number of such drops per unit volume having diameters in the infinitesimal range ($D - dD/2$, $D + dD/2$) of size dD centred on D.

Several researchers have proposed various standard classical statistical distributions for the non-negative continuous DSD, $N(D)$. Marshall and Palmer [1] proposed the relationship

$$N(D) = N_0 exp(-\Lambda D),\ 0 < D \le D_{max} \tag{1}$$

where D_{max} is the maximum drop diameter, where $\Lambda = \alpha R^{\beta}$ (in mm^{-1}) is a function of the rainfall rate R (mm/h). This however fails for small diameters ($D < 1.5$ mm).

Other later researchers also modelled rainfall data using the lognormal distribution [2–5]. The lognormal distribution for the number of drops in a given volume is given in the general form

$$N(D) = \frac{N_T}{\sqrt{2\pi}\sigma_g(D-\theta)} \exp\left[-\frac{\left(\ln(D-\theta)-\mu_g\right)^2}{2\sigma_g^2}\right] \tag{2}$$

Ulbrich and Atlas [6] showed a gamma distribution yielded better rainfall rate computations when combined with radar data. They used a gamma distribution expressed in the form

$$N(D) = N_T D^{\mu} \exp(-\Lambda D) \qquad 0 \le D \le D_{max} \tag{3}$$

with Λ, μ, and N_T as the slope, shape and scaling parameters respectively, and these allow for the characterization of a wide range of rainfall scenarios, though in the paper they use $\mu = 2$, and the exponential distribution is a special case of the gamma distribution with $\mu = 0$. Ulbrich and Atlas [6] do not actually claim the DSD is a gamma distribution, but simply that a gamma distribution yields more accurate rainfall rate computations. They accept that other distributions might serve equally well. It should be noted that these three standard models for DSDs are all unimodal.

A typical example of rainfall drop size distribution modelled with lognormal, gamma (with the method of moments) and gamma (with the method of likelihood estimates) is shown in Fig. 1a below for 6th January, 2007 at 14:24, with 11.1 mm/h rain rate. A goodness of fit is done with the data and the various statistical distributions using Pearson's chi square, and the data did not reject any of the distributions tested. Figure 1b, however shows a clear case of multimodality, and the chi square statistic show that none of the distributions can be fitted to the data.

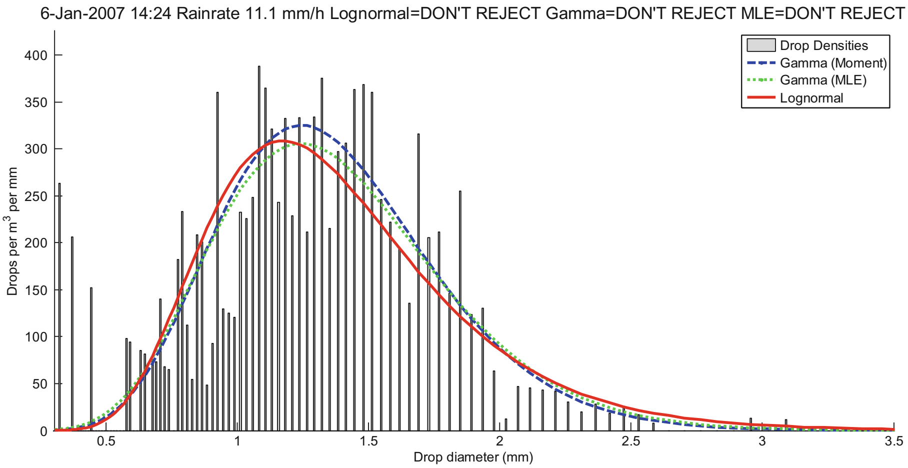

Fig. 1a: DSD for 6th Jan, 2007 at 14:24, modelled with three statistical distributions.

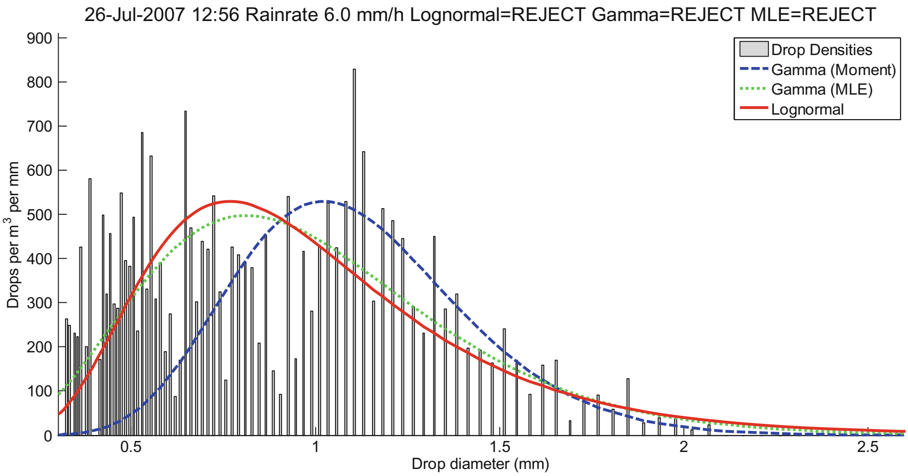

Fig. 1b: DSD for 26th July, 2007 at 12:56, modelled with three statistical distributions.

