Detection of Temporally Correlated Primary User Signal with Multiple Antennas

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Abstract. In this paper, we address the problem of multiple antenna spectrum sensing in cognitive radios (CRs) when the samples of the primary user (PU) signal as well as samples of noise are assumed to be temporally correlated. We model and formulate this multiple antenna spectrum sensing problem as a hypothesis testing problem. First, we derive the optimum Neyman-Pearson (NP) detector for the scenario in which the channel gains, the PU signal and noise correlation matrices are assumed to be known. Then, we derive the sub-optimum generalized likelihood ratio test (GLRT)-based detector for the case when the channel gains and aforementioned matrices are assumed to be unknown. Approximate analytical expressions for the false-alarm probabilities of the proposed detectors are given. Simulation results show that the proposed detectors outperform some recently-purposed algorithms for multiple antenna spectrum sensing.

1 Introduction

Using multiple antennas at the secondary user (SU) receiver is one efficient approach to overcome deleterious effects of noise uncertainty, fading and shadowing. Moreover, it improves the performance of spectrum sensing by exploiting available observations in the spatial domain [1–10]. [1] considers a blind spectrum sensing approach where the empirical characteristic function of the multiantenna samples is used in the formulation of the statistical test. In [2], the authors derive the optimum NP and sub-optimum GLRT-based multiantenna detectors of an orthogonal frequency division multiplexing (OFDM) signal with a cyclic prefix of known length. In [3–7], GLRT eigenvalues-based detectors of spatial rank-one PU signals robust to noise variance uncertainty are derived. In addition, some GLRT eigenvalue-based detectors for multiantenna spectrum sensing are proposed in [8,9], for PU signals with spatial rank larger than one. In CR networks, signals from far PUs arrive at the SU base station within a small beamwidth, which results in a high correlation between the channel gains of different antennas. The Roa test is applied to derive sub-optimum multiantenna detectors under the correlated receiving antennas model in [10].

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All the detectors proposed in [1–10] do not consider any temporal correlation between the samples of the received signal. Nevertheless, in practice, the PU signal samples as well as noise samples may be temporally correlated, which causes degradation in the performance of detectors proposed in [1–10]. In [11], the detection of temporally correlated signals over multipath fading channels is discussed and a modified energy detector (ED) is proposed for such a scenario. However, as known, the performance of the ED is susceptible to errors in the noise variance and it has been shown that in order for ED to achieve a desired probability of detection under noise (or in more general terms, under model uncertainties) the signal-to-noise ratio (SNR) must be above a certain threshold [12] (SNR wall).

In this paper, we consider multiple antenna spectrum sensing when there are temporal correlation between the PU signal samples in the presence of additive temporally correlated Gaussian noise. First, for benchmarking purposes, we obtain the optimum NP detector for the case when the SU receiver has complete knowledge about the channel gains, the PU signal and noise covariance matrices. Then, we derive the sub-optimum GLRT-based detector when the channel gains, the PU signal and noise covariance matrices are assumed to be unknown to the SU receiver. Approximate analytical expressions for the false-alarm probabilities of proposed detectors are also given. The simulation results are provided to evaluate the impact of the different parameters on the performance of the proposed detectors and, moreover, to compare the performance of the proposed detectors with some recently-proposed detectors.

The rest of the paper is organized as follows. In Section 2, we describe the system model and the basic assumptions about the PU signal and noise. In Section 3, we derived the optimum NP detector. The sub-optimum GLRT-based detector is obtained in Section 4. Asymptotic expressions for the false-alarm probabilities of the proposed detectors are evaluated in Sections 5. The simulation results and related discussions are given in Section 6. Finally, Section 7 concludes the paper.

