Downlink Scheduling and Power Allocation in Cognitive Femtocell Networks

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Abstract. We consider resource assignment and power allocation problem in femtocells under channel estimation errors. Our formulation is to maximize the throughput of femtocell users that share spectrum resources with macrocell base station (MBS) while limiting interference between macrocell and femtocells. Using cognitive capabilities, femtocell basestations (FBS) can acquire the needed information about the neighboring MBS users to reduce cross-tier interference between FBS and MBS users. We analyze the distributions of signal to interference and noise ratio (SINR) of MBS users and signal to interference ratio (SIR) of FBS users. Based on the analytical results, we present resource assignment and power allocation solutions to maximize the mean sum rate subject to SINR and SIR outage constraints, along with simulation verifications.

Keywords: Cognitive femtocell · Cross-tier interference · Resource assignment · Outage constraint · Power allocation

1 Introduction

For nearly a century, wireless capacity has doubled every 30 months. Capacity analysis shows that the capacity increased 25x due to wider spectrum, 5x from dividing spectrum into smaller portions, 5x from enhancements in modulation techniques, and 1600x through reducing the cell sizes and accordingly the communication distances [1]. Despite such high capacity growth, consumer demand for capacity rises even higher. Recent studies show that nearly 50% of voice traffic and 70% of data traffic take place from indoor consumers and it is predicted that this indoor traffic will increase to 60% and 90% for voice and data traffic respectively [1][2]. Femtocell is one promising solution to the traffic growth problem under limited spectrum. Femtocell basestation (FBS) is a short range, low-power and low-cost basestation, installed by users with internet connection, in order to provide better service for local or indoor users.

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A number of other existing works have focused on the interference problem that arises because of spectrum sharing between MBS and FBS [3]. Among various solutions, cognitive radio (CR) may effectively add the needed spectrum awareness functions to the FBS [4][5]. Such FBS with cognitive capabilities may obtain spectrum information needed to control interference level on the shared resources. The authors of [6] presented an algorithm for optimal power allocation in order to solve the downlink interference problem, requiring prior knowledge of all the system channel gains collected by a fusion center. In [7] the authors presented a decentralized interference management method for LTE-A femtocells by sharing measured pathloss information among neighboring femtocells. The authors in [8] formulated the optimization problem of the resource allocation as a Stackelberg game. They focused on the energy efficiency aspect of the shared spectrum in heterogeneous networks. Further, authors of [9] used game theory to model the resource allocation problem and introduced cognitive radio resource management and strategic game based radio resource management schemes to solve the given problem.

In this paper, we study the underlay femtocell scheduling and power assignment problem. Our main objective is to derive a decentralized technique for FBS resource scheduling so as to maximize the total capacity of home user equipments (HUEs) served by the FBS while keeping SINR of nearby macro-user equipments (MUEs) above a given threshold when sharing the same resources. We incorporate some sensing capabilities at the FBS for collecting needed information on the shared resources and for measuring the femtocell impact on nearby cochannel MUEs. Our main contribution is in considering an estimation error in FBS-to-HUE and MBS-to-HUE channel gains. By formulating the problem based on this channel uncertainty, we analyze the distributions of HUEs signal to interference ratio (SIR) and MUEs SINR. Another main contribution in this work is the analytical reduction of the given problem into a much simpler problem [10]. We further present two decentralized methods to solve the resulting channel assignment and power allocation problem based on the optimal Hungarian algorithm and a greedy suboptimal algorithm.

We organize our manuscript as follows: in section 2 we introduce the system model and our problem formulation to maximize the average sum rate subject to SINR and SIR outage constraints. section 3 provides the distribution analysis on SIR of HUEs and SINR of the MUEs. We present our proposed solution for the given optimization problem in section 4 and simulation results in section 5.

2 System Model

2.1 Network Architecture

Our underlying heterogeneous network consists of two tiers: a central macrocell and several femtocells. Each femtocell shares assigned bandwidth (BW) with the macrocell without intra-tier interference with other femtocells. This can be achieved by orthogonal bandwidth assignments for adjacent femtocells. The femtocells operate in closed access mode.