2.2 Multimodality in the DSD

Despite the standard models for DSDs all being unimodal, raindrop data often strongly suggest underlying multimodal distributions instead [7–10]. Steiner and Waldvogel [8] define an interval of drop sizes to be a mode if "… the concentration of raindrops per unit volume and unit diameter interval of a given interval was significantly larger than the concentrations of the two neighbouring diameter intervals". Sauvageot and Koffi [9] and Radhakrishna and Rao [10] simply implement this as $N(D_{i-1}) < N(D_i) > N(D_{i+1})$.

The challenge in detecting modes in the underlying distribution is that the data collected are necessarily a finite sample – if we have fine enough diameter measurements then every drop will appear in an interval by itself, with no drops in the intervals on either side, so there will be as many modes as there are drops. This is clearly unsatisfactory. In practice drop size data is collected using an instrument such as an impact disdrometer (see Sect. 3.1 for details) that counts the number of drops in a set interval of drop diameters or "bin". The counts in a number of adjoining bins are merged to produce smoother data; however the optimal number to merge is not clear.

Sauvageot and Koffi [9] attribute the presence of multimodality in DSDs to the overlapping of different rain shafts resulting from cloud volumes at different heights, and they also show that the number of peaks, N_m, of a DSD depends on the rain rate variations, and not on the mean rain rate, but this was for rain with $D_i > 2$ mm, where large N_m are inversely related to values of the slope parameter, λ and with large values of the intercept, N_0 of the exponential distribution. Steiner and Waldvogel [8] equally studied multimodal behaviours in DSDs and report that these modes existed for different drop size diameters in convective rain regimes. Radhakrishna and Rao's [10] study indicated that the appearance of multimodal distribution in the DSDs are dependent on the height, and varies with different rain systems, with multimodal distributions frequently encountered in convective rain systems. They classified the rain systems as convection, stratiform, and transition. This work however classes rain regimes as light, moderate, heavy and very heavy, as shown in Sect. 4.

3 Data and Procedure Used in This Study

3.1 Data Collection

This work utilised data captured by the RD-69 Joss-Waldgovel Impact Disdrometer connected to an ADA90 analyser at the Chilbolton Observatory in southern England (51.1° N, 1.4° W) between April 2003 and December 2009. Data was not captured from July 2005 to May 2006 and for 73 other days in this period. The disdrometer works by converting the vertical momentum of an impacting raindrop into an electrical pulse, and estimating the diameter of the raindrop from the amplitude of the pulse. The disdrometer has a surface area of 50 cm^2 and measures raindrop diameters from 0.3 mm to 5.0 mm in 127 gradations, or bins, sampling at 10 seconds intervals. The 127 size classes are distributed more or less exponentially over the range of drop diameters and the accuracy rate of the readings is ± 5 % of measured drop diameter [11]. A picture of the disdrometer at chilbolton (with a co-located rain guage) is shown in Fig. 2. This study aggregated six 10 seconds samples into a one minute samples to achieve a larger sample. This implicitly assumes that the underlying distribution is approximately stationary over a 1 min timescale. This is the approach taken by previous workers [12–14].

Furthermore, five adjoining bins were merged to smooth the data. This number was adopted as the best value after experimenting with different numbers – merging fewer bins results in too noisy a sample, whereas merging more bins results in too great a loss of detail. *Radhakrishna and Rao,* [10] merged over five minute interval, instead of five bins. This however requires stronger assumptions on the stationarity of the distribution in order to be confident that a multimodal distribution is not simply the result of merging different unimodal distributions (Fig. 3).

Fig. 2. The disdrometer and a co-located rain gauge at Chilbolton Observatory.

This work however looks at a different method in the determination of modes in a multimodal distribution by identifying each individual cluster (or distribution) from the troughs separating each cluster, here assuming that each cluster has a peak (mode). A trough (the end of a cluster, with an assumed peak) is thus defined as $N(D_i)$, when $N(D_{i-1}) > N(D_i) < N(D_{i+1}) < N(D_{i+2})$. This ensures a steady rise to determine the beginning of the next cluster. To ascertain the reliability of the method compared to that used by others [8–10], an inspection of the data for 27th July, 2007 showed that from 170 one-minute samples, 40 samples showed the same number of modes for both methods, while a visual inspection seem to agree with 38 samples for the earlier method and 93 for the current method.

4 Results

Using the data and procedure described in Sect. 3, distributions were fitted to a total of 166,065 one-minute samples, with 5 consecutive bins merged. Figure 3 shows a typical one-minute sample of data (26th July, 2007 at 17:45) which clearly suggests a trimodal distribution. The bar chart shows the distribution of the rain drop sizes, the drop densities in mm^{-1} m^{-3} plotted against the log of the drop diameters in mm. A log scale has been used simply to provide equally spaced bars (since as explained in Sect. 3.1, the bin sizes increase exponentially) – the use of a log scale has no effect on the multimodality.