Notation: Lightface letters denote scalars. Boldface lower-and upper-case letters denote column vectors and matrices, respectively. x[.] are the entries and $\mathbf{x}_i$ is sub-vector of vector $\mathbf{x}$. The determinant and inverse of matrix $\mathbf{A}$ are $|\mathbf{A}|$ and $\mathbf{A}^{-1}$, respectively. vec[$\mathbf{A}$] is the column-wise vectorization of matrix $\mathbf{A}$. The $M \times M$ identity matrix is $\mathbf{I}_M$ and the $M \times M$ zeros matrix is $\mathbf{0}_M$. Superscripts $*$, T and H are the complex conjugate, transpose and Hermitian (conjugate transpose) operations, respectively. $\mathbb{E}[\cdot]$ is the statistical expectation. $\mathbf{A} \otimes \mathbf{B}$ is kronecker product of matrices $\mathbf{A}$ and $\mathbf{B}$. $\mathcal{CN}(\mathbf{m}, \mathbf{P})$ denotes circularly symmetric complex Gaussian distribution with mean $\mathbf{m}$ and covariance matrix $\mathbf{P}$. $Q(x)$ is Q-function $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty exp\Big(\frac{-u^2}{2}\Big) du$.

2 System Model

We consider a CR network including a single-antenna PU and a mutiantenna SU with M receiving antennas. We assume the received signal is downconverted to baseband and sampled at the frequency $f_s = \frac{1}{T_s}$ at each antenna to generate N consecutive time blocks, each of which contains L consecutive temporal samples. Define $\mathbf{y}_{i,j} \in \mathbb{C}^M$ as the vector of the received signal samples from M different antennas of the ith time block at the jth time instant. The observation vector $\mathbf{y}_{i,j}$ is given as

$$\mathbf{y}_{i,j} = \mathbf{h}_i s_j + \mathbf{n}_{i,j},\ i = 1, \ldots, N;\ j = 1, \ldots, L \tag{1}$$

where $\mathbf{h}_i \in \mathbb{C}^M$ is the baseband equivalent of channel gains vector at ith time block, which is assumed to be constant during each time block. $s_j \in \mathbb{C}$ denotes the baseband samples of the PU signal, which is assumed to be distributed as a zero-mean complex Gaussian random variable with autocorrelation function $r_s[k] = \mathbb{E}[s_j s_{j-k}^*]$. We assume, in general, s_j exhibits temporal correlation, i.e. $r_s[k] \neq 0$ for $k \neq 0$. $\mathbf{n}_{i,j} \in \mathbb{C}^M$ denotes the baseband equivalent of noise samples which is assumed to be distributed as a zero-mean complex Gaussian random vector. In addition, $\mathbf{n}_{i,j}$ is assumed to be spatially uncorrelated but temporally correlated with autocorrelation function $r_n[k]$. We assume noise and the PU signal samples are mutually independent.

Let us define the matrix $\mathbf{Y}_i \doteq [\mathbf{y}_{i,1}, \ldots, \mathbf{y}_{i,L}] \in \mathbb{C}^{M\times L}$ containing L time samples of the ith time block. In addition, let us define $\mathbf{y}_i = \text{vec}[\mathbf{Y}_i] \in \mathbb{C}^{LM\times 1}$ and $\mathbf{y} = \text{vec}[\mathbf{y}_1, \ldots, \mathbf{y}_N] \in \mathbb{C}^{NLM\times 1}$. We denote the hypotheses of the presence and absence of the PU signal by $\mathcal{H}_1$ and $\mathcal{H}_0$, respectively. By defining the correlation matrix of $\mathbf{y}$: $\mathbf{\Sigma}_\nu = \mathbb{E}\{\mathbf{y}\mathbf{y}^H|\mathcal{H}_\nu\}$, $\nu = 0, 1$, it can be easily shown that,

$$\mathbf{\Sigma_0} = \mathbf{I}_N \otimes \mathbf{I}_M \otimes \mathbf{\Sigma}_n = \mathbf{I}_{NM} \otimes \mathbf{\Sigma}_n, \tag{2}$$