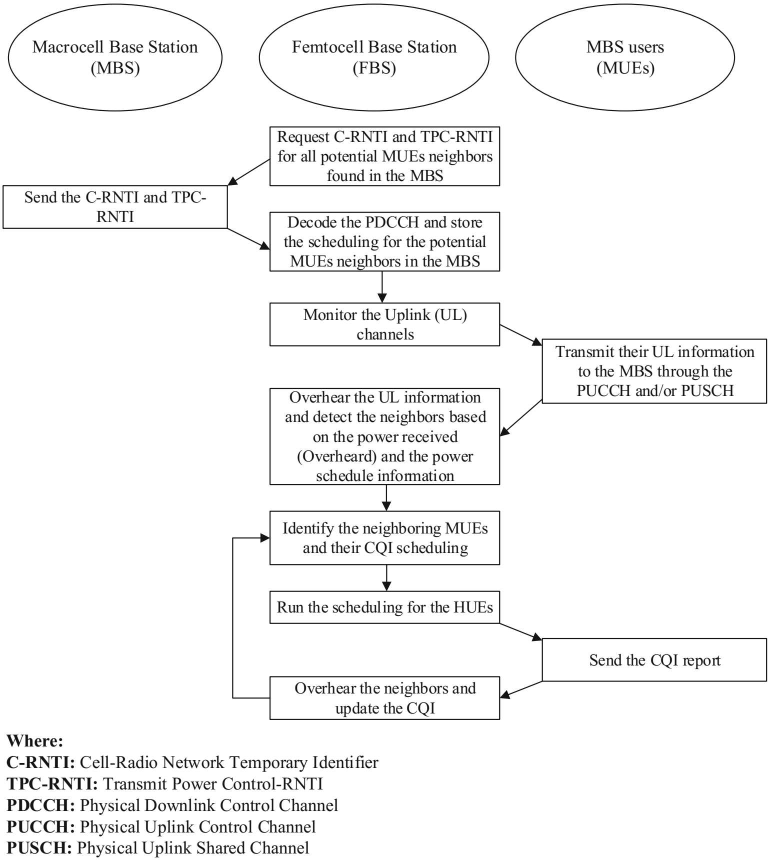

Fig. 1. The action sequence to identify the femtocell neighbors.

We assume cognitive capabilities in each FBS in order to assist in the scheduling and power assignment process. Given the cognitive capability plus indirect coordination of the macrocell base station (MBS), FBS can identify the neighboring MUEs as well as their power and channel assignments. Fig. 1 illustrates the actions of FBS, MBS, and MUEs in order for the FBS to acquire the needed information. Each FBS schedules its actions separately. In this paper, resource block (RB) and channel are synonymous. Before proceeding, here are some important notations we use:

- $\gamma_u^c(t)/\gamma_v^c(t)$: SINR of the HUE u / MUE v on channel c at time t.
- $CQI_v^c(t)$: The overheard channel quality information (CQI) report of the MUE v on channel c at time t.
- $\theta_u^c(t)$: SIR of the HUE u on channel c at time t.
- $\theta_{HUELB}/\gamma_{MUELB}$: The minimum SIR / SINR for the HUE / MUE that can guarantee reliable connection with the FBS / MBS.

- $H^c_{F-u}(t)/H^c_{M-u}(t)/H^c_{M-v}(t)/H^c_{F-v}(t)$: Complex channel gains for the FBS-HUE / MBS-HUE / MBS-MUE / FBS-MUE.
- $P^c_M(t)/P^c_F(t)$: MBS / FBS assigned power on channel c at time t.
- $P_{u,c}(t)$: Power assigned for the HUE u by the FBS on channel c at time t.
- $C_{out}(t)/C^I_{out}(t)/C^N_{out}(t)$: Normalized total capacity of HUEs at time t over all / overlapped / non-overlapped assigned resources.
- N_{HUE}/N_{MUE}: Number of HUEs / MUEs.
- α, β: Probability values from 0 to 1.
- $\mathbf{E}[g(t)]/\mathbf{V}[g(t)]$: Mean / Variance of $g(t)$.
- $\chi'^2(k, \lambda)$: Non-central chi square distribution with k degrees of freedom and non-centrality parameter λ .

2.2 Problem Formulation

Assume the MBS has N_{MUE} users (neighboring the FBS) and N_1 available RBs while the FBS has N_{HUE} users assigned with N_2 RBs ($N_2 < N_1$). Normally not all the N_2 RBs are occupied with neighboring MUEs. We can divide the total HUEs capacity according to

$$C_{out}(t) = C^I_{out}(t) + C^N_{out}(t), \tag{1}$$

where overlapped resources are RBs occupied by neighboring MUEs and assigned to HUE by the FBS for sharing whereas non-overlapped resources are either empty or occupied by far away MUEs.

