Non-uniform Quantized Distributed Sensing in Practical Wireless Rayleigh Fading Channel

Sina Mohammadi Fard[1], Hadi Hashemi[1], Abbas Taherpour[1], and Tamer Khattab[2(✉)]

[1] Department of Electrical Engineering, Imam Khomeini International University, Qazvin, Iran
[2] Electrical Engineering, Qatar University, Doha, Qatar
tkhattab@ieee.org

Abstract. In this paper, we study non-uniform multilevel quantization problem in cognitive radio networks (CRNs). We consider a practical collaborative spectrum sensing (CSS) scenario in which secondary users (SUs) cooperate with each other to decide about the presence of the primary user (PU). We consider a cooperative parallel access channel (CPAC) scheme in reporting channels in which SUs transmit their quantized data to fusion center (FC) for the final decision. Also, we evaluate the final summation-based decision statistic and Kullback-Leibler (KL) divergence performance criterion in the Rayleigh fading channel and additive Gaussian noise. We compare the non-uniform quantization scheme performance with the uniform one and illustrate the sensitivity of the provided quantization scheme to average error probability of symbols. Furthermore, the effect of the collaboration in the CPAC scheme on performance of the distributed sensing compared with non-cooperative scheme is investigated.

Keywords: Collaborative spectrum sensing · Non-uniform quantization · Rayleigh fading channel

1 Introduction

Cooperative spectrum sensing (CSS) is one of the ways to improve the performance of spectrum sensing algorithms in the shadowing and channel fading situations [1–6]. In [1] the impact of sensors collaboration to achieve optimal performance is studied. Authors in[2], investigate a CSS problem in which the energy sensed by each SU is transmitted to the others and each SU attempts to decide based on its own information and received signals. In [3–5], cooperation in CRNs with assumption of independent channels is considered. The authors in [3] and [4] assume the same distribution for all users, while [5] assumes different distributions which is more realistic for practical scenarios. In [6], the authors propose a method which uses spatial diversity to deal with the devastating effects of a fading channel. However, due to the increasing wireless network users, the CSS algorithms are often faced with the problem of limited bandwidth. Thus, information received by the SUs must be properly quantized before transmitting

M. Weichold et al. (Eds.): CROWNCOM 2015, LNICST 156, pp. 78–91, 2015.
DOI: 10.1007/978-3-319-24540-9_7

to FC in order to occupy less bandwidth, while maintaining the accuracy of the observations [7–15]. In [7], the M-level quantization problem for a distributed detection system is investigated by assuming interfering nodes and Byzantine attacks. In [8] the problem of SUs binary decisions fusion in Rayleigh channels is studied. Also in [9], authors analyze quantizer design which is robust to link outages and/or sensor failure in a multi-user system. In [10], comparison of the performance of single-user and collaborative multi-user quantized system has been studied. In [11] and [12], authors review the performance of relay based collaborative distributed detection system for the quantize and forward scheme. The same problem with the assumption of orthogonal multiple access channels is considered in [12]. In [13], the authors provide a quantization system with multiple non-uniform threshold levels, while the impact of channel errors on the performance of the quantized detection system has been examined in [14]. In [15], a simulation-based investigation of energy quantization effect on proposed detector performance is provided.

In this paper, we investigate a multilevel quantization problem in a practical collaborative distributed detection system. We review an M-level non-uniform quantization procedure. In order to make the final decision, the SUs transmit their quantized data to FC in CPAC protocol. We assume a practical wireless report channel with Rayleigh fading and analytically evaluate the corresponding performances. A remarkable point of this paper is the comprehensive study of the SUs cooperation impact on practical wireless networks by deploying a non-uniform quantization technique. The rest of the paper is organized as follows. In Section 2, we introduce the system model and assumptions on SUs detector and applied transmission protocol. In Section 3, we provide boundaries calculations of uniform and non-uniform multilevel quantization scheme which we have used. Derivation and evaluation of final decision performance in FC and KL divergence performance criterion are investigated in Section 4. Simulation results are provided in Section 5 and finally, Section 6 summarizes the conclusions.

Notation: Lightface letters denote scalars. Boldface lower-and upper-case letters denote column vectors and matrices, respectively. x(.) is the entries and $\mathbf{x}_i$ is sub-vector of vector $\mathbf{x}$ and $[\mathbf{A}]_{.,.}$ is the entries of matrix $\mathbf{A}$. $\mathcal{N}(\mu, \sigma^2)$ denotes Gaussian distribution with mean μ and variance σ^2. Superscript T is transpose and $Q(x)$ is Q-function $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} exp\left(\frac{-u^2}{2}\right) du$.

2 System Model

Suppose a CSS system in CRNs which K SUs detect presence or absence of PU signals in certain frequency range. We assume each SU has been equipped with a single antenna. Thus, the final goal of this spectrum sensing system is decision between the two following hypotheses,

$$y_i(t) = \begin{cases} w_i(t) & \mathcal{H}_0 \\ s(t) + w_i(t) & \mathcal{H}_1 \end{cases}, i = 1, 2, .., K, \tag{1}$$

where $y_i(t)$ is observed signal in ith SU, $s(t)$ is PU's transmitted signal and $w_i(t)$ is additive white Gaussian noise of the channel between the PU antenna and ith SU.