where

$$\mathbf{\Sigma}_n \doteq \begin{pmatrix} r_n[0] & r_n^*[1] & \ldots & r_n^*[L-1] \\ r_n[1] & r_n[0] & \ldots & r_n^*[L-2] \\ \vdots & \vdots & \ddots & \vdots \\ r_n[L-1] & r_n[L-2] & \ldots & r_n[0] \end{pmatrix}, \tag{3}$$

is the temporal correlation matrix of noise, and

$$\mathbf{\Sigma}_1 \doteq \begin{pmatrix} \mathbf{\Sigma}_{1,1} & \mathbf{0} & \ldots & \mathbf{0} \\ \mathbf{0} & \mathbf{\Sigma}_{2,2} & \ldots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \ldots & \ldots & \mathbf{\Sigma}_{N,N} \end{pmatrix}, \tag{4}$$

where

$$\mathbf{\Sigma}_{i,i} = \mathbf{h}_i \mathbf{h}_i^H \otimes \mathbf{\Sigma}_s + \mathbf{I}_M \otimes \mathbf{\Sigma}_n \tag{5}$$

with

$$\mathbf{\Sigma}_s \doteq \begin{pmatrix} r_s[0] & r_s^*[1] & \dots & r_s^*[L-] \\ r_s[1] & r_s[0] & \dots & r_s^*[L-2] \\ \vdots & \vdots & \ddots & \vdots \\ r_s[L-1] & r_s[L-2] & \dots & r_s[0] \end{pmatrix}. \tag{6}$$

Accordingly, the distribution of observations under each hypothesis is given as

$$\begin{cases} \mathcal{H}_0: & \mathbf{y} \sim \mathcal{CN}(\mathbf{0}_{NLM}, \mathbf{\Sigma_0}) \\ \mathcal{H}_1: & \mathbf{y} \sim \mathcal{CN}(\mathbf{0}_{NLM}, \mathbf{\Sigma_1}). \end{cases} \tag{7}$$

3 Optimum Detector

In this section, we derive the NP detector for the case of completely known $\mathbf{h}_i$, $\mathbf{\Sigma}_n$ and $\mathbf{\Sigma}_s$. From (7) the probability density function (PDF) of the observations vector $\mathbf{y}$ under each hypothesis is given by,

$$\begin{aligned} f(\mathbf{y}|\mathcal{H}_0, \mathbf{\Sigma}_0) &= \frac{\exp\left\{-\mathbf{y}^H \mathbf{\Sigma}_0^{-1} \mathbf{y}\right\}}{\pi^{NLM} |\mathbf{\Sigma}_0|} \\ &= \frac{\exp\left\{-L\text{tr}(\mathbf{\Sigma}_n^{-1} \sum_{i=1}^N \sum_{m=1}^M \mathbf{R}_{i,mm})\right\}}{\pi^{NLM} |\mathbf{\Sigma}_n|^{NM}}. \end{aligned} \tag{8}$$

and

$$\begin{aligned} f(\mathbf{y}|\mathcal{H}_1, \mathbf{\Sigma}_1) &= \frac{\exp\left\{-\mathbf{y}^H \mathbf{\Sigma}_1^{-1} \mathbf{y}\right\}}{\pi^{NLM} |\mathbf{\Sigma}_1|} \\ &= \frac{\exp\left\{-L\text{tr}(\sum_{i=1}^N \mathbf{R}_i \mathbf{\Sigma}_{i,i}^{-1})\right\}}{\pi^{NLM} \prod_{i=1}^N |\mathbf{\Sigma}_{i,i}|}, \end{aligned} \tag{9}$$

where

$$\mathbf{R}_i \doteq \frac{1}{L} \mathbf{y}_i \mathbf{y}_i^H = \begin{pmatrix} \mathbf{R}_{i,11} & \mathbf{R}_{i,12} & \dots & \mathbf{R}_{i,1M} \\ \mathbf{R}_{i,21} & \mathbf{R}_{i,22} & \dots & \mathbf{R}_{i,2M} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{R}_{i,M1} & \dots & \dots & \mathbf{R}_{i,MM} \end{pmatrix}. \tag{10}$$

in which $\mathbf{R}_{i,mk}$'s are the corresponding sub-matrices.