In our problem formulation, we will consider maximizing HUEs capacity over shared RBs, in other words we are just considering optimizing the scheduling and power assignment over the interfered channels.

Let N_0 be the background noise power. Eqs. (2) and (3) define the SINR and SIR of HUE u respectively as

$$\gamma^c_u(t) = \frac{P_{u,c}(t)\left|H^c_{F-u}(t)\right|^2}{P^c_M(t)\left|H^c_{M-u}(t)\right|^2 + N_0}, \tag{2}$$

$$\theta^c_u(t) = \frac{P_{u,c}(t)\left|H^c_{F-u}(t)\right|^2}{P^c_M(t)\left|H^c_{M-u}(t)\right|^2}. \tag{3}$$

Because we only consider the overlapped channels, $P^c_M(t) > 0$ and consequently $N_0 \ll P^c_M(t)\left|H^c_{M-u}(t)\right|^2$, Hence $\gamma^c_u(t) \approx \theta^c_u(t)$, from which we can define the normalized capacity of the HUEs over the overlapped channels by

$$C^I_{out}(t) = \sum_{i=1}^{N_{HUE}} (\sum_{j=1}^{N_{Ch}} a_{i,j}(t) \log_2(1 + \theta^j_i(t))) \tag{4}$$

where N_{Ch} is the number of overlapped channels and $a_{i,j}(t)$ is the action by the FBS such that:

$$a_{i,j}(t) = \begin{cases} 0, & \text{if Channel } j \text{ is not assigned to HUE } i \\ 1, & \text{if Channel } j \text{ is assigned to HUE } i \end{cases}.$$

Since channel gains are estimated by the HUE u before being sent to the FBS, according to estimation error model provided in [11], we can represent the channel gains as Random Variables (RVs) as shown in (5)

$$H^c_{F-u}(t) = \hat{H}^c_{F-u}(t) + \tilde{H}^c_{F-u}, \tag{5}$$

where $\hat{H}^c_{F-u}(t)$ is a constant complex value representing the estimated channel at time t and $\tilde{H}^c_{F-u}$ is a complex normal distributed RV represent the estimation error such that $\tilde{H}^c_{F-u} \sim \mathcal{CN}(0, 2\sigma^2_{F-u})$. Therefore $H^c_{F-u}(t)$ can be modeled as a complex normally distributed RV such that $H^c_{F-u}(t) \sim \mathcal{CN}(\hat{H}^c_{F-u}(t), 2\sigma^2_{F-u})$ Similarly we have

$$H^c_{M-u}(t) = \hat{H}^c_{M-u}(t) + \tilde{H}^c_{M-u}, \tag{6}$$

where $H^c_{M-u}(t)$ can be modeled as a complex normally distributed RV such that $H^c_{M-u}(t) \sim \mathcal{CN}(\hat{H}^c_{M-u}(t), 2\sigma^2_{M-u})$. We also define the SINR of the MUE v

$$\gamma^c_v(t) = \frac{P^c_M(t)\left|H^c_{M-v}(t)\right|^2}{P^c_F(t)\left|H^c_{F-v}(t)\right|^2 + N_0}. \tag{7}$$

We assume that the MBS shares its power assignment information with the FBS in order to reduce the interference from FBS frequency reuse. In order to gain some information about $H^c_{M-v}(t)$ and $H^c_{F-v}(t)$, we assume that the channel is slow-fading channel, such that the channel gain is approximately constant in 3 consecutive time slots.
From this assumption, for $t-2 \leq T \leq t$ we have

$$H^c_{M-v}(T) = H^c_{M-v}, \tag{8}$$

$$H^c_{F-v}(T) = H^c_{F-v}. \tag{9}$$

Therefore,

$$\gamma^c_v(t) = \frac{P^c_M(t)\left|H^c_{M-v}\right|^2}{P^c_F(t)\left|H^c_{F-v}\right|^2 + N_0}, \tag{10}$$

and

$$\gamma^c_v(t-1) = \frac{P^c_M(t-1)\left|H^c_{M-v}\right|^2}{P^c_F(t-1)\left|H^c_{F-v}\right|^2 + N_0}, \tag{11}$$

$$\gamma^c_v(t-2) = \frac{P^c_M(t-2)\left|H^c_{M-v}\right|^2}{P^c_F(t-2)\left|H^c_{F-v}\right|^2 + N_0}. \tag{12}$$

