Optimization of Collaborative Spectrum Sensing with Limited Time Resource

Fariba Mohammadyan[1(✉)], Zahra Pourgharehkhan[1], Abbas Taherpour[1], and Tamer Khattab[2(✉)]

[1] Department of Electrical Engineering, Imam Khomeini International University, Qazvin, Iran
{f_mohammadyan,pourgharehkhan}@edu.ikiu.ac.ir, taherpour@eng.ikiu.ac.ir
[2] Electrical Engineering, Qatar University, Doha, Qatar
tkhattab@ieee.org

Abstract. In this paper, Cognitive Radios (CRs) collaborate in spectrum sensing to detect random signals corrupted by Gaussian noise. Our analysis is based on a limited time resource assumption. This implies that the time resource dedicated for cooperative spectrum sensing process is constrained and shared between spectrum sensing time and results reporting time, which depends on the number of sensing users. We use common weighted gain combining detector to detect presence or absence of Primary User (PU). In order to find optimum gains, number of users and detection threshold, we maximize the achievable throughput with two approaches so that the predefined constraints on detection and false alarm probabilities are satisfied to protect the cooperative network performance quality. Analytical results in addition to simulation results show that the proposed schemes significantly outperform similar traditional detectors.

1 Introduction

The collaboration or cooperation among multiple Secondary Users (SUs) is one of the efficient approaches to make a reliable and accurate spectrum sensing in wireless channels where a single SU's sensing capability will be limited due to the deleterious channel effects such as shadowing [1–3]. Although, collaboration of SUs has a significant impact on decreasing the error probability of identifying the accurate status of the spectrum, it has some challenges: a large delay occurs for making final decision and CR network may be more affected by external attacks [4] and especially, the energy consumed in CR network is increased. Therefore, the analysis of the energy efficiency of cooperative spectrum sensing must be investigated before making any conclusions on the actual benefits of this approach. The energy efficiency of cooperative spectrum sensing has been investigated in many papers. However, the results available in the literature are not often directly comparable since the analysis is performed under different assumptions. Many works have investigated the optimization of the number of sensing users for several objectives. The problem was firstly formulated by [5],

M. Weichold et al. (Eds.): CROWNCOM 2015, LNICST 156, pp. 109–122, 2015.
DOI: 10.1007/978-3-319-24540-9_9

where the number of users is optimized to maximize a target function combining the detection performance and the usage efficiency of the resources. In [6] a new energy-efficient CSS scheme is investigated which implies that only a SU will broadcast its local decision among the whole network and other SUs will object to the fusion center, or agree with the announced decision. Using the proposed scheme, the broadcasting SU is selected so that to maximize energy efficiency. Several robust collaborative spectrum sensing schemes are presented in [7] wherein a trust value for each secondary user is obtained to reflect its suspicious level and mitigate its harmful effect on cooperative sensing. The motivation of the paper is to investigate the problem of the cooperative spectrum sensing considering limitation on time resources to make CR network more practically efficient. In order to have an efficient spectrum sensing with a controlled time, we suppose that a synchronous slotted communication protocol with duration T is employed by PU, in which SUs should perform sensing, result reporting and transmitting data operations according to PU's time slot. It is supposed that a fixed part of total time frame is dedicated for data transmission, while the rest is divided between local sensing and results reporting. The reporting channel between SUs and FC is considered Time Division Multiple Access (TDMA). So, the restriction on time duration of cooperative spectrum sensing causes relationship between number of SUs and their sample numbers. Unlike to the most of other works which assume a fixed sensing time and variable data/reporting times [8], our model does not affect data transmission and thus, makes cooperation a less ineffective process. In our approach, it is assumed that Fusion Center (FC) uses a useful and popular detector known as weighted gain combining (WGC)[9,10]. The WGC has better performance than the other energy detection-based detectors. In order to find optimum number of SUs, weighting gains vector and decision threshold, we maximize total achievable throughput which has an important role in efficiency of data transmission. Unlike to similar studied works available in literature, we analytically prove that our optimization problems are convex to make sure that derived optimal solutions are global. In studied optimization problems, we consider some constraints on number of SUs, predefined detection and false alarm probabilities to protect network requirements. We show that our proposed method outperforms the conventional WGC detectors in considered problems.

2 Basic Assumptions and System Model

Suppose there are N SUs available which are interested to detect presence or absence of the PU signal in a special frequency band and each SU receives M independent samples from the PU signal. A centralized topology is considered for secondary network in which the distance between users in secondary network is negligible compared to the distance between PU and SUs. Individual SUs use energy detector and send their sensing test statistics to FC through a control channel and in FC, final decision on presence or absence of PU is taken and then shared between SUs. We consider two basic assumptions for hypothesis testing problem and frame duration as follows:

Hypothesis Testing Problem. Regardless of any collaboration among the SUs, each SU has to decide individually based on its own received samples. In this case, the spectrum sensing for each SU in a wireless channel at m^{th} time instant can be modeled as a binary hypothesis testing problem as

$$\mathbf{y}_i(m) = \begin{cases} \mathbf{v}_i(m) & , \mathcal{H}_0 \\ h_i\ \mathbf{x}(m) + \mathbf{v}_i(m) & , \mathcal{H}_1 \end{cases} ; i = 1, 2, ..., N\ , \tag{1}$$

where $\mathbf{y}_i \in \mathbb{C}^M$ is the complex signal received by i^{th} SU, $\mathbf{h}_i$ is the channel gain between the PU and the i^{th} SU which is assumed that changes slowly such that it can be considered to be constant during each operation period of interest [8]. Also, we assume that $\mathbf{x} \sim \mathcal{CN}(\mathbf{0}, \sigma_x^2 \mathbf{I}_M)$ is the vector of the PU signal samples, and $\mathbf{v}_i \sim \mathcal{CN}(\mathbf{0}, \sigma_v^2 \mathbf{I}_M)$ is the vector of additive noise samples at the i^{th} SU. In order to do more accurate and faster spectrum sensing, the N SUs collaborate with each other by sharing information between themselves and after collaboration, the final collaborative decision about the absence or presence of the PU signal is made by the FC. Thus, for final collaborative decision at FC, we can write following binary hypothesis testing problem