Taking logarithm from (8) and (9) and defining $\mathcal{L}_\nu(\mathbf{y}) \doteq \ln f(\mathbf{y}|\mathcal{H}_\nu, \mathbf{\Sigma}_\nu)$, $\nu = 0, 1$, we obtain,

$$\mathcal{L}_0(\mathbf{y}) = -L\text{tr}(\mathbf{\Sigma}_n^{-1} \sum_{i=1}^{N} \sum_{m=1}^{M} \mathbf{R}_{i,mm}) - NLM \ln \pi - NM \ln |\mathbf{\Sigma}_n|, \quad (11)$$

$$\mathcal{L}_1(\mathbf{y}) = -L\text{tr}(\sum_{i=1}^{N} \mathbf{R}_i \mathbf{\Sigma}_{i,i}^{-1}) - NLM \ln \pi - \sum_{i=1}^{N} \ln |\mathbf{\Sigma}_{i,i}|. \quad (12)$$

By constituting the logarithm of likelihood ratio (LLR) function from (11) and (12) and comparing it with a threshold, the optimum detector is given by

$$\text{LLR} = L\text{tr}(\sum_{i=1}^{N} \mathbf{R}_i [(\mathbf{I}_M \otimes \mathbf{\Sigma}_n)^{-1} - \mathbf{\Sigma}_{i,i}^{-1}]) \gtrless_{H_0}^{H_1} \tau', \quad (13)$$

Now by using matrix inversion lemma, we have,

$$\begin{aligned} &(\mathbf{h}_i \mathbf{h}_i^H \otimes \mathbf{\Sigma}_s + \mathbf{I}_M \otimes \mathbf{\Sigma}_n)^{-1} \quad (14) \\ &= (\mathbf{I}_M \otimes \mathbf{\Sigma}_n)^{-1} - (\mathbf{I}_M \otimes \mathbf{\Sigma}_n)^{-1} (\mathbf{h}_i \mathbf{h}_i^H \otimes \mathbf{\Sigma}_s)(\mathbf{h}_i \mathbf{h}_i^H \otimes \mathbf{\Sigma}_s + \mathbf{I}_M \otimes \mathbf{\Sigma}_n)^{-1} \end{aligned}$$

Therefore, by substituting (14) in (13), we find,

$$\mathrm{T_{opt}} = \mathrm{Ltr}(\sum_{\mathrm{i=1}}^{\mathrm{N}} \mathbf{R}_\mathrm{i} \mathbf{C}_\mathrm{i}^{-1}) \gtrless_{\mathrm{H_0}}^{\mathrm{H_1}} \tau, \quad (15)$$

where $\mathbf{C}_i \doteq (\mathbf{h}_i \mathbf{h}_i^H \otimes \mathbf{\Sigma}_s + \mathbf{I}_M \otimes \mathbf{\Sigma}_n)(\mathbf{h}_i \mathbf{h}_i^H \otimes \mathbf{\Sigma}_s)^{-1}(\mathbf{I}_M \otimes \mathbf{\Sigma}_n)$.

In order to simplify the optimum detector more, we can use the singular value decomposition (SVD) of $\mathbf{C}_i$: $\mathbf{C}_i = \mathbf{U}\mathbf{\Lambda_i}\mathbf{U}^H$, where $\mathbf{\Lambda}_i = \text{diag}\{\lambda_{i,1}, \lambda_{i,2}, ..., \lambda_{i,LM}\}$ contains eigenvalues of $\mathbf{C}_i$ and the columns of $\mathbf{U}$ its the corresponding eigenvectors. Therefore,

$$T_{\text{opt}} = \text{tr}(\sum_{i=1}^{N} \mathbf{y}_i \mathbf{y}_i^H \mathbf{U} \mathbf{\Lambda_i^{-1}} \mathbf{U}^H) = \sum_{i=1}^{N} \text{tr}(\mathbf{\Lambda}_i^{-1} \mathbf{U}^H \mathbf{y}_i \mathbf{y}_i^H \mathbf{U}) \gtrless_{H_0}^{H_1} \tau. \quad (16)$$