As a result of FBS's cognitive capabilities, the FBS can overhear $CQI^c_v(t-1)$ and $CQI^c_v(t-2)$, which represent quantized versions of $\gamma^c_v(t-1)$ and $\gamma^c_v(t-2)$, respectively. Therefore, $CQI^c_v(T)$ indicates the interval of $\gamma^c_v(T)$ such that

$$At\ CQI^c_v(T) = K \rightarrow \gamma^c_v(T) \in [a, b], \tag{13}$$

where K represent the overheard CQI value and a, b represent the interval boundaries corresponding to K that $\gamma_v^c(T)$ lies in. Therefore at time t, we can estimate H_{M-v}^c and H_{M-v}^c (assuming $CQI_v^c(t-1) \neq CQI_v^c(t-2)$) from Eqs. (11) and (12), thereby allowing us to find $P(\gamma_v^c(t)|CQI_v^c(t-1), CQI_v^c(t-2)) \geq \beta$.

We now formulate the maximization of HUE capacity:

$$\max_{P_{u,c}} \mathbf{E}[C_{out}^I(t)] = \max_{P_{u,c}}(\sum_{i=1}^{N_{HUE}}(\sum_{j=1}^{N_{Ch}} a_{i,j}(t)\mathbf{E}[\log_2(1+\theta_i^j(t))])) \tag{14a}$$

$$s.t. \quad \sum_{i=1}^{N_{HUE}} a_{i,c}(t) \leq 1 \tag{14b}$$

$$\sum_{j=1}^{N_{Ch}} a_{u,j}(t) = 1 \tag{14c}$$

$$\mathbf{P}(\sum_{j=1}^{N_{Ch}} a_{u,j}(t)\theta_u^j(t) \geq \theta_{HUELB}) \geq \alpha \tag{14d}$$

$$\mathbf{P}(\sum_{j=1}^{N_{Ch}} \xi_{v,j}(t)\gamma_v^j(t) \geq \gamma_{MUELB} \Big| \sum_{j=1}^{N_{Ch}} \xi_{v,j}(t)CQI_v^j(t-1), \sum_{j=1}^{N_{Ch}} \xi_{v,j}(t)CQI_v^j(t-2)) \geq \beta \tag{14e}$$

$$u = 1, 2...N_{HUE}, \quad v = 1, 2, ...N_{MUE}, \quad c = 1, 2, ...N_{Ch},$$

where $\xi_{v,j}(t)$ is the participation indicator at time t based on overheard scheduling information:

$$\xi_{v,j}(t) = \begin{cases} 0, & \text{if Channel } j \text{ is not scheduled to MUE } v \\ 1, & \text{if Channel } j \text{ is scheduled to MUE } v \end{cases}.$$

Note that [12] provides a Gaussian approximation for the objective function as:

$$\mathbf{E}[\log_2(1+\theta_u^c(t))] \approx \log_2(1+\mathbf{E}[\theta_u^c(t)]) - \frac{\mathbf{V}[\theta_u^c(t)]}{2(1+\mathbf{E}[\theta_u^c(t)])^2}, \tag{15}$$

which provides our approximate objective function:

$$\max_{P_{u,c}} \mathbf{E}[C_{out}^I(t)] = \max_{P_{u,c}}(\sum_{i=1}^{N_{HUE}}(\sum_{j=1}^{N_{Ch}} a_{i,j}(t)(\log_2(1+\mathbf{E}[\theta_u^c(t)]) - \frac{\mathbf{V}[\theta_u^c(t)]}{2(1+\mathbf{E}[\theta_u^c(t)])^2}))) \tag{16}$$

The constraints shown in Eqs. (14b) and (14c) aim to ensure that each channel is occupied once and that each HUE gets only one channel respectively.

While the constraints in Eqs. (14d) and (14e) are to guarantee that there exists a minimum acceptable SIR and SINR levels for each HUE and MUE to sustain a reliable transmission with the FBS and MBS respectively.

In order to solve the shown problem, we need to analyze the distributions of $\gamma_v^c(t)$ and $\theta_u^c(t)$ as well as calculating the first order statistics of $\theta_u^c(t)$.