$$\begin{cases} \mathcal{H}_0 : W < \eta, \text{ PU is absent} \\ \mathcal{H}_1 : W > \eta, \text{ PU is present.} \end{cases} \tag{2}$$

where η is the decision threshold and

$$W = \sum_{i=1}^{N} \mathbf{w}_i \mathbf{z}_i = \mathbf{w}^T \mathbf{z}. \tag{3}$$

is our total decision statistic at FC, where $\mathbf{w} = [\mathbf{w}_1, \mathbf{w}_2, ..., \mathbf{w}_\mathbf{N}]^T$ is the combining coefficients vector and the elements of $\mathbf{z} = [\mathbf{z}_1, \mathbf{z}_2, ..., \mathbf{z}_N]^T$ are our local test statistics which are defined as

$$\mathbf{z}_i = \sum_{m=1}^{M} |\mathbf{y}_i(m)|^2 = \|\mathbf{y}_i\|^2 \tag{4}$$

Additionally, we assume that $\|\mathbf{w}\| = 1$. In accordance with [8] and [11], since the local test statistics ($\mathbf{z}_i$) are normally distributed, their linear combination would also be distributed normally. Consequently, for the performance of the proposed cooperative spectrum detection scheme at the FC, we have

$$P_{\text{fa}} = P[W > \eta \mid \mathcal{H}_0] = Q(\frac{\eta - M\sigma_v^2 \mathbf{w}^T \mathbf{1}}{\sqrt{2M\sigma_v^4 \mathbf{w}^T \mathbf{w}}}). \tag{5}$$

where $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp\left(-\frac{u^2}{2}\right) du$ is the tail probability of the standard normal distribution and $\mathbf{1}$ is a vector with all elements equal to one. In addition,

$$P_{\text{d}} = P[W > \eta \mid \mathcal{H}_1] = Q(\frac{\eta - M\mathbf{w}^T(\sigma_x^2 \mathbf{h} + \sigma_v^2 \mathbf{1})}{\sqrt{2M\sigma_v^4 \mathbf{w}^T \mathbf{C} \mathbf{w}}}). \tag{6}$$

where $\mathbf{h} = [|h_1|^2, |h_2|^2, ..., |h_N|^2]^T$ and $\mathbf{C} = diag\{[\mathbf{1} + 2\boldsymbol{\gamma}]\}$ so that $\boldsymbol{\gamma} = [\gamma_1^2, \gamma_2^2, ..., \gamma_N^2]^T$ and $\gamma_i^2 \triangleq \frac{|h_i|^2\sigma_x^2}{\sigma_v^2}$ is the received Signal-to-Noise Ratio (SNR) at i^{th} SU.

Frame Duration Structure. The transmission is organized in frames of fixed time duration. The frame duration T is divided into three sub-frames: i) the sensing sub-frame of duration T_s, during which local sensing is performed; ii) the reporting sub-frame of duration T_r, where local results are reported to the FC; and iii) the data transmission sub-frame of duration T_t, where data transmission occurs if the channel is identified as free. As a consequence, $T = T_s + T_r + T_t$. We assume that T_t is given and fixed, while T_s and T_r are chosen in order to trade-off sensing and reporting reliability, respectively, such that T is kept fixed. The frame duration structure has been shown in Figure 1. If t_r is the time needed by each SU to report the sensed result to the FC, then $Tr = Nt_r$. It means that we have supposed the channel between SUs and the FC to be TDMA. Since T_t is assumed fixed, sensing duration can be expressed as

$$T_s = \underbrace{(T - T_t)}_{fixed} - T_r = T_{cte} - Nt_r \tag{7}$$

If $M = f_s T_s$ where f_s is sampling frequency, the number of sensing samples is expressed as a function of the number of SUs as follows

$$M = f_s(T_{cte} - Nt_r) \tag{8}$$

It can be observed that as N increases, sensing samples decreases.

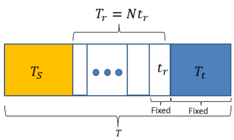


Fig. 1. The frame duration structure for cooperative spectrum sensing. Increasing the number of users yields decreasing sensing samples.

3 Optimization of Throughput

In this section, optimization of throughput function will be obtained with two distinct approaches. At first, we investigate joint optimization problem and then, we solve the optimization problem through optimizing a measure called *modified deflection coefficient*.

3.1 Joint Optimization

In order to create more chances for the SUs to send their data by higher rate when the frequency band is idle, the achievable throughput must be increased. The achievable throughput is defined here as [12]:

$$R(N, \mathbf{w}, \eta) = \pi_0 D_t T_t (1 - P_{\text{fa}}(N, \mathbf{w}, \eta)) \tag{9}$$

where π_0 and D_t $[bit/sec]$ are respectively the probability of the primary user being absent in the channel and the transmission rate. In addition, P_{fa} is the false alarm probability after replacing M from (8) which is shown as follow

$$P_{\text{fa}}(N, \mathbf{w}, \eta) = Q(\frac{\eta - f_s(T_{cte} - Nt_r)\sigma_v^2 \mathbf{w}^T \mathbf{1}}{\sqrt{2 f_s (T_{cte} - Nt_r)\sigma_v^4 \mathbf{w}^T \mathbf{w}}}) \tag{10}$$