There are several methods for received signal detection in CRNs that each of them require different information about PU signal parameters. Generally, it is assumed that PU signal and priori probabilities of transmitted symbols are unknown for SU and the traditional and efficient detection method is energy detector (ED) in this situation. For this reason and also better expression, we use ED in this paper. Input band-pass filter of detector selects the center frequency f_c, and the bandwidth of interest, W. By assuming sampling time interval T, each SU takes $P = 2TW$ samples. Thus, ED for each user can be declared, as follows,

$$\mathrm{T}_i = \sum_{t=1}^{P} |y_i(t)|^2. \tag{2}$$

Then, statistical distributions of the derived detector in each SU are,

$$\mathrm{T}_i \sim \begin{cases} \chi_P^2 & \mathcal{H}_0 \\ \chi_P^2(\lambda_i) & \mathcal{H}_1 \end{cases} , i = 1, 2, \ldots, K, \tag{3}$$

where χ_P^2 and $\chi_P^2(\lambda_i)$ denote central and non-central chi squared distributions, respectively, each with P degrees of freedom and non-centrality parameter of λ_i for the latter distribution. λ_i is the instantaneous signal to noise ratio (SNR) of ith SU. To overcome the bandwidth constraint problem of reporting channels, which link SUs and FC, calculated statistic in each SU is quantized into M levels. The M quantized symbols are transmitted to FC over non-ideal channel. In different papers, cooperative and non-cooperative schemes are used for transmission scheme. The point we consider in this paper is that we benefit the whole capacity of the report channels to transmit the SUs data to FC. One scheme that has this feature is CPAC, which is described in the following. According to the results derived in [6], CPAC scheme has the best performance among the analogous ones. Thus, transmission scheme which is used in this network is CPAC, where sensors are assigned orthogonal channels for transmission.

CPAC transmitting protocol, as is shown in Fig. 1, involves K^2 phases that each SU transmits during K phases, despite the non-cooperative one, which is called PAC scheme, where each SU transmits data only in one phase. Thus, the symbol of each SU is transmitted over all of report channels between SUs and FC. Therefore, received vector in FC is,

$$\mathbf{y}_{\mathrm{FC}} = \mathbf{H}_{eq}\mathbf{u}_{\mathrm{SU}} + \mathbf{n}, \tag{4}$$

where $\mathbf{u}_{\mathrm{SU}} = (u_{\mathrm{SU}\,1}, u_{\mathrm{SU}\,2}, ..., u_{\mathrm{SU}\,K})^T$ is transmitted vector from SUs to FC, $\mathbf{n}$ is additive noise vector in which $n_i,\ i = 1, ..., K$ is a Gaussian random variable with distribution $\mathcal{N}(0, \sigma_n^2)$ and $\mathbf{H}_{eq}$ is the equivalent channel matrix as follows,

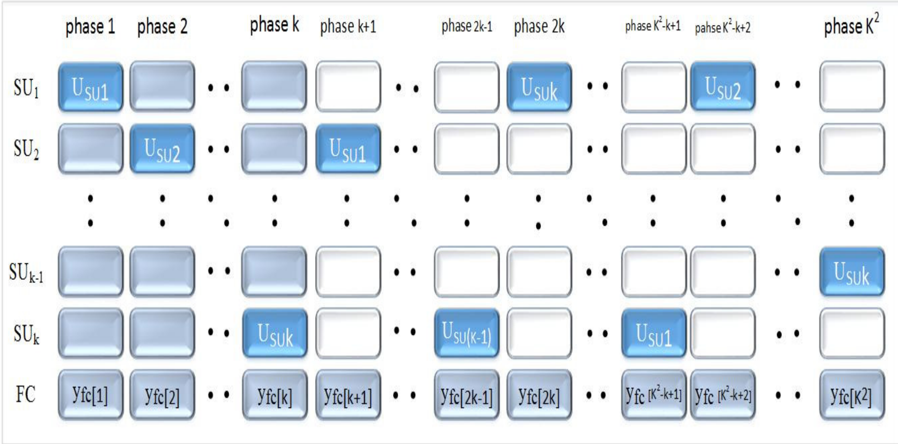

Fig. 1. Structure of CPAC scheme. White blocks represent inactive slots, blue blocks represent active slots (dark blue for transmitting and light blue for receiving).

$$\mathbf{H}_{eq} = \begin{pmatrix} h_1 & 0 & \dots & 0 \\ 0 & h_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & h_K \\ h_2 & 0 & \dots & 0 \\ 0 & h_3 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & h_1 \\ \vdots & \vdots & \ddots & \vdots \\ h_K & 0 & \dots & 0 \\ 0 & h_1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & h_{K-1} \end{pmatrix}, \tag{5}$$

where $h_i,\ i = 1, ..., K$ is ith SU's channel fading gain which is assumed has Rayleigh distribution. Also channel gain and additive noise is assumed independent. Remarkable thing is that in this scheme the cooperative channels between sensors are assumed to be error-free. In other words, we assume that the symbols trasmitted from each SU, is received completely and correctly in others SUs.

3 Multilevel Quantization Scheme

In this section, we provide a multilevel quantization scheme to transmit statistic values of SUs $\mathrm{T}_i, i = 1, ..., K$ to FC to make final decision on presence or absence of PU signal. In the multilevel quantization scheme, we need to partition T_i into multiple regions. Thus, we must determine multiple boundaries. In the following subsections, we investigate on uniform and non-uniform procedures on boundaries determination.