3 SIR and SINR Distribution Analysis

3.1 SIR Distribution Analysis

According to the channel model of (5), we define

$$\bar{H}_{F-u}^c(t) = H_{F-u}^c(t)/\sigma_{F-u}, \tag{17}$$

$$\bar{H}_{M-u}^c(t) = H_{M-u}^c(t)/\sigma_{M-u}, \tag{18}$$

Therefore

$$\theta_u^c(t) = \frac{P_{u,c}(t)\sigma_{F-u}^2\left|\bar{H}_{F-u}^c(t)\right|^2}{P_M^c(t)\sigma_{M-u}^2\left|\bar{H}_{M-u}^c(t)\right|^2}, \tag{19}$$

where the random variable $\bar{H}_{F-u}^c(t) \sim \mathcal{CN}(\frac{\hat{H}_{F-u}^c(t)}{\sigma_{F-u}}, 2)$ and accordingly

$$\left|\bar{H}_{F-u}^c(t)\right|^2 \sim \chi'^2(2, \left|\frac{\hat{H}_{F-u}^c(t)}{\sigma_{F-u}}\right|^2). \tag{20}$$

And similarly we have

$$\left|\bar{H}_{M-u}^c(t)\right|^2 \sim \chi'^2(2, \left|\frac{\hat{H}_{M-u}^c(t)}{\sigma_{M-u}}\right|^2). \tag{21}$$

Then we will have

$$\theta_u^c(t) = m\underbrace{\frac{\left|\bar{H}_{F-u}^c(t)\right|^2}{\left|\bar{H}_{M-u}^c(t)\right|^2}}_{\bar{H}}. \tag{22}$$

Therefore, from [13] we conclude that $\bar{H}$ has a doubly non-central F-Distribution with parameters $(2, 2, \left|\hat{H}_{F-u}^c(t)/\sigma_{F-u}\right|^2, \left|\hat{H}_{M-u}^c(t)/\sigma_{M-u}\right|^2)$, from which the probability density function (PDF) of $\theta_u^c(t)$ is also known.

3.2 SINR Distribution Analysis

In order to evaluate the constraint shown in equation (14e), we need to calculate the cumulative distribution function (CDF) of $\gamma_v^c(t)$. To do so we will start by substituting in Eq. (10) by Eqs. (11) and (12), to get the form in equation (23)

$$\gamma_v^c(t) = \frac{K_1\gamma_v^c(t-1)\gamma_v^c(t-2)}{K_2\gamma_v^c(t-1) + K_3\gamma_v^c(t-2)}, \tag{23}$$

where

$$K_1 = P_M^c(t)(P_F^c(t-2) - P_F^c(t-1))$$
$$K_2 = P_M^c(t-2)(P_F^c(t) - P_F^c(t-1))$$
$$K_3 = P_M^c(t-1)(P_F^c(t-2) - P_F^c(t))$$

and since we do not know the exact values of $\gamma_v^c(t-1)$ and $\gamma_v^c(t-2)$, we only can overhear their CQI level as we mentioned earlier. Therefore we can model $\gamma_v^c(t-1)$ and $\gamma_v^c(t-2)$ as random variables uniformly distributed within the known interval based on the CQI level.

Starting from equation (23), at $K_2 \neq 0$

$$\gamma_v^c(t) = (\frac{K_1}{K_2})\frac{\gamma_v^c(t-1)\gamma_v^c(t-2)}{\gamma_v^c(t-1) + \frac{K_3}{K_2}\gamma_v^c(t-2)}, \tag{24}$$

$$\gamma_v^c(t) = (\frac{K_1}{K_2})S, \tag{25}$$

where

$$S = \frac{\gamma_v^c(t-1)\gamma_v^c(t-2)}{\gamma_v^c(t-1) + k\gamma_v^c(t-2)}, \tag{26}$$

and $k = K_3/K_2$. Applying Eq. (26) and PDFs of $\gamma_v^c(t-1)$ and $\gamma_v^c(t-2)$, we evaluated a closed-form PDF of S which is then used to determine $\gamma_v^c(t)$ PDF and CDF.

4 Proposed Solution

In this section we will introduce a solution to the given problem by first focusing on a power selection policy.

4.1 Optimum Power Level Selection

Considering the objective function (14a), Eqs. (14b) and (14c) guarantee that no channel assigned to more than one HUE and that every HUE gets only one channel, while equations (14d) and (14e) specify the minimum and maximum power limits respectively. Thus, for a valid assignment we can rewrite our problem as follows:

$$\max_{P_{u,c}} \mathbf{E}[C_{out}^I(t)] = \max_{P_{u,c}}(\sum_{i=1}^{N_{HUE}}(\sum_{j=1}^{N_{Ch}} a_{i,j}(t)\mathbf{E}[\log_2(1+\theta_i^j(t))])) \tag{27a}$$

s.t.

$$P_{u,c}^{min} \leq P_{u,c} \leq P_{u,c}^{max} \tag{27b}$$

Lemma 1: For the objective function (27a) with any valid channel assignment, if there exists $P_{u,c}$ for HUE u to satisfy (27b), then its optimum power assignment equals $P_{u,c}^{max}$.