Clearly, the higher throughput is achieved if the false alarm probability is decreased. On the other hand, more accurate and reliable cooperative spectrum sensing will be resulted when higher overall detection probability is provided. Therefore, we should make a compromise between the higher achievable throughput and more reliable sensing. From all above, the optimization problem can be defined as

$$\max_{\mathbf{w},\eta,N} : \quad R(N, \mathbf{w}, \eta) \tag{11a}$$

$$s.t. : \quad P_{\text{fa}} \leq \beta \tag{11b}$$

$$P_{\text{d}} \geq \bar{P}_{\text{d}} \tag{11c}$$

$$1 \leq N \leq N_{\max} \tag{11d}$$

$$\|\mathbf{w}\| = 1 \tag{11e}$$

$$\mathbf{w}, \eta > 0 \tag{11f}$$

where P_{d} is the detection probability and $N_{\max} = \frac{T_{cte}}{t_r} - 1$ is obtained when we assum $T_s = 0$. Additionally, β and $\bar{P}_{\text{d}}$ are respectively the predefined constraints of the false alarm and detection probabilities to protect network performance quality, which are desired as $0 < \beta < \frac{1}{2}$ and $\frac{1}{2} < \bar{P}_{\text{d}} < 1$. From (8), it is obvious that the optimization of N is equal to optimization of M. Also, note that when transmission time T_t is constant, the minimization of the false alarm probability is equal to maximization of the achievable throughput. Thus, the optimization problem can be replaced by

$$\min_{\mathbf{w},\eta,M} : \quad P_{\text{fa}}(M, \mathbf{w}, \eta) \tag{12a}$$

$$s.t. : \quad P_{\text{fa}} \leq \beta \tag{12b}$$

$$P_{\text{d}} \geq \bar{P}_{\text{d}} \tag{12c}$$

$$t_r f_s \leq M \leq f_s(T_{cte} - t_r) \tag{12d}$$

$$\|\mathbf{w}\| = 1 \tag{12e}$$

$$\mathbf{w}, \eta > 0 \tag{12f}$$

In order to solve the optimization problem (12), it is easily realized that the decision threshold should meet (12b) and (12c). Therefore, from (5) and (6) we have (13). On the other hand, P_{fa} and P_{d} are decreasing functions of η and so, to find the minimum value of false alarm probability, η should be maximized that causes reduction of P_{d}. As a consequence, the maximum value of the decision threshold which can satisfy $P_{\mathrm{d}} = \bar{P}_{\mathrm{d}}$ and minimize the objective function of problem (12) is

$$Q^{-1}(\beta)\sqrt{2M\sigma_v^4\mathbf{w}^T\mathbf{w}} + M\sigma_v^2\mathbf{w}^T\mathbf{1} \leq \eta \leq Q^{-1}(\bar{P}_{\mathrm{d}})\sqrt{2M\sigma_v^4\mathbf{w}^T\mathbf{C}\mathbf{w}} + M\mathbf{w}^T(\sigma_x^2\mathbf{h} + \sigma_v^2\mathbf{1}) \tag{13}$$

$$\eta_{\mathrm{opt}} = Q^{-1}(\bar{P}_{\mathrm{d}})\sqrt{2M\sigma_v^4\mathbf{w}^T\mathbf{C}\mathbf{w}} + M\mathbf{w}^T(\sigma_x^2\mathbf{h} + \sigma_v^2\mathbf{1}) \tag{14}$$

Thus, the optimization problem can be rewritten as

$$\min_{\mathbf{w},M} : \quad Q\left(\frac{Q^{-1}(\bar{P}_{\mathrm{d}})\sqrt{2M\sigma_v^4\mathbf{w}^T\mathbf{C}\mathbf{w}} + M\sigma_x^2\mathbf{w}^T\mathbf{h}}{\sqrt{2M\sigma_v^4\mathbf{w}^T\mathbf{w}}}\right) \tag{15a}$$

$$s.t. : \quad P_{\mathrm{fa}} \leq \beta \tag{15b}$$

$$t_r f_s \leq M \leq f_s(T_{cte} - t_r) \tag{15c}$$

$$\|\mathbf{w}\| = 1 \tag{15d}$$

$$\mathbf{w} > 0 \tag{15e}$$

To find the minimum value of objective function, one approach is to use convex optimization methods. Since we encounter with a complicated optimization problem, an efficient suboptimal method to solve (15) is to minimize the upper bound of its objective function. Using Rayleigh-Ritz theorem and (15d) and by noticing the fact that $Q^{-1}(\bar{P}_{\mathrm{d}}) < 0$ (since $\bar{P}_{\mathrm{d}} > \frac{1}{2}$), we have

$$\begin{aligned} &Q\left(\frac{Q^{-1}(\bar{P}_{\mathrm{d}})\sqrt{2M\sigma_v^4\mathbf{w}^T\mathbf{C}\mathbf{w}} + M\sigma_x^2\mathbf{w}^T\mathbf{h}}{\sqrt{2M\sigma_v^4\mathbf{w}^T\mathbf{w}}}\right) \\ &= Q\left(Q^{-1}(\bar{P}_{\mathrm{d}})\sqrt{\frac{\mathbf{w}^T\mathbf{C}\mathbf{w}}{\mathbf{w}^T\mathbf{w}}} + \frac{M\sigma_x^2\mathbf{w}^T\mathbf{h}}{\sqrt{2M\sigma_v^4\mathbf{w}^T\mathbf{w}}}\right) \\ &\leq Q\left(Q^{-1}(\bar{P}_{\mathrm{d}})\sqrt{\lambda_{\max}\mathbf{C}} + \frac{M'\mathbf{w}^T\boldsymbol{\gamma}}{\sqrt{\mathbf{w}^T\mathbf{w}}}\right) \\ &= Q\left(Q^{-1}(\bar{P}_{\mathrm{d}})\sqrt{\lambda_{\max}\mathbf{C}} + M'\mathbf{w}^T\boldsymbol{\gamma}\right) \end{aligned} \tag{16}$$

where $M' \triangleq \sqrt{\frac{M}{2}}$ and $\lambda_{\max}\mathbf{C}$ denotes maximum eigenvalue of $\mathbf{C}$. Since matrix $\mathbf{C}$ is diagonal, the eigenvalues are simply recognizable on the diagonal of matrix, and when we assume the SNRs in descending order of their γ_i so that the 1^{st} SU in the list has the highest received SNR ($\gamma_1 \geq \gamma_2 \geq ... \geq \gamma_N$), $\lambda_{\max}\mathbf{C}$ equals $1 + 2\gamma_1$. By minimizing the upper bound of the objective function, a good approximation to the optimal solution of the original problem is achieved.