Let $\mathbf{w}_i \doteq \mathbf{U}^H \mathbf{y}_i$ and $w_i(n)$ be the nth elements of $\mathbf{w}_i$, then the optimum detector can be written as

$$T_{\text{opt}} = \sum_{i=1}^{N} \sum_{n=1}^{LM} \lambda_{i,n}^{-1} |w_i(n)|^2 \gtrless_{H_0}^{H_1} \tau. \quad (17)$$

4 Sub-Optimum GLRT-Based Detector

In this section, we assume the channel gains, the PU signal and noise covariance matrices are unknown to the SU receiver. We derive the GLRT-based detector for such a scenario. We first maximize (9) with respect to $\mathbf{\Sigma}_1$ in order to compute the maximum likelihood estimate (MLE) of $\mathbf{\Sigma}_1$. By setting the derivative of (9) with respect to $\mathbf{\Sigma}_1$ equal to zero, we obtain

$$L(\mathbf{\Sigma}_{i,i}^{-1}\mathbf{R}_i\mathbf{\Sigma}_{i,i}^{-1})^T = (\mathbf{\Sigma}_{i,i}^{-1})^T, \tag{18}$$

which results to $\hat{\mathbf{\Sigma}}_{i,i} = L\mathbf{R}_i$.

In addition, in order to compute the MLE of $\mathbf{\Sigma}_n$, we should take derivative of (8) with respect to $\mathbf{\Sigma}_n$ and set it equal to zero, which yields $\hat{\mathbf{\Sigma}}_n = \frac{L}{NM}\sum_{i=1}^{N}\sum_{m=1}^{M}\mathbf{R}_{i,mm}$.

By constituting the LR function, the GLRT-based detector is obtained as,

$$\mathrm{T}_{\mathrm{sub}} = \frac{f(\mathbf{y}|\mathcal{H}_1,\hat{\mathbf{\Sigma}}_1)}{f(\mathbf{y}|\mathcal{H}_0,\hat{\mathbf{\Sigma}}_0)} = \frac{|\hat{\mathbf{\Sigma}}_n|^{NM}}{\prod_{i=1}^{N}|\hat{\mathbf{\Sigma}}_{i,i}|} = \frac{1}{(NM)^{NML}}\prod_{i=1}^{N}\frac{|\sum_{i=1}^{N}\sum_{m=1}^{M}\mathbf{R}_{i,mm}|^M}{|\mathbf{R}_i|}. \tag{19}$$

5 Analytical Performance

In the following section, we evaluate the performance of the proposed optimum and sub-optimum detectors in terms of the detection and false-alarm probabilities, i.e. P_d and P_{fa}, respectively.

5.1 Performance of the Optimum Detector

Performance of the optimum detector is evaluated in this sub-section. We can rewrite (17) in the null hypothesis as,

$$\begin{aligned}\mathrm{T}_{\mathrm{opt}|\mathcal{H}_0} &= \sum_{i=1}^{N}\sum_{n=1}^{LM}\lambda_{i,n}^{-1}|w_i(n)|^2 = \sum_{i=1}^{N}\lambda_{i,1}^{-1}\lambda_{0_{i,1}}\frac{|w_i(1)|^2}{\lambda_{0_{i,1}}} + \dots \\ &\quad + \lambda_{i,LM}^{-1}\lambda_{0_{i,LM}}\frac{|w_i(LM)|^2}{\lambda_{0_{i,LM}}} \\ &= \sum_{i=1}^{N}\sum_{n=1}^{LM}\lambda_{i,n}^{-1}\lambda_{0_{i,n}}z_i(n),\end{aligned} \tag{20}$$

where $\lambda_{0_{i,n}}$'s are eigenvalues of ith time block covariance matrix and $w_i(n)$ is a zero-mean Gaussian random variable with variance $\lambda_{0_{i,n}}$. Thus, from [13], $z_i(n) = \frac{|w_i(n)|^2}{\lambda_{0_{i,n}}}$ has the chi-squared distribution with one degree of freedom.