Proof: We can show that our objective function is monotonically increasing in $P_{u,c}$. The details are omitted here.

4.2 Main Structure of the Solution Algorithm

In order to explain the proposed solution, we will first describe the reduction/ transformation used to transfer the given problem equivalently into an assignment problem. The term "problem reduction" is very popular in complexity theory. The main idea is in transform underlying problem from an unknown form (non-convex optimization problem) to a known one such that there exists an optimal and efficient algorithm to solve it. One common use of problem reduction is to show that a specific problem belongs to a certain class of complexity like P, NP and NP-complete. This reduction is based on the analytical results from the previous sections and it is described in the algorithm given below. These steps should be made regardless of the method we will use later to solve the assignment problem.

Algorithm: Optimum Channel Allocation (main structure)

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1. Combining the calculated CDF of $\gamma_v^c(t)$ (section 3.2) and the constraint in (14e), we will be able to evaluate the maximum power ($P_{u,c}^{max}$) for all the available RBs.
2. The results of section 3.1, enable us to calculate the distribution of the random variable $\theta_{u,c}^{max}(t)$ as well as its first order statistics for each HUE at each RB.
3. Using the first order statistics of $\theta_{u,c}^{max}(t)$, we will be able to calculate the maximum capacity for each HUE on every channel.
4. In order to apply the constraint in equation (14d) we will use the $\theta_{u,c}^{max}(t)$ CDF to verify that all HUEs SIR exceeds θ_{HUELB}, otherwise exclude this channel assignment from the result.

Result: a lookup (rate) table $r(i,j)$ representing the maximum capacity for each HUE at each channel ($i = 1, ...N_{HUE}$ and $j = 1, ...N_{Ch}$)

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where $\theta_{u,c}^{max}(t)$ is the maximum SIR of the HUE u on channel c at time t

$$\theta_{u,c}^{max}(t) = \frac{P_{u,c}^{max}(t)\left|H_{F-u}^{c}(t)\right|^2}{P_M^c(t)\left|H_{M-u}^{c}(t)\right|^2}. \tag{28}$$

Basically we start our solution by using the results in section 3 to calculate the lookup (rate) table or matrix $\mathbf{R}$ (where $\mathbf{R} = [r(i,j)]$ for $i = 1, ...N_{HUE}$ and $j = 1, ..N_{Ch}$). After completing the 4 steps, we will proceed to find the channel assignment to maximize the objective function.

4.3 Channel Assignment Algorithms

Given matrix $\mathbf{R}$ which represents the lookup table, we can also view this as the edge weight matrix of a bipartite graph. On one end of the bipartite graph are the user nodes, while on the opposite end of the bipartite graph are the available channels. To find the best pairing to maximum the sum rate, we can either use a simpler greedy algorithm or resort to the well known Hungarian Algorithm designed to solve such assignment problem optimally in shorter time.

Greedy Algorithm. The lookup table is a matrix $\mathbf{R}$ with HUEs as rows and channels as columns. We can determine the suboptimum channel assignment by applying a greedy algorithm to find the maximum pairing in each iteration. Let a matrix $\mathbf{P_1} = \mathbf{R}$. For the i-th iteration, our greedy algorithm find the maximum element in matrix $\mathbf{P_i}$ as a pairing choice before forming the next matrix $\mathbf{P_{i+1}}$ by removing the corresponding row and column of the maximum element from $\mathbf{P_i}$. We continue until all HUEs or channels are exhausted.

In the greedy algorithm, successful HUE acquires the maximum capacity from the available channels regardless the remaining HUEs. Although the complexity of greedy solution is very low and its time consumption grows linearly with increasing problem size, it is generally not optimal.

Hungarian Algorithm. Starting from the rate lookup table, our problem is viewed as an assignment problem in which the Hungarian algorithm has proven to solve optimally and in polynomial time [14][15]. The Hungarian algorithm is a combinatorial optimization algorithm first introduced in 1955 to solve an equivalent assignment problem [14].