3.1 Uniform Quantization Scheme

In the uniform method, which is a common approach for M-level quantization, the process is such that each user determine the maximum and minimum values of the derived statistic during N observations, as the upper and lower quantization bounds, respectively. In the next step, the range between these two values is divided into M interval and finally quantized value for each interval is obtained, as follows,

$$\tau_{m,i} = \left(\frac{\theta_{max,i} - \theta_{min,i}}{M}\right) m + \theta_{min,i},\ m = 0, \ldots, M, \tag{6}$$

where $\theta_{max,i}$ and $\theta_{min,i}$ are maximum and minimum of $\mathrm{T}_i^{(j)}$, $j = 1, \ldots, N$, respectively. And thus, quantized values are,

$$\nu_{m,i} = \left(\frac{\tau_{m-1,i} + \tau_{m,i}}{2}\right),\ m = 1, \ldots, M, \tag{7}$$

3.2 Non-uniform Quantization Scheme

Since detection error usually occur in low SNR and nearby thresholds and in higher SNR detection of PU presence is much easier, it is more efficient that quantization boundaries density be higher nearby thresholds. In this study, we use a procedure to create non-uniform boundaries. In this procedure, quantization boundaries of each SU are derived based on the threshold value $\eta_i, i = 1, \ldots, K$ which has been calculated by consideration of Neyman-Pearson method for binary hypotheses and acceptable false alarm probability [13]. The M quantization levels of ith SU quantizer is represented by $U_{\mathrm{SU}\,i} = \{u_{1,i}, u_{2,i}, \ldots, u_{M,i}\}$ and its quantization boundaries are $\mathbf{t}_i = \{t_{0,i}, t_{1,i}, \ldots, t_{M,i}\}$. In fact, when $\mathrm{T}_i \in (t_{m+1,i}, t_{m,i}]$ for $m = 1, \ldots, M$, quantizer decides $U_{\mathrm{SU}\,i} = u_{m,i}$. First, we determine an upper and lower bounds for $t_{m,i}$, in order to cover all possible values of each SU's statistic. Then, determination of boundaries values is in a way that number of boundaries near the binary hypothesis threshold η_i be greater. Therefore, boundaries are determined as follows,

$$\begin{cases} t_{M,i} = \mathcal{T}_i + \eta_i \\ t_{\frac{M}{2},i} = \eta_i \\ t_{0,i} = \eta_i - \mathcal{T}_i. \end{cases} \tag{8}$$

where,

$$\mathcal{T}_i = \max_j |\mathrm{T}_i^{(j)} - \eta_i|,\ j = 1, \ldots, N, \tag{9}$$

is the maximum distance of ith SU statistic from threshold in N observations and also $\mathrm{T}_i^{(j)}$ is ith SU statistic in jth observation. For middle values of boundaries, we have,

$$\begin{cases} t_{m,i} = \frac{t_{m+1,i}+\eta_i}{2}, & m > \frac{M}{2} \\ t_{n,i} = \frac{t_{n-1,i}+\eta_i}{2}, & n < \frac{M}{2}. \end{cases} \tag{10}$$

Therefore, quantized value of each level is defined as,

$$u_{m,i} = \frac{t_{m-1,i} + t_{m,i}}{2},\ m = 1, ..., M. \tag{11}$$

The probability mass function (PMF) of U_{SU_i}, which is the transmitted value to FC from ith SU, can be expressed as,

$$P(U_{\mathrm{SU}_i} = u_{m,i}|\mathcal{H}_j) = \int_{t_{m-1,i}}^{t_{m,i}} f(\mathrm{T}_i = t|\mathcal{H}_j)dt,\ j = 0, 1. \tag{12}$$

According to cumulative distribution function (CDF) of central and non-central chi-square random variables and (3), we have,

$$P(U_{\mathrm{SU}_i} = u_{m,i}|\mathcal{H}_0) = \frac{\gamma\left(\frac{N}{2}, \frac{t_{m,i}}{2}\right) - \gamma\left(\frac{N}{2}, \frac{t_{m-1,i}}{2}\right)}{\Gamma\left(\frac{N}{2}\right)}, \tag{13}$$

where $\Gamma(.)$ and $\gamma(.,.)$ are gamma and lower incomplete gamma function, respectively.

$$P(U_{\mathrm{SU}_i} = u_{m,i}|\mathcal{H}_1) = Q_{\frac{N}{2}}(\sqrt{\lambda_i}, \sqrt{t_{m-1,i}}) - Q_{\frac{N}{2}}(\sqrt{\lambda_i}, \sqrt{t_{m,i}}), \tag{14}$$

for $m = 1, ..., M$, where $Q_{\frac{N}{2}}(.,.)$ is Marcum Q-function.