Therefore, (15) can be reformulated into an equivalent form with an objective function upper bounded by a convex function as

$$\min_{\mathbf{w},M'}: \quad f(M',\mathbf{w}) = Q\left(Q^{-1}(\bar{P_{\mathrm{d}}})\sqrt{\lambda_{\max}\mathbf{C}} + M'\mathbf{w}^T\boldsymbol{\gamma}\right) \tag{17a}$$

$$s.t.: \quad P_{\mathrm{fa}} \leq \beta \tag{17b}$$

$$\sqrt{\frac{t_r f_s}{2}} \leq M' \leq \sqrt{\frac{f_s(T_{cte}-t_r)}{2}} \tag{17c}$$

$$\|\mathbf{w}\| = 1 \tag{17d}$$

$$\mathbf{w} > 0 \tag{17e}$$

Here, we have an optimization problem with $N+1$ variables which is proved to be convex through following lemma:

Lemma 1. *The optimization problem (17) is convex with respect to M' and coefficient vector $\mathbf{w}$ if*

$$0 < \beta \leq Q\left(\frac{1}{-Q^{-1}(\bar{P_{\mathrm{d}}})\sqrt{\lambda_{\max}\mathbf{C}} + \sqrt{Q^{-1}(\bar{P_{\mathrm{d}}})^2\lambda_{\max}\mathbf{C}+2}}\right)$$

Proof. See Appendix A.

Moreover, as mentioned before, in the throughput function all the elements are constant values, except P_{fa}. Thus, convexity of $P_{\mathrm{fa}}(M',\mathbf{w})$, means concavity of $R(M',\mathbf{w})$ and so, the maximum value of throughput can be achieved easily. By applying the following proposed algorithm, the optimum values of $\mathbf{w}$ and N_{opt} are achieved. According to (8) and relation between M and M', there is

$$N_{opt} = \frac{T_{cte}}{t_r} - \frac{2M'^2_{opt}}{f_s t_r} \tag{18}$$

In this algorithm, the variable N is each time selected respectively from 1 to N_{max}, and every time for selected N, we have a vector variable with specified size, which is found from (17). Then, the objective function f is calculated every time and the values of N, $\mathbf{w}$ correspond to minimum one are interpreted as optimum values.

3.2 Optimization of Throughput by Maximizing Modified Deflection Coefficient

Here, we present an approach to solve optimization problem (11) via maximizing modified deflection coefficient. This measure is used for evaluating detection performance at the FC. When the test statistic W is normally distributed under both hypotheses, for a determined probability of false alarm, maximizing $d_N^2(\mathbf{w})$ leads to an increment of detection probability. Although the method incurs small performance degradation, due to its less computational complexity has been

Algorithm 1. Joint optimization algorithm for problem

Set $N_{max} = \frac{T_{cte}}{t_r} - 1$ and $f_0(M', \mathbf{w}) \triangleq \infty$
for $N = 1 : N_{max}$
    Find M_N from (8)
    Set $M'_N \triangleq \sqrt{\frac{M_N}{2}}$
    Find optimum N-dimensional vector ($\mathbf{w_N^{opt}}$) by (17)
    Put M'_N and $\mathbf{w}_N^{opt}$ in $f_N(M', \mathbf{w})$
    if $f_N(M', \mathbf{w}) > f_{N-1}(M', \mathbf{w})$ then
        $f_{opt}(M', \mathbf{w}) = f_{N-1}(M', \mathbf{w})$,
        $\mathbf{w}_{opt} = \mathbf{w}_{N-1}^{opt}, \ M'_{opt} = M'_{N-1}$
        Using M'_{opt}, obtain N_{opt} from (18).
    end if
end for

interesting in the literature. This method is completely interpreted in [8] and [11]. By applying this method, we are able to find optimum weight vector value and replace it in the optimization problem. The modified deflection coefficient is defined as

$$d_N^2(\mathbf{w}) = \frac{\left(\mathbb{E}[W|\mathcal{H}_1] - \mathbb{E}[W|\mathcal{H}_0]\right)^2}{Var[W|\mathcal{H}_1]} = \frac{f_s(T - Nt_r)(\mathbf{w}^T\gamma)^2}{2\mathbf{w}^T\mathbf{C}\mathbf{w}} \tag{19}$$

Hence, we have to maximize $d_N^2(\mathbf{w})$ while having a constraint on the weight vector to be on the unit-norm ball. So

$$\max_{\mathbf{w}} : \quad d_N^2(\mathbf{w}) \tag{20a}$$

$$s.t. : \quad \| \mathbf{w} \|_2 = 1 \tag{20b}$$

We can rewrite equation (19) to obtain

$$\frac{f_s(T - Nt_r)\mathbf{w}^T\gamma\gamma^T\mathbf{w}}{2\mathbf{w}^T\mathbf{C}\mathbf{w}} = \frac{f_s(T - Nt_r)\mathbf{w'}^T\mathbf{C}^{-\frac{T}{2}}\gamma\gamma^T\mathbf{C}^{-\frac{1}{2}}\mathbf{w}'}{2\mathbf{w'}^T\mathbf{w}'}$$
$$\leq \frac{f_s(T - Nt_r)}{2}\lambda_{max}(\mathbf{C}^{-\frac{T}{2}}\gamma\gamma^T\mathbf{C}^{-\frac{1}{2}}) \tag{21}$$

where, $\mathbf{w}'$ is defined as $\mathbf{w}' = \mathbf{C}^{\frac{1}{2}}\mathbf{w}$, and inequality results from Rayleigh-Ritz. Equality incurs when $\mathbf{w}'$ equals to eigenvector which is corresponded to maximum eigenvalue. Noting that $\mathbf{w}$ is a normalized vector, we can obtain it as

$$\mathbf{w'}_{opt} = \mathbf{C}^{-\frac{T}{2}}\gamma \rightarrow \mathbf{w}_{opt} = \frac{\mathbf{C}^{-1}\gamma}{\|\mathbf{C}^{-1}\gamma\|_2} \tag{22}$$