In order to achieve the optimum channel assignment we add one more step on the algorithm main structure in section 4.2 by adopting the Hungarian algorithm. Unlike [16], we did not use the Hungarian algorithm to work on the original scheduling problem which may result near optimal solutions. Instead, we used Hungarian algorithm to determine the optimal combination from the rate lookup table.

5 Performance Evaluation

We will present our simulation results in three parts, in the first part we verify the analysis in section 3 with numerical examples. We will compare the two proposed solutions in the second part. The third part compares the results according to our channel gain estimation error assumption and the assumption of zero channel gain estimation error.

Fig. 2 shows both numerical and analytical distributions of MUE SINR, from which we can see excellent verification of the analytical results in section 3. We also presents both numerical and analytical distributions of HUE SIR, from which we also observe evident verification.

In Fig. 3, we present the maximum capacity against the estimation error standard deviation. We plot the Hungarian algorithm solution along with the solution from the greedy algorithm. Clearly, the Hungarian algorithm achieves optimal solution. The results from the greedy algorithm show sub-optimality but require lower complexity ($O(n)$) while the Hungarian algorithm requires $O(n^3)$ [17]. In Fig. 4 we compare two results: one from being ignorant of the channel estimation error existing in the channel gain by assigning resources based on purely the estimated channel (assuming estimation error = 0), another account for the channel estimation error and assign the resources based on this consideration as in our proposed algorithms.

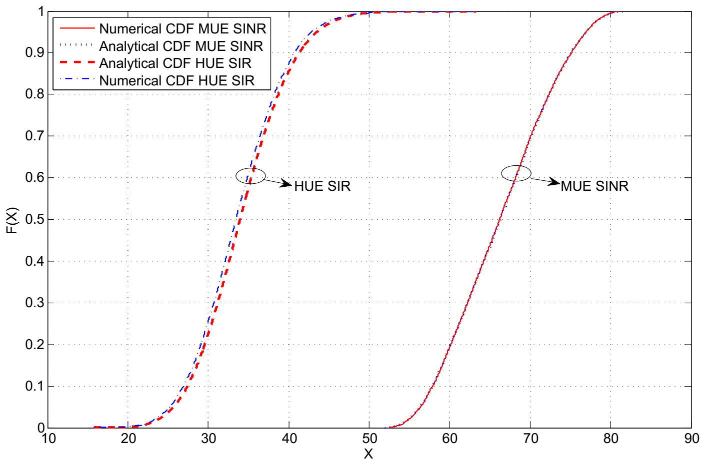

Fig. 2. The numerical and analytical distributions of the $\gamma_v^c(t)$ and $\theta_u^c(t)$.

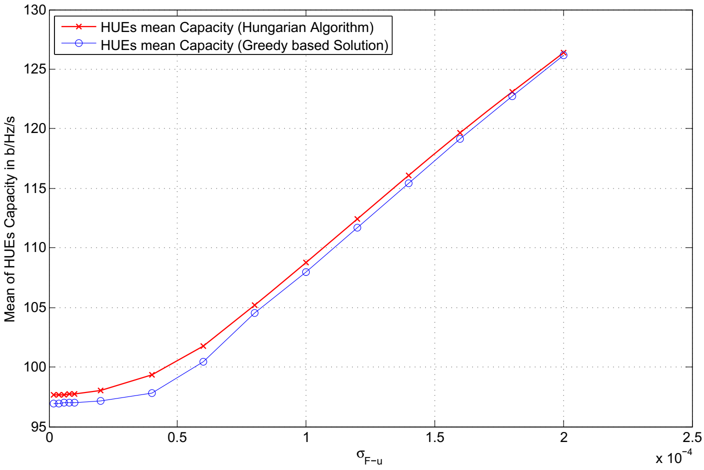

Fig. 3. The Hungarian and Greedy algorithms results.

For small variance in channel estimation error, the results of both cases are nearly the same. However, as channel estimation error variance grows, the first result starts to deteriorate to less than optimal while the second results remains optimum (as circled and diamond curves).