From central limit theorem (CLT), with NLM sufficiently large and, also, from [13], the distribution of the optimum detector under the null hypothesis is Gaussian. Hence, for evaluating performance of the optimum detector, we should compute its mean and variance as,

$$\mu_{\mathrm{T}_{\mathrm{opt}}|\mathcal{H}_0} = \sum_{i=1}^{N}\sum_{n=1}^{LM}\lambda_{i,n}^{-1}\lambda_{0_{i,n}}\mathbb{E}[z_i(n)|\mathcal{H}_0] = \sum_{i=1}^{N}\sum_{n=1}^{LM}\lambda_{i,n}^{-1}\lambda_{0_{i,n}}, \tag{21}$$

and

$$\sigma^2_{\mathrm{T_{opt}}|\mathcal{H}_0} = \sum_{i=1}^{N}\sum_{n=1}^{LM} \lambda_{i,n}^{-1}\lambda_{0_{i,n}} Var[z_i(n)|\mathcal{H}_0] = 2\sum_{i=1}^{N}\sum_{n=1}^{LM} \lambda_{i,n}^{-1}\lambda_{0_{i,n}}. \tag{22}$$

Similarly, under $\mathcal{H}_1$, $\mathrm{T_{opt}}$ has a Gaussian distribution with mean and variance as,

$$\mu_{\mathrm{T_{opt}}|\mathcal{H}_1} = \sum_{i=1}^{N}\sum_{n=1}^{LM} \lambda_{i,n}^{-1}\lambda_{1_{i,n}} \mathbb{E}[z_i(n)|\mathcal{H}_1] = \sum_{i=1}^{N}\sum_{n=1}^{LM} \lambda_{i,n}^{-1}\lambda_{1_{i,n}}, \tag{23}$$

and

$$\sigma^2_{\mathrm{T_{opt}}|\mathcal{H}_1} = \sum_{i=1}^{N}\sum_{n=1}^{LM} \lambda_{i,n}^{-1}\lambda_{1_{i,n}} Var[z_i(n)|\mathcal{H}_1] = 2\sum_{i=1}^{N}\sum_{n=1}^{LM} \lambda_{i,n}^{-1}\lambda_{1_{i,n}}, \tag{24}$$

where $\lambda_{1_{i,n}}$'s are eigenvalues of ith time block covariance matrix under $\mathcal{H}_1$.

Thus, the false-alarm and detection probabilities can be calculated as,

$$P_{\mathrm{fa}} = P\{\mathrm{T_{opt}} > \tau|\mathcal{H}_0\} = \mathrm{Q}\left(\frac{\tau - \mu_{\mathrm{T_{opt}}|\mathcal{H}_0}}{\sigma_{\mathrm{T_{opt}}|\mathcal{H}_0}}\right), \tag{25}$$

$$P_d = P\{\mathrm{T_{opt}} > \tau|\mathcal{H}_1\} = \mathrm{Q}\left(\frac{\tau - \mu_{\mathrm{T_{opt}}|\mathcal{H}_1}}{\sigma_{\mathrm{T_{opt}}|\mathcal{H}_1}}\right). \tag{26}$$

5.2 Performance of the Sub-Optimum Detector

In this sub-section, we derive the asymptotic distribution of the proposed detectors under $\mathcal{H}_0$ by using the results existing for the asymptotic distribution of the GLRT.