As mentioned before, the report channel is assumed fading channel with Rayleigh distribution and additive Gaussian noise. Therefore, according to (5) and Fig. 1, all of SUs quantized data are transmitted to FC over K different independent channels. i.e. for each SU's transmitted symbol u_{SU_i}, we have K observations in K different time phases. If variable l be index of observations time phases, it can be seen in Fig. 1 that transmitted symbol related to ith SU, u_{SU_i}, is received in $l = i + nK,\ n = 0, ..., K-1$, time phases. Thus, the received signal from ith SU in lth time phase is,

$$y_{\mathrm{FC}_i}[l] = u_{\mathrm{SU}_i} h_{i,l} + n_i = x_{\mathrm{SU}_{i,l}} + n_i, \tag{15}$$

where $h_{i,l} = [\mathbf{H}_{eq}]_{l,i}$ has Rayleigh distribution with unit power as follows,

$$f_H(h_{i,l}) = 2h_{i,l}e^{-h_{i,l}^2}, \quad h_{i,l} > 0. \tag{16}$$

For $x_{\mathrm{SU}\,i,l}$ distribution, we have,

$$F_{X_{\mathrm{SU}\,i,l}}(x_{\mathrm{SU}\,i,l}) = P(X_{\mathrm{SU}\,i,l} < x_{\mathrm{SU}\,i,l}) = P(H_{i,l}U_{\mathrm{SU}\,i} < x_{\mathrm{SU}\,i,l}) \\ = P(H_{i,l} < \frac{x_{\mathrm{SU}\,i,l}}{u_{\mathrm{SU}\,i}}) = F_H(\frac{x_{\mathrm{SU}\,i,l}}{u_{\mathrm{SU}\,i}}). \tag{17}$$

Then,

$$f_{X_{\mathrm{SU}\,i,l}}(x_{\mathrm{SU}\,i,l}) = \frac{1}{u_{\mathrm{SU}\,i}} f_H(\frac{x_{\mathrm{SU}\,i,l}}{u_{\mathrm{SU}\,i}}) \tag{18}$$
$$= \frac{2}{u^2_{\mathrm{SU}\,i}} x_{\mathrm{SU}\,i,l} \exp\left(-\frac{x^2_{\mathrm{SU}\,i,l}}{u^2_{\mathrm{SU}\,i}}\right), \quad \frac{x_{\mathrm{SU}\,i,l}}{u_{\mathrm{SU}\,i}} > 0.$$

4 Decision in Fusion Center

In this section, we derive a posteriori probability of received symbols in FC from each user. Since, additive noise is assumed Gaussian, distribution of observed signal in FC from ith SU in lth time phase can be calculated as,

$$f(y_{\mathrm{FC}\,i}[l]|u_{\mathrm{SU}\,i}, \mathcal{H}_j) = f(y_{\mathrm{FC}\,i}[l]|u_{\mathrm{SU}\,i}) \tag{19}$$
$$= \int_{-\infty}^{+\infty} \frac{2x_{\mathrm{SU}\,i,l}}{\sqrt{2\pi} u^2_{\mathrm{SU}\,i}} e^{-\left(\frac{x^2_{\mathrm{SU}\,i,l}}{u^2_{\mathrm{SU}\,i}} + \frac{(y_{\mathrm{FC}\,i}[l] - x_{\mathrm{SU}\,i,l})^2}{2\sigma_n^2}\right)} dx_{\mathrm{SU}\,i,l}.$$

By replacement $\tau_{\mathrm{SU}\,i,l} = \frac{x_{\mathrm{SU}\,i,l}}{u_{\mathrm{SU}\,i}}$, and also from Section 3.462 in [16], (19) might be simplified as,

$$f(y_{\mathrm{FC}\,i}[l]|u_{\mathrm{SU}\,i}) = \frac{\sqrt{2}\sigma_n}{\sqrt{\pi}(u^2_{\mathrm{SU}\,i} + 2\sigma_n^2)} e^{\left(\frac{-y_{\mathrm{FC}\,i}[l]^2}{2\sigma_n^2}\right)} \tag{20}$$
$$\times \left[1 + \frac{\sqrt{2\pi} u_{\mathrm{SU}\,i} y_{\mathrm{FC}\,i}[l]}{\sigma_n \sqrt{u^2_{\mathrm{SU}\,i} + 2\sigma_n^2}} e^{\left(\frac{u^2_{\mathrm{SU}\,i} y_{\mathrm{FC}\,i}[l]^2}{2\sigma_n^2 \left(u^2_{\mathrm{SU}\,i} + 2\sigma_n^2\right)}\right)} \left\{\frac{1}{2} - \frac{1}{2}\mathrm{erf}\left(\frac{1}{\sqrt{2}} \frac{-u_{\mathrm{SU}\,i} y_{\mathrm{FC}\,i}[l]}{\sigma_n \sqrt{u^2_{\mathrm{SU}\,i} + 2\sigma_n^2}}\right)\right\}\right].$$

In (20), $\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$ is Gaussian error function of random variable x.

By assumption $A_{\mathrm{SU}\,i} = \frac{1}{u^2_{\mathrm{SU}\,i} + 2\sigma_n^2}$, then (20) can be rewritten as follows,

$$f(y_{\mathrm{FC}\,i}[l]|u_{\mathrm{SU}\,i}) = \frac{\sqrt{2}}{\sqrt{\pi}} \sigma_n A_{\mathrm{SU}\,i} \; e^{\left(\frac{-y_{\mathrm{FC}\,i}[l]^2}{2\sigma_n^2}\right)}$$
$$+ 2u_{\mathrm{SU}\,i} A^{\frac{3}{2}}_{\mathrm{SU}\,i} y_{\mathrm{FC}\,i}[l] e^{-A_{\mathrm{SU}\,i} y_{\mathrm{FC}\,i}[l]^2} Q\left(-\frac{u_{\mathrm{SU}\,i} A^{\frac{1}{2}}_{\mathrm{SU}\,i}}{\sigma_n} y_{\mathrm{FC}\,i}[l]\right). \tag{21}$$