Now, we aim to solve problem (11). After we put $\mathbf{w}_{opt}$ in objective function, problem will change into

$$\max_{\eta,N}: \quad R(N,\eta) \tag{23a}$$
$$s.t.: \quad P_{\text{fa}} \leq \beta \tag{23b}$$
$$P_{\text{d}} \geq \bar{P}_{\text{d}} \tag{23c}$$
$$1 \leq N \leq N_{\max} \tag{23d}$$
$$\eta > 0 \tag{23e}$$

As seen in Section A, we can minimize P_{fa} instead of maximizing throughput and so, the problem is equal to

$$\min_{\eta,N}: \quad Q\left(\frac{\eta\|\mathbf{C}^{-1}\gamma\| - f_s(T_{cte}-Nt_r)\sigma_v^2(\gamma^{\mathbf{T}}\mathbf{C}^{-T})\mathbf{1}}{\|\mathbf{C}^{-1}\gamma\|\sqrt{2f_s(T_{cte}-Nt_r)\sigma_v^4}}\right) \tag{24a}$$
$$s.t.: \quad P_{\text{fa}} \leq \beta \tag{24b}$$
$$P_{\text{d}} \geq \bar{P}_{\text{d}} \tag{24c}$$
$$1 \leq N \leq N_{\max} \tag{24d}$$
$$\eta > 0 \tag{24e}$$

After rewriting constraint (24b) and (24c), we have (25) and (26).

$$Q^{-1}(\beta)\sqrt{2f_s(T_{cte}-Nt_r)\sigma_v^4} + f_s(T_{cte}-Nt_r)\sigma_v^2\frac{\gamma^{\mathbf{T}}\mathbf{C}^{-T}\mathbf{1}}{\|\mathbf{C}^{-1}\gamma\|} \leq \eta \tag{25}$$

and

$$\eta \leq \frac{Q^{-1}(\bar{P}_{\text{d}})\sqrt{2f_s(T_{cte}-Nt_r)\sigma_v^4\gamma^{\mathbf{T}}\mathbf{C}^{-T}\gamma}}{\|\mathbf{C}^{-1}\gamma\|} + \frac{f_s(T_{cte}-Nt_r)\gamma^{\mathbf{T}}\mathbf{C}^{-\mathbf{T}}(\sigma_x^2\mathbf{h}+\sigma_v^2\mathbf{1})}{\|\mathbf{C}^{-1}\gamma\|} \tag{26}$$

Similar to Section A, the optimum value of η occurs when η equals to its upper bound. Hence, by putting η in (24), the optimization problem turns into a single variable problem

$$\min_{N}: \quad Q\left(\frac{Q^{-1}(\bar{P}_{\text{d}})\sqrt{\gamma^{\mathbf{T}}\mathbf{C}^{-T}\gamma}}{\|\mathbf{C}^{-1}\gamma\|} + \frac{\sqrt{f_s(T_{cte}-Nt_r)}\gamma^{T}\mathbf{C}^{-T}\gamma}{\sqrt{2}\|\mathbf{C}^{-1}\gamma\|}\right) \tag{27a}$$
$$s.t.: \quad 1 \leq N \leq N_{\max} \tag{27b}$$

Lemma 2. *The optimization problem (27) is convex in N and so, $R(N)$ is concave.*

To proof the lemma, the second derivative of objective function is here

$$\frac{\partial^2 P_{\text{fa}}}{\partial N^2} = \frac{f_s^2 t_r^2(\gamma^T\mathbf{C}^{-T}\gamma)^2(\rho)\exp\left(-\frac{\rho^2}{2}\right)}{8\sqrt{2\pi}f_s(T_{cte}-Nt_r)\|\mathbf{C}^{-1}\gamma\|^2} \geq 0 \tag{28}$$

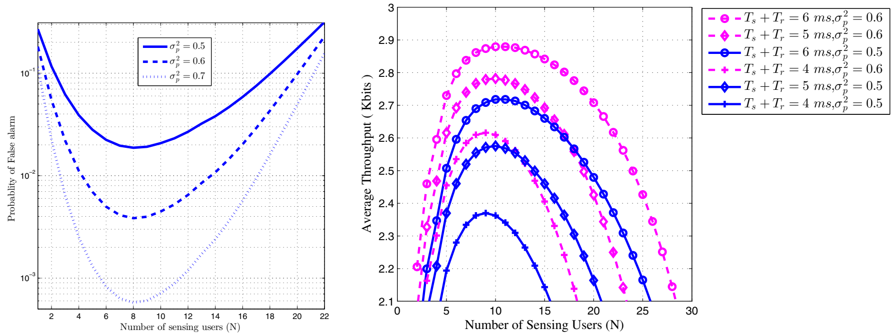
where, $\rho = \frac{Q^{-1}(\bar{P_{\rm d}})\sqrt{\gamma^{\mathbf{T}}\mathbf{C}^{-T}\gamma}}{\|\mathbf{C}^{-1}\gamma\|} + \frac{\sqrt{f_s(T_{cte}-Nt_r)}\gamma^T\mathbf{C}^{-T}\gamma}{\sqrt{2}\|\mathbf{C}^{-1}\gamma\|}$

and also, $\frac{\partial^2 R(N)}{\partial N^2} = -\pi_0 D_t T_t \frac{\partial^2 P_{\rm fa}}{\partial N^2} \leq 0$ which proves concavity of $R(N)$.