The work investigated the variation of the modes with the rain rate and seasons of the year. Table 1 shows a summary of both unimodal and multimodal distributions for different rain rates. When the rainfall is classified as light (less than 2 mm/h), moderate (2 mm/h to 10 mm/h), heavy (10 mm/h to 50 mm/h), and very heavy (more than 50 mm/h) then it is noticeable that multimodality is more common at higher rain rates.

Table 1. Summary of results for unimode and multimode at different rain rates

Rain type	Rain rate	Unimode	Multimode			Unimode Total	Multimode Total	Grand Total
	(mm/h)	1	2	3	4+			
Light	<2	53 %	38 %	8 %	1 %	72,249	64,520	**136,769**
Moderate	2-10	32 %	48 %	18 %	2 %	8,872	18,446	**27,318**
Heavy	10-50	18 %	48 %	29 %	4 %	349	1,567	**1,916**
Very Heavy	> 50	16 %	47 %	34 %	3 %	10	52	**62**
Total		**81,480**	**66,395**	**16,653**	**1,537**	**81,480**	**84,585**	**166,065**

Results show that of the 166,065 samples, 136,769 (82 %) were classified as light rain (with rain rates less than 2 mm/h) and 72,249 samples in this rain regime were unimodal whilst 64,520 samples (39 %: 38 % bimodal, 8 % trimodal, and 1 % with 4 modes and above) were multimodally distributed as shown in Table 1. Considering all rain rates, 81,480 (49 %) samples were unimodal, whilst 84,585 (51 %) samples were multimodal.

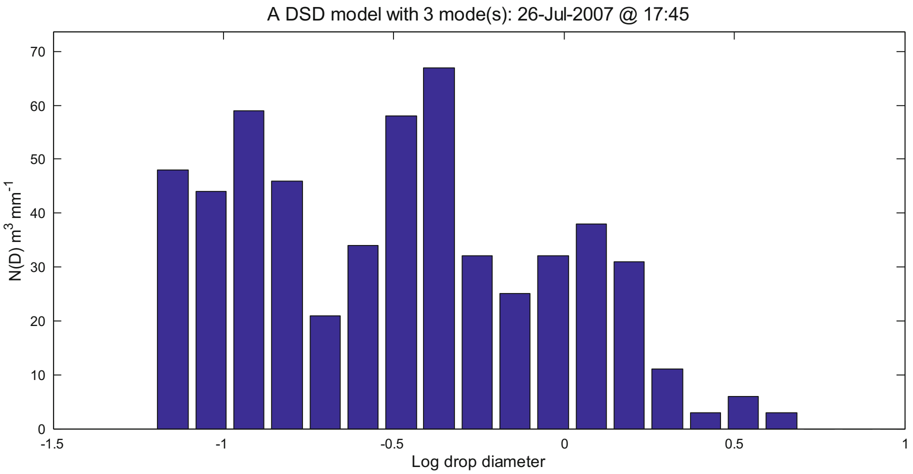

Fig. 3. DSD with three clusters for 26th July, 2007 at 17:45.

Considering the seasons; Spring (March-May), Summer (June-August), Autumn (September-November), and Winter (December to February), there does not seem to be a marked variation between the seasons. Results (Table 2) show that 22,794 (50 %) samples were unimodal in Winter, whilst 22,373 (39 % bimodal, 10 % trimodal, and 1 % with four modes and above) were multimodal in the same season. In all modes, the seasons were distributed as follows: Winter (27 %), Spring (24 %), Summer (24 %), and Autumn (24 %). The detailed distributions are as shown in the table below.

The fundamental result here is that multimodality does occur significantly often, particularly at higher rain rates.

Table 2. Summary of results for unimode and multimodes at different seasons

Season	Unimode	Multimode			Unimode Total	Multimode Total	Grand Total
	1	2	3	4+			
Winter	50 %	39 %	10 %	1 %	22,794	22,373	**45,167**
Spring	46 %	43 %	11 %	1 %	18,320	21,633	**39,953**
Summer	49 %	40 %	10 %	1 %	19,833	20,455	**40,288**
Autumn	51 %	39 %	10 %	1 %	20,533	20,124	**40,657**
Total	**81,480**	**66,395**	**16,653**	**1,537**	**81,480**	**84,585**	**166,065**

5 Interpretations, Further Work and Conclusions

This work has shown that whilst there is no discernible variation between the seasons, the different rain regimes however show a high occurrence of unimodality in light rains and high bimodality in the other rain types (moderate, heavy and very heavy). However, we should note that a large portion of the data was classified as light rain, and very little of the sample was classified as very heavy.

Although moderate, heavy and very heavy rains are less common than light rain, it is precisely these types of rain that cause the most attenuation to signals at frequencies above 10 GHz. Hence it is of concern that multimodality is common here, but none of the standard DSD models are multimodal. Ekerete et al. [7] highlighted the fact that the standard models often do not fit the data well, as assessed using the chi-square goodness of fit test. It would appear that a good explanation for this is that the data is multimodal but the models unimodal. More work is needed to find appropriate multimodal models for these multimodal situations.

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