As we can see, by assumption of known noise variance, (21) is function of $u_{\mathrm{SU}\,i}$ and $y_{\mathrm{FC}\,i}[l]$ and is independent from instantaneous channel value. Since transmitted data from SUs are discrete random variables, we have to calculate PMF of received symbols as a posteriori probability of quantized data from distribution of received signals in FC. Thus,

$$P(U_{\mathrm{FC}\,i}[l] = u_{m,i} | u_{\mathrm{SU}\,i} = u_{k,i}) = \int_{\Omega_m} f(y_{\mathrm{FC}\,i}[l] = t | u_{k,i}) dt, \tag{22}$$

where Ω_m is decision region of $u_{m,i}$ symbol. Calculation of integral in (22) is given in Appendix. Thus, from (12) and (A-4), a posteriori probability of received symbol from ith SU under both hypotheses can be represented as,

$$P(U_{\mathrm{FC}\,i}[l] = u_{m,i} | \mathcal{H}_j) = \sum_{k=1}^{M} P(U_{\mathrm{FC}\,i}[l] = u_{m,i} | u_{\mathrm{SU}\,i} = u_{k,i}) P(U_{\mathrm{SU}\,i} = u_{k,i} | \mathcal{H}_j), \tag{23}$$

for $j = 0, 1$. By averaging on K time phases in which ith SU's symbol is transmitted, we have, for $j = 0, 1$,

$$P(U_{\mathrm{FC}\,i} = u_{m,i} | \mathcal{H}_j) = \frac{1}{K} \sum_{l=1}^{K} P(U_{\mathrm{FC}\,i}[l] = u_{m,i} | \mathcal{H}_j). \tag{24}$$

To determine transmitted symbol of the ith SU, maximum a posteriori probability (MAP) is used such that the received symbol with greater a posteriori probability is selected. Thus, for equal priori probabilities of both hypotheses, we have,

$$\widehat{u}_{\mathrm{SU}\,i} = \max_{m=1,\ldots,M} \{u_{m,i} : P(U_{\mathrm{FC}\,i} = u_{m,i})\}. \tag{25}$$

4.1 Decision Rule Based on Received Symbol Summation

In this subsection, we consider summation based fusion scheme in FC in which FC sums the determined symbols of all SUs to make final decision. i.e.,

$$\mathrm{T_{FC}} = \sum_{i=1}^{K} \widehat{u}_{\mathrm{SU}\,i} \lesseqgtr_{\mathcal{H}_0}^{\mathcal{H}_1} \eta_{\mathrm{FC}}. \tag{26}$$

To evaluate performance of the expressed decision rule, we have to derive the PMF of $\mathrm{T_{FC}}$. Thus, From [17], we have,

$$p(\mathrm{T_{FC}} | \mathcal{H}_j) = p(\widehat{U}_{\mathrm{SU}\,1} | \mathcal{H}_j) * \ldots * p(\widehat{U}_{\mathrm{SU}\,K} | \mathcal{H}_j). \tag{27}$$

Where $p(\widehat{U}_{\mathrm{SU}\,i} | \mathcal{H}_j)$ denotes the PMF of ith received symbol estimation and $*$ denotes the convolution operator of discrete random variables. If $\mathcal{L}_{\mathrm{T_{FC}}}$, which has M^K elements, be the set of values which $\mathrm{T_{FC}}$ can take, then according to

Neyman-Pearson test for discrete random variables, decision rule at the FC is given by,

$$\begin{cases} \mathrm{T_{FC}} < \eta_{_\mathrm{FC}}, & \mathcal{H}_0 \\ \mathrm{T_{FC}} = \eta_{_\mathrm{FC}}, & \mathcal{H}_1 \text{ with probability } \gamma \\ \mathrm{T_{FC}} > \eta_{_\mathrm{FC}}, & \mathcal{H}_1, \end{cases} \tag{28}$$

where for maximum acceptable false alarm probability α, $\eta_{_\mathrm{FC}}$ is the threshold given by,

$$\eta_{_\mathrm{FC}} = \min_{t_{_\mathrm{FC}} \in \mathcal{L}_{\mathrm{T_{FC}}}} \{t_{_\mathrm{FC}} : P(\mathrm{T_{FC}} > t_{_\mathrm{FC}} | \mathcal{H}_0) < \alpha\}. \tag{29}$$

and γ is randomization parameter as follows,

$$\gamma = \frac{\alpha - P(\mathrm{T_{FC}} > \eta_{_\mathrm{FC}} | \mathcal{H}_0)}{P(\mathrm{T_{FC}} = \eta_{_\mathrm{FC}} | \mathcal{H}_0)}. \tag{30}$$

Detection probability can be calculated as,

$$P_\mathrm{d} = P(\mathrm{T_{FC}} > \eta_{_\mathrm{FC}} | \mathcal{H}_1) + \gamma P(\mathrm{T_{FC}} = \eta_{_\mathrm{FC}} | \mathcal{H}_1). \tag{31}$$