4 Numerical Results and Discussion

In this section, some simulation results are provided to evaluate the optimization problems and ensure the accuracy of the calculations. We have assumed that SNR of j^{th} SU in dB domain equals to γ_j. The basic parameters which are determined fixed in simulation results are as :

1. Frequency of sampling in each SU: $f_s = 10\ kHz$
2. Time of reporting the results to the FC by each SU: $t_r = 0.2\ ms$
3. Time of transmission if frequency is detected as idle: $T_t = 3\ ms$



(a) $T = 7\ ms$. (b) $D_t = 2\ Mbit/sec$ and $T_t = 3\ ms$.

Fig. 2. False alarm probability and Throughput with respect to number of SUs.

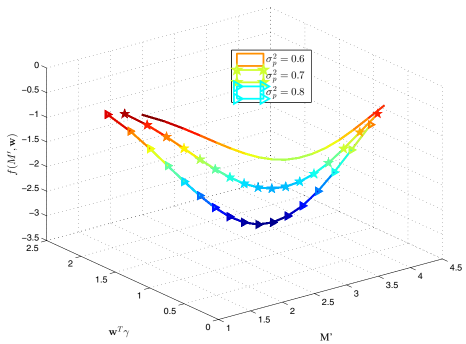
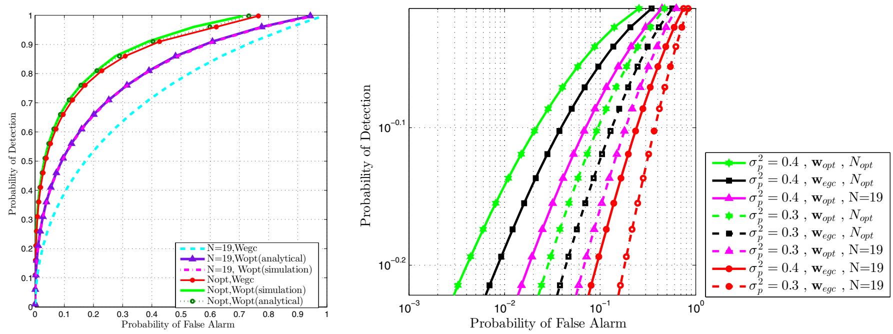


Fig. 3. Cost function of problem (17) versus M' and $\mathbf{w}^T\boldsymbol{\gamma}$ in different values of σ_p^2 when $T = 7\ ms$ and $P_{\rm d} = 0.9$.



(a) Comparison between ROCs when $\sigma_p^2 = 0.3$ and $T = 9\ ms$.

(b) Logarithmic ROCs with $N_{opt} = 10$, $T = 11\ ms$ and $P_{\rm d}$ changes between $(0.6, 1)$.

Fig. 4. Receiver Operation Characteristic (ROC)

4. Achievable throughput rate: $D_t = 2\ Mbit/sec$
5. Desired false alarm and detection probabilities to protect the QoS: $\beta = 0.1$ and $\bar{P}_{\rm d} = 0.9$
6. The prior probabilities are both assumed: $\pi_1 = \pi_0 = 0.5$.
7. Finally, it is assumed that spectrum sensing performs in low SNR regime.

Channel gain between PU and SUs is assumed such a slow flat fading channel. We note that the number of SUs is limited by $N_{\max} = \frac{T_{\rm cte}}{t_r} - 1$, that changes in different figures by changing the whole frame of time, So the optimum value of N is also changed. In Figure 2(a), using above algorithm, we depict the probability of false alarm versus number of SUs for three different cases of $\sigma_p^2 = \frac{\sigma_x^2}{\sigma_v^2} \triangleq 0.5,\ 0.6,\ 0.7$ when we have set $T_{\rm cte} = 4\ ms$. In each curve, the change in the number of users, causes variation in size of channel gain vector and as a result the SNR vector ($\boldsymbol{\gamma}$) is also changed. So, the SNR of SUs are respectively arranged in these ranges:$(-12.5, -4.5)dB$, $(-12, -3.5)dB$, $(-11, -3)dB$ respectively. Now, looking at Figure 2(b), we can evaluate achievable throughput in three different cases where $T_s + T_r$ varies from 4 ms to 6 ms and as a result, the frame duration changes: $T = 7\ ms$, $T = 8\ ms$, $T = 9\ ms$. Also, it is supposed that $\sigma_p^2 = 0.8$. From this Figure, we find out that, increasing the value of sensing and reporting time, leads to higher achievable throughput. In Figure 3, using above algorithm, we depict the logarithmic cost function of problem (17) versus parameters M' and $\mathbf{w}^T\boldsymbol{\gamma}$ for three different cases of $\sigma_p^2 = 0.6,\ 0.7,\ 0.8$ when we have set $T_{\rm cte} = 4\ ms$ and $P_{\rm d} = 0.9$. Having these parameters it is obvious that the minimum point is $(3.3, 1.57, -3.5)$ for $\sigma_p^2 = 0.8$ as an example. We can see that with a bit increase in the range of SNRs, $f(M', \mathbf{w})$ decreases significantly. We have illustrated the receiver operating characteristics (ROC) scheme of spectrum sensing in different states in Figure 4(a). It is assumed that $\sigma_p^2 = 0.3$ and $T = 9\ ms$. A comparison between the optimum state –which uses optimum values for variables N and $\mathbf{w}$– with two other cases is shown. One of them is obtained by allocating $N_{\rm opt}$