Moreover in the first case, the total capacity estimation is constant (Asterisk line) regardless of the value of the real capacity (diamond line) and the total

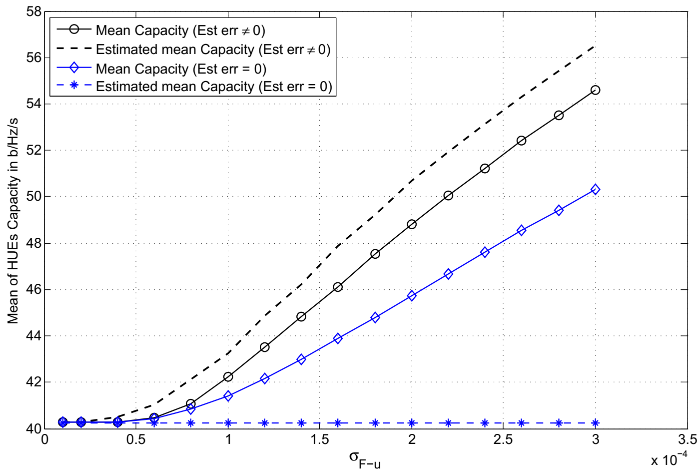

Fig. 4. Results due to zero and non-zero estimation error assumptions.

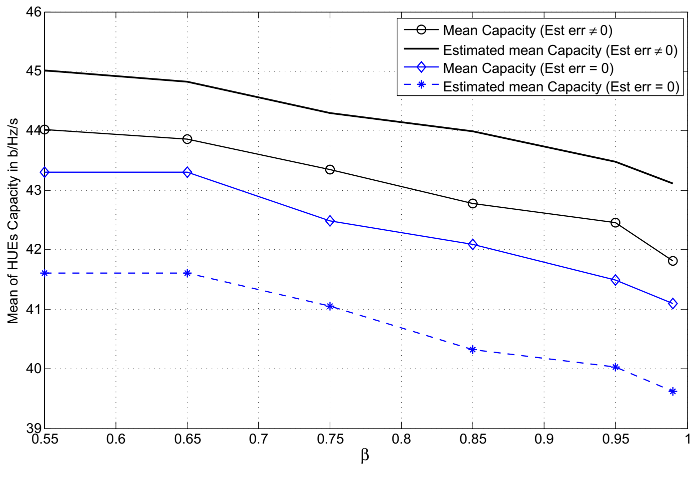

Fig. 5. Results according to error assumptions for different β.

capacity estimation error increase with growing channel error variance. On the other hand, for the second case, the total capacity estimation (dashed line) tracks the actual capacity (circled line).

Finally, in Fig. 5 and Fig. 6 we compare the performance of the two error assumptions illustrated earlier for different β. In Fig. 5, we can see that as β increases the total capacity decreases. This follows from (14e), as β affects the

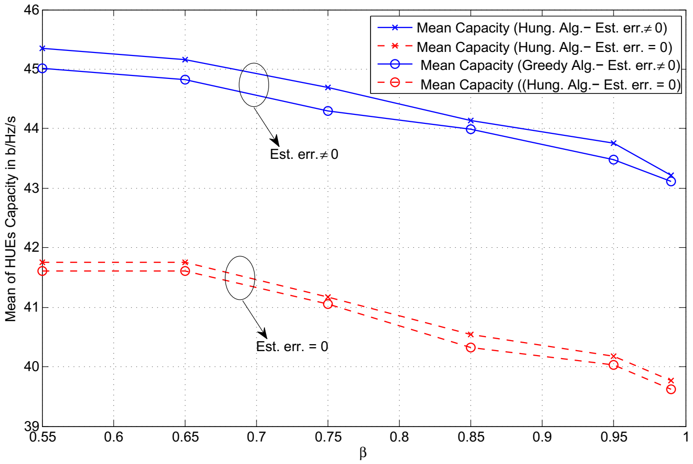

Fig. 6. Solutions results according to error assumptions for different β.

maximum allowed power on the occupied channel. Still our performance is optimal whereas the performance according to the zero estimation error assumption is not. Moreover the performance gap between the two results remains almost constant regardless the value of β. In Fig. 6, we compare performances of the two solutions according to the different error assumptions.

From the simulation results thus far, we have verified our analytical results in section 3, and established the optimality of applying the reduction algorithm followed by the Hungarian algorithm for the given problem. Lastly we illustrated the importance to account for the channel estimation error assumption instead of ignoring the error.

6 Conclusion

In this work we focused on downlink cross-tier interference problem in a two-tier heterogeneous network. In order to control the cross-tier interference while maximizing the femtocell capacity, we exploit the cognitive capabilities of FBS to acquire nearby MUE scheduling. Our problem formulation take into account channel estimation error and we provide full analysis for the distribution of the HUE SIR and MUE SINR. Both analytical results are verified via simulations. Based on our analysis we developed a problem reduction method for the given problem. We also suggested two solutions for the reduced problem based on the Hungarian and greedy algorithms, respectively, with demonstrated simulation results.

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