4.2 Kullback-Leibler Divergence Criterion in FC

In this subsection, we benefit KL divergence criterion for evaluation of detector performance in FC, which in probability, is a criterion of the difference between two probability distributions. The KL divergence is a fundamental equation to quantify the proximity of two probability distributions[18]. While, it may be used as criterion of divergence between the two hypotheses in detection, since it is the expected log-likelihood ratio[19]. In this paper, we express the KL divergence criterion in the following form,

$$D_{KL_{FC}} = \sum_{\mathbf{m}} P(\widehat{\mathbf{u}} = \mathbf{m} | \mathcal{H}_1) \ln \frac{P(\widehat{\mathbf{u}} = \mathbf{m} | \mathcal{H}_1)}{P(\widehat{\mathbf{u}} = \mathbf{m} | \mathcal{H}_0)}, \tag{32}$$

where $P(\widehat{\mathbf{u}} = \mathbf{m} | \mathcal{H}_j), j = 0, 1$, is PMF of the received symbols estimations vector, $\widehat{\mathbf{u}} = \{\widehat{u}_{_{\mathrm{SU}\,1}}, \widehat{u}_{_{\mathrm{SU}\,2}}, ..., \widehat{u}_{_{\mathrm{SU}\,K}}\}$ and $\mathbf{m} \in \{u_{1,1}, ..., u_{M,1}\} \times \ldots \times \{u_{1,K}, ..., u_{M,K}\}$. As mentioned before, we assume that SUs are independent and the PMF of them are independent too. Therefore, (32) can be expressed as follows,

$$D_{KL_{FC}} = \sum_{i=1}^{K} \sum_{m=1}^{M} P(\widehat{u}_{_{\mathrm{SU}\,i}} = u_{m,i} | \mathcal{H}_1) \ln \frac{P(\widehat{u}_{_{\mathrm{SU}\,i}} = u_{m,i} | \mathcal{H}_1)}{P(\widehat{u}_{_{\mathrm{SU}\,i}} = u_{m,i} | \mathcal{H}_0)}. \tag{33}$$

5 Simulation Results

In this section, we provide a comparative simulation-based performance of the system which is introduced in this paper, based on Monte-Carlo simulations.

Fig. 2 depicts comparison of the performance in FC for different quantization levels under uniform and non-uniform schemes based on detection probability P_d versus SNR at false alarm probability rate $P_\mathrm{fa} = 0.01$ and $K = 4$ in the

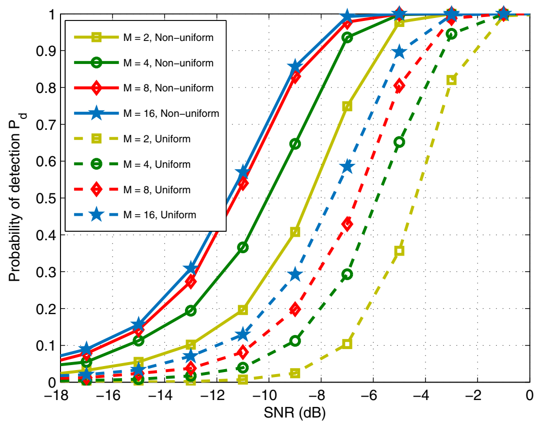

Fig. 2. Probability of detection P_d of the CPAC scheme versus SNR for $P_\mathrm{fa} = 0.01$, $K = 4$ and different uniform and non-uniform quantization levels.

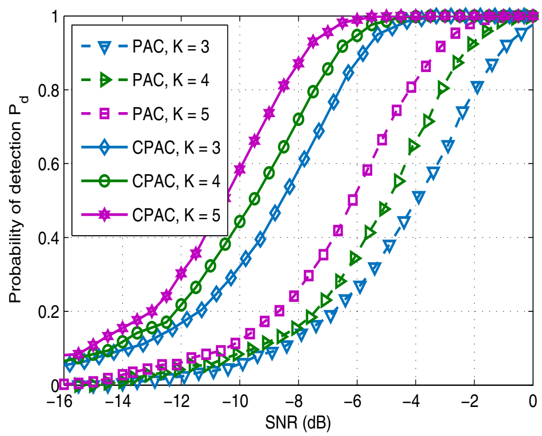

Fig. 3. Probability of detection P_d of the CPAC and PAC schemes versus SNR for $P_\mathrm{fa} = 0.01$, $M = 8$ and different number of users.

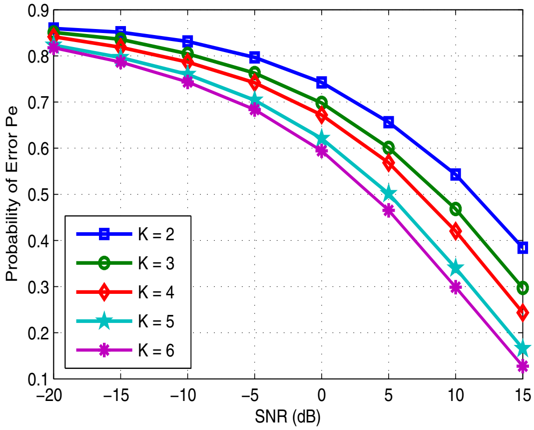

Fig. 4. Average error probability of FC detector versus SNR in the CPAC scheme for $M = 8$.