and weight vector with uniform elements such as equal gain combining (EGC) technique, And the other state is when the value of N is selected randomly between interval $[1, N_{max}]$, but the weight vector with N elements is set to $\mathbf{w}_{\text{opt}}$, which is obtained from analytical results in (22) and simulation both. As realized from the Figure, the curves which use $\mathbf{w}_{opt}$ from simulation and analysis are almost overlapped. For an example, $N = 19$ is depicted. What ever the selected N is adjacent to N_{opt}, the curve is closer to the optimum one. With given values for parameters in this Figure, the optimum number of SUs is achieved $N_{opt} = 7$. In Figure 4(b), two distinct ROCs are represented in different N and $\mathbf{w}$ values. Frame duration is considered $T = 11\ ms$ and the ranges for P_{d} is assumed between $[0.6, 1]$. By assigning $\sigma_p^2 = 0.3, 0.4$ the SNR vector($\boldsymbol{\gamma}$) is also changed. In each curve, the change in the number of users, causes variation in size of channel gain vector and as a result the SNR vector ($\boldsymbol{\gamma}$) is also changed. So, for curves with $N = 19$ and $\sigma_p^2 = 0.3$, the SNR elements are arranged in interval $[-31.5\ dB, -6.5\ dB]$, for $N = 19$ and $\sigma_p^2 = 0.4$: $[-30.5\ dB, -5.5\ dB]$, for curves with $N_{opt} = 10$, $\sigma_p^2 = 0.3$: $[-13.5\ dB, -6.5\ dB]$ and for $N_{opt} = 10$, $\sigma_p^2 = 0.4$: $[-12.5\ dB, -5.5\ dB]$. Looking at this Figure, it is obvious that increment of SNR ratio for all of the SUs leads to an enhancement in ROC as expected.

5 Conclusion

In this paper, we proved that to have an efficient cooperative spectrum sensing with limited time resource, while applying the optimum number and consuming lower energy resources, we would have a better performance such as higher achievable throughput. Moreover, by constraining the whole frame time of collaborative spectrum sensing, a relationship between number of samples and number of sensing users was obtained. From that, we could find the optimum number of samples too. Furthermore, through analytical results and also simulation results, it was shown that the upper bound of false alarm probability (and then the achievable throughput) is a convex (concave) function of SUs number and weighting vector with some constraints. So, the proposed scheme which uses jointly optimized number of SUs and weighting vector, outperforms significantly other traditional detectors that use one of the optimum values.

Acknowledgments. This publication was made possible by the National Priorities Research Program (NPRP) award NPRP 6-1326-2-532 from the Qatar National Research Fund (QNRF) (a member of the Qatar Foundation). The statements made herein are solely the responsibility of the authors.

A Appendix

In order to show that cost function of the optimization problem is convex with respect to $\mathbf{w}$ and M', we should prove that its Hessian matrix is positive semi definite. From (17), if $f(M', \mathbf{w}) = Q(\xi)$ where $\xi \triangleq Q^{-1}(\bar{P}_\mathrm{d})\sqrt{\lambda_{\max}\mathbf{C}} + M'\mathbf{w}^T\boldsymbol{\gamma}$, we can obtain hessian matrix as (29).

$$\mathbf{H} = \nabla^2 f(M', \mathbf{w}) = \begin{pmatrix} a & \mathbf{b}^T \\ \mathbf{b} & \mathbf{D} \end{pmatrix} = \tag{29}$$

$$\frac{1}{\sqrt{2\pi}} \begin{pmatrix} (\mathbf{w}^T\boldsymbol{\gamma})^2 \xi \exp(-\xi^2/2) & -\boldsymbol{\gamma}^T \exp(-\xi^2/2)[1 - M'(\mathbf{w}^T\boldsymbol{\gamma})\xi] \\ -\boldsymbol{\gamma} \exp(-\xi^2/2)[1 - M'(\mathbf{w}^T\boldsymbol{\gamma})\xi] & M'^2 \xi \exp(-\xi^2/2)(\boldsymbol{\gamma}\boldsymbol{\gamma}^T) \end{pmatrix}$$

To prove positiveness we have: If $a \succ 0 \Longrightarrow$ then, $\mathbf{H} \succeq 0 \Leftrightarrow \mathbf{S} = \mathbf{D} - \mathbf{b}a^{-1}\mathbf{b}^T \succeq 0$. In other words , since a is positive, the Hessian matrix $\mathbf{H}$ is positive semi-definite if and only if its Schur complement is positive semi-definite. Its Schur complement is $\mathbf{S} = \frac{\mathbf{D}a - \mathbf{b}\mathbf{b}^T}{a}$. Thus, we should have

$$\mathbf{S} = \frac{M'^2 \boldsymbol{\gamma}\boldsymbol{\gamma}^T \xi \exp(-\xi^2/2)}{\sqrt{2\pi}} - \frac{\boldsymbol{\gamma}\boldsymbol{\gamma}^T \exp(-\xi^2)(M'\xi\mathbf{w}^T\boldsymbol{\gamma} - 1)^2}{\sqrt{2\pi}(\mathbf{w}^T\boldsymbol{\gamma})^2 \exp(-\xi^2/2)\xi} \succeq \mathbf{0} \tag{30}$$

Therefore

$$\frac{(2M'\xi\mathbf{w}^T\boldsymbol{\gamma} - 1)\exp(-\xi^2/2)\boldsymbol{\gamma}\boldsymbol{\gamma}^T}{\sqrt{2\pi}(\mathbf{w}^T\boldsymbol{\gamma})^2\xi} \succeq \mathbf{0} \tag{31}$$