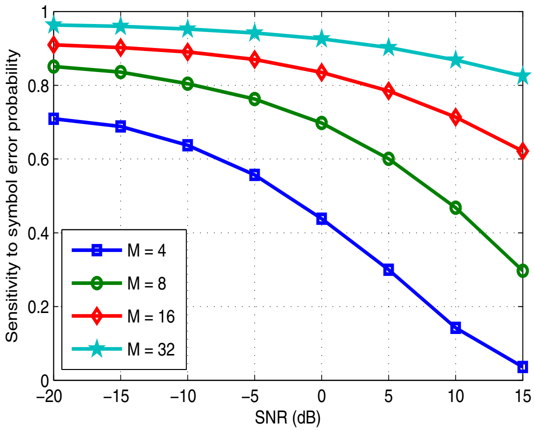

Fig. 5. sensitivity of FC to average error probability of symbols received from each SU versus SNR in the CPAC scheme for $K = 4$.

CPAC scheme. As can be seen, statistic performance is more efficient in the provided non-uniform scheme compared with the uniform one and also with increasing quantization levels, performance will be better, but this improvement in the levels change from 2 to 4 is more than 4 to 8 or 8 to 16 in the non-uniform one. Because values of bounds in this method are very close to each other near the threshold and so increasing of levels will not have noticeable effect. Therefore, non-uniform method does not require high quantization levels for better performance that will be effective for less occupied bandwidth.

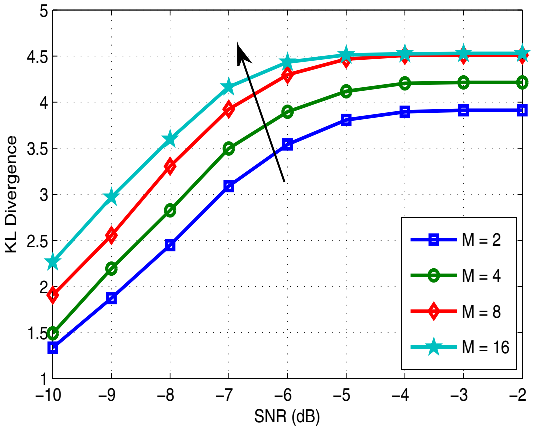

Fig. 6. KL divergence performance metric in FC versus SNR in CPAC scheme for $K = 2$.

In Fig. 3, we investigate the impact of CPAC protocol on performance of the FC statistic based on detection probability P_{d} versus SNR for $M = 8$ and $P_{\mathrm{fa}} = 0.01$. As can be seen, in addition to better performance of the detector for more SUs, performance improvement in cooperative scheme compared with non-cooperative one, PAC, is also evident.

In Fig. 4, decreasing average error probability P_{e} of FC detector versus SNR in CPAC scheme for $M = 8$ and the higher number of SUs is depicted. Fig. 5 shows sensitivity of FC detector to average symbol error probability for $K = 4$ and cooperative scheme. In this figure, the maximum numerical value for sensitive criterion is equal 1, thus the values closer to one are more sensitive, which means in higher quantization levels, receiver is more sensitive to average symbol error and thus its decision is more accurate.

Finally, in Fig. 6 we provide simulation-based evaluation of the KL divergence criterion versus SNR for different quantization levels and $K = 2$ in CPAC scheme. Higher values of KL divergence which means more accurate decision can be observed in figure with increasing quantization levels.

6 Conclusion

In this paper, we investigated a practical CSS problem in which SUs use multilevel non-uniform quantization to transmit their data to FC based on cooperative scheme in Rayleigh fading wireless links. We detected symbols received in FC from each SU in different time phases of CPAC scheme based on MAP pattern. In addition, we provided summation-based final decision statistic and KL divergence metric to evaluate the system performance in FC. Simulation results revealed the effect of SUs cooperation and also applying non-uniform quantization on the performance improvement of the derived detector in FC. Also, we showed that sensitivity of the detector to average symbol error probability and KL divergence criterion will increase in higher quantization levels.

Acknowledgments. This publication was made possible by the National Priorities Research Program (NPRP) award NPRP 6-1326-2-532 from the Qatar National Research Fund (QNRF) (a member of the Qatar Foundation). The statements made herein are solely the responsibility of the authors.

Appendix

In this section we provide calculation of integral in (22) as follows,

$$\int_{\Omega_m} f(y_{\text{FC}_i}[l]=t|u_{\text{SU}_i})dt \tag{A-1}$$
$$= \int_{\Omega_m} \frac{\sqrt{2}\sigma_n A_{\text{SU}_i}}{\sqrt{\pi}}\, e^{\left(\frac{-t^2}{2\sigma_n^2}\right)} dt + \int_{\Omega_m} 2u_{\text{SU}_i} A_{\text{SU}_i}^{\frac{3}{2}} t e^{-A_{\text{SU}_i}t^2} Q\left(-\frac{u_{\text{SU}_i}\sqrt{A_{\text{SU}_i}}}{\sigma_n}t\right) dt.$$

First and second parts of (A-1) can be calculated, respectively, as,

$$I:\quad \int \frac{\sqrt{2}\sigma_n A_{\text{SU}_i}}{\sqrt{\pi}} e^{\left(\frac{-t^2}{2\sigma_n^2}\right)} dt = \sigma_n^2 A_{\text{SU}_i}\text{erf}\left(\frac{t}{\sqrt{2}\sigma_n}\right). \tag{A-2}$$

Also, by integration by parts method, for second part we have,

$$II:\quad \int 2u_{\text{SU}_i} A_{\text{SU}_i}^{\frac{3}{2}} t e^{-A_{\text{SU}_i}t^2} Q\left(-\frac{u_{\text{SU}_i}\sqrt{A_{\text{SU}_i}}}{\sigma_n}t\right) dt$$
$$= \frac{\sqrt{\pi}u_{\text{SU}_i}^2 A_{\text{SU}_i}}{2}\text{erf}\left(\frac{t}{\sigma_n}\right) - u_{\text{SU}_i}\sqrt{A_{\text{SU}_i}} Q(-\frac{u_{\text{SU}_i}\sqrt{A_{\text{SU}_i}}}{\sigma_n}t)e^{-A_{\text{SU}_i}t^2}. \tag{A-3}$$

Then, from (A-2), (A-3) and (22), we have,

$$P(U_{\text{FC}_i}[l]=u_{m,i}|u_{\text{SU}_i}=u_{k,i}) \tag{A-4}$$
$$= \left[\sigma_n^2 A_{k,i}\text{erf}\left(\frac{t}{\sqrt{2}\sigma_n}\right) + \frac{\sqrt{\pi}u_{k,i}^2 A_{k,i}}{2}\text{erf}\left(\frac{t}{\sigma_n}\right) - u_{k,i}\sqrt{A_{k,i}}Q(-\frac{u_{k,i}\sqrt{A_{k,i}}}{\sigma_n}t)e^{-A_{k,i}t^2}\right]_{\Omega_m}.$$

References

1. Akyildiz, I., Lee, W., Vuran, M., Mohanty, S.: Next generation dynamic spectrum access cognitive radio wireless networks: A survey. Computer Networks **50**(13) (2006)

2. Taherpour, A., Nasiri-Kenari, M., Jamshidi, A.: Efficient cooperative spectrum sensing in cognitive radio networks. In: IEEE International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC) (2007)
3. Sun, C., Zhang, W., Letaief, K.: Cluster-based cooperative spectrum sensing in cognitive radio systems. In: Proc. of the IEEE International Conference on Communications (ICC), pp. 2511–2515, June 2007
4. Zhang, W., Mallik, R., Letaief, K.: Cooperative spectrum sensing optimization in cognitive radio networks. In: Proc. of the IEEE International Conference on Communications (ICC), pp. 3411–3415, May 2008
5. Rao, A., Alouini, M.-S.: Cooperative spectrum sensing over non-identical nakagami fading channels, pp. 1–4, May 2011
6. Salvo Rossi, P., Ciuonzo, D., Romano, G.: Orthogonality and cooperation in collaborative spectrum sensing through MIMO decision fusion. IEEE Transactions on Wireless Communications **12**(11), 5826–5836 (2013)
7. Nadendla, V., Han, Y., Varshney, P.: Distributed inference with M-Ary quantized data in the presence of byzantine attacks. IEEE Transactions on Signal Processing **62**(10), 2681–2695 (2014)
8. Niu, R., Chen, B., Varshney, P.: Fusion of decisions transmitted over Rayleigh fading channels in wireless sensor networks. IEEE Transactions on Signal Processing **54**(3), 1018–1027 (2006)
9. Lin, Y.: Quantization for distributed detection under link outages. In: 42nd Asilomar Conference on Signals, Systems and Computers, pp. 1948–1952, October 2008
10. Chen, H., Tse, C., Zhao, F.: Optimal quantization bit budget for a spectrum sensing scheme in bandwidth-constrained cognitive sensor networks. IET Wireless Sensor Systems **1**, 144–150 (2011)
11. Schwandter, S., Matz, G.: A practical forwarding scheme for wireless relay channels based on the quantization of log-likelihood ratios. In: IEEE International Conference on Acoustics Speech and Signal Processing (ICASSP), pp. 2502–2505, March 2010
12. Zeitler, G., Bauch, G., Widmer, J.: Quantize-and-forward schemes for the orthogonal multiple-access relay channel. IEEE Transactions on Communications **60**(4), 1148–1158 (2012)
13. Liang, H., Chen, Y., Li, S.: A log-likelihood ratio non-uniform quantization scheme for cooperative spectrum sensing. In: 8th International Conference on Wireless Communications, Networking and Mobile Computing (WiCOM), pp. 1–4, September 2012
14. Chaudhari, S., Koivunen, V.: Effect of quantization and channel errors on collaborative spectrum sensing. In: Conference Record of the Forty-Third Asilomar Conference on Signals, Systems and Computers, pp. 528–533, November 2009
15. Taherpour, A., Norouzi, Y., Nasiri-Kenari, M., Jamshidi, A., Zeinalpour-Yazdi, Z.: Asymptotically optimum detection of primary user in cognitive radio networks. IET Communication **1**, 1138–1145 (2007)
16. Gradshteyn, I., Ryzhik, I.: Table of integrals, series, and products
17. Evans, D., Leemis, L.: Algorithms for computing the distributions of sums of discrete random variables. Mathematical and Computer Modelling **40**, 1429–1452 (2014)
18. Shlens, J.: Notes on Kullback-Leibler divergence and likelihood (2014). CoRR, abs/1404.2000
19. Eguchi, S., Copas, J.: Interpreting Kullback-Leibler divergence with the Neyman-Pearson lemma. Journal of Multivariate Analysis **97**(9), 2034–2040 (2006)