The matrix $\boldsymbol{\gamma}\boldsymbol{\gamma}^T$ has rank 1 and all the eigenvalues are zero except its maximum eigenvalue which equals $\boldsymbol{\gamma}^T\boldsymbol{\gamma}$ and is positive. So $\boldsymbol{\gamma}\boldsymbol{\gamma}^T \succeq \mathbf{0}$. Thus $\mathbf{S} \succeq \mathbf{0}$ if $2M'\xi\mathbf{w}^T\boldsymbol{\gamma} - 1 \geq 0$. In fact, after manipulation the inequality, the condition $P_\mathrm{fa} \leq Q(\frac{1}{2M'\mathbf{w}^T\boldsymbol{\gamma}})$ should be satisfied. But $\Theta = Q(\frac{1}{2M'\mathbf{w}^T\boldsymbol{\gamma}})$ depends on $\mathbf{w}$. So, we should find a lower bound for Θ to be ensure that P_fa is lower than this term and condition is satisfied. To this end, we can see the condition above as

$$2M'\xi\mathbf{w}^T\boldsymbol{\gamma} - 1 \geq 0$$
$$2M'\mathbf{w}^T\boldsymbol{\gamma}\left(Q^{-1}(\bar{P}_\mathrm{d})\sqrt{\lambda_{\max}\mathbf{C}} + M'\mathbf{w}^T\boldsymbol{\gamma}\right) \geq 1 \tag{32}$$

So,

$$2Q^{-1}(\bar{P}_\mathrm{d})\sqrt{\lambda_{\max}\mathbf{C}}(M'\mathbf{w}^T\boldsymbol{\gamma}) + 2(M'\mathbf{w}^T\boldsymbol{\gamma})^2 - 1 \geq 0 \tag{33}$$

which is a quadratic function of $M'\mathbf{w}^T\boldsymbol{\gamma}$. It can be easily shown that for this quadratic function to be positive, just one of the answers is acceptable. So, we should have

$$2M'\mathbf{w}^T\boldsymbol{\gamma} \geq -Q^{-1}(\bar{P}_\mathrm{d})\sqrt{\lambda_{\max}\mathbf{C}} + \sqrt{(Q^{-1}(\bar{P}_\mathrm{d}))^2\lambda_{\max}\mathbf{C} + 2} \tag{34}$$

which is equal to

$$Q\left(\frac{1}{2M'\mathbf{w}^T\boldsymbol{\gamma}}\right) \geq Q\left(\frac{1}{-Q^{-1}(\bar{P}_\mathrm{d})\sqrt{\lambda_{\max}\mathbf{C}} + \sqrt{(Q^{-1}(\bar{P}_\mathrm{d}))^2\lambda_{\max}\mathbf{C} + 2}}\right)$$

Therefore, for satisfying condition $P_\mathrm{fa} \leq Q(\frac{1}{2M'\mathbf{w}^T\boldsymbol{\gamma}})$ from (35), we obtain the condition for convexity of cost function of the optimization problem as [13]:

$$P_\mathrm{fa} \leq Q\left(\frac{1}{-Q^{-1}(\bar{P}_\mathrm{d})\sqrt{\lambda_{\max}\mathbf{C}} + \sqrt{Q^{-1}(\bar{P}_\mathrm{d})^2\lambda_{\max}\mathbf{C} + 2}}\right) \tag{35}$$

References

1. Ahsant, B., Viswanathan, R.: A review of cooperative spectrum sensing in cognitive radios. In: Mukhopadhyay, S.C., Jayasundera, K.P., Fuchs, A. (eds.) Advancement in Sensing Technology. SSMI, vol. 1, pp. 69–80. Springer, Heidelberg (2013)
2. Sedighi, S., Pourgharehkhan, Z., Taherpour, A., Khattab, T.: Distributed spectrum sensing of correlated observations in cognitive radio networks. In: 7th IEEE GCC Conference and Exhibition (GCC), IEEE 2013, pp. 483–488 (2013)
3. Pourgharehkhan, Z., Sedighi, S., Taherpour, A. Uysal, M.: Spectrum sensing of correlated subbands with colored noise in cognitive radios. In: Wireless Communications and Networking Conference (WCNC). IEEE, pp. 1017–1022 (2012)
4. Akyildiz, I.F., Lo, B.F., Balakrishnan, R.: Cooperative spectrum sensing in cognitive radio networks: a survey. Physical Communication **4**(1), 40–62 (2011)
5. Chen, Y.: Optimum number of secondary users in collaborative spectrum sensing considering resources usage efficiency. IEEE Communications Letters **12**(12), 877–879 (2008)
6. Althunibat, S., Granelli, F.: An objection-based collaborative spectrum sensing for cognitive radio networks. IEEE Communications Letters **18**(8), 1291–1294 (2014)
7. Li, H., Cheng, X., Li, K., Hu, C., Zhang, N., Xue, W.: Robust collaborative spectrum sensing schemes for cognitive radio networks. IEEE Transactions on Parallel andDistributed Systems **25**(8), 2190–2200 (2014)
8. Quan, Z., Cui, S., Sayed, A.H.: Optimal linear cooperation for spectrum sensing in cognitive radio networks. IEEE Journal of Selected Topics in SignalProcessing **2**(1), 28–40 (2008)
9. Quan, Z., Cui, S., Sayed, A.: An optimal strategy for cooperative spectrum sensing in cognitive radio networks. In: Global Telecommunications Conference, GLOBECOM 2007. IEEE, pp. 2947–2951 (2007)
10. Peh, E., Liang, Y., Guan, Y., Zeng, Y.: Cooperative spectrum sensing in cognitive radio networks with weighted decision fusion schemes. IEEE Transactions on Wireless Communications **9**(12), 3838–3847 (2010)
11. Quan, Z., Cui, S., Sayed, A.H., Poor, H.V.: Optimal multiband joint detection for spectrum sensing in cognitive radio networks. IEEE Transactions on SignalProcessing **57**(3), 1128–1140 (2009)
12. Peh, E.C.Y., Liang, Y.-C., Guan, Y.L., Zeng, Y.: Optimization of cooperative sensing in cognitive radio networks: a sensing-throughput tradeoff view. IEEE Transactions on Vehicular Technology **58**(9), 5294–5299 (2009)
13. Hoseini, P.P., Beaulieu, N.C.: An optimal algorithm for wideband spectrum sensing in cognitive radio systems. In: Communications (ICC). IEEE, pp. 1–6 (2010)