Lemma 1. *Let $\boldsymbol{\Theta} = [\boldsymbol{\mu}_r, \boldsymbol{\mu}_s]^T$, with $\boldsymbol{\mu}_r \in \mathbb{R}^r$ and $\boldsymbol{\mu}_s \in \mathbb{R}^s$, be the set of unknown parameters under $\mathcal{H}_1$ and $\mathcal{H}_0$. For a composite hypothesis test of the form,*

$$\begin{cases} \mathcal{H}_0 : \boldsymbol{\mu}_r = \boldsymbol{\mu}_{r_0}, \boldsymbol{\mu}_s \\ \mathcal{H}_1 : \boldsymbol{\mu}_r \neq \boldsymbol{\mu}_{r_0}, \boldsymbol{\mu}_s \end{cases}, \tag{27}$$

the asymptotic distribution of GLRT statistic, T_{GLRT}, under $\mathcal{H}_0$, as $N \to \infty$, is as

$$2\ln T_{GLR} \sim \chi_r^2, \tag{28}$$

where χ_n^2 denotes the central chi-squared distribution with n degrees of freedom.

Proof. See [14]

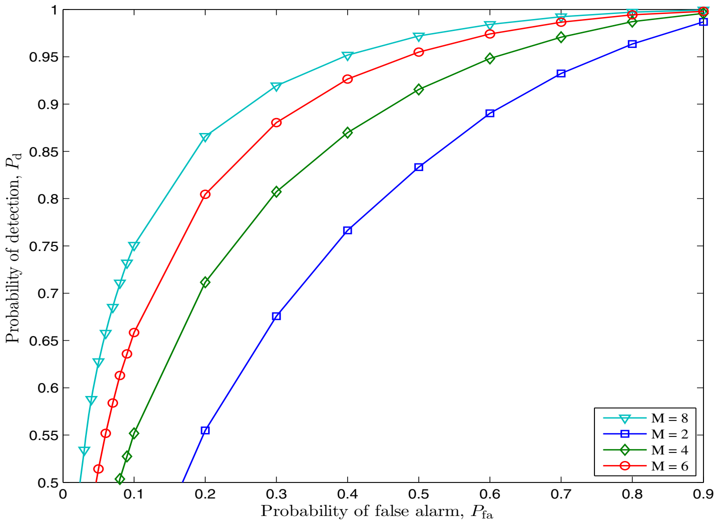

Fig. 1. The complementary ROC of the proposed GLRT-based detector for $SNR = -8$, $P_{fa} = 0.01$, $N = 32$, $L = 10$ and different value of M.

According to Lemma 1, as $N \to \infty$, we have

$$2 \ln T_{\text{sub}} \sim \chi_f^2, \tag{29}$$

where $f = NL^2(M^2 - 1)$. Thus, the false-alarm probability of Λ_{GLR1} can be obtained as

$$P_{\text{fa}} = \mathbb{P}[T_{\text{sub}} > \eta|\mathcal{H}_0] = \mathbb{P}[2\ln T_{\text{sub}} > 2\ln\eta|\mathcal{H}_0] = \frac{\gamma\left(\frac{1}{2}NL^2(M^2-1), \ln\eta\right)}{\Gamma\left(\frac{1}{2}NL^2(M^2-1)\right)}, \tag{30}$$

where $\gamma(k, z) \doteq \int_0^z t^{k-1}e^{-t}dt$ is the lower incomplete Gamma function.

6 Simulation Results

In this section, we provide simulations in order to evaluate the impact of the different parameters on the performance of the proposed detectors and, moreover, to compare the performance of the proposed detectors with other previously reported detectors used as a benchmark. Specifically, the benchmark detectors are: the AGM method [8, Eqn.(14)], the maximum eigenvalue to trace (MET) detector [3, Eqn.(39)], and maximum to minimum eigenvalue (MME) detector [15, Algorithm1].

The complementary ROC (receiver operating characteristics) of the proposed GLRT-based detector for $SNR = -8dB$, $P_{fa} = 0.01$, $N = 32$, $L = 10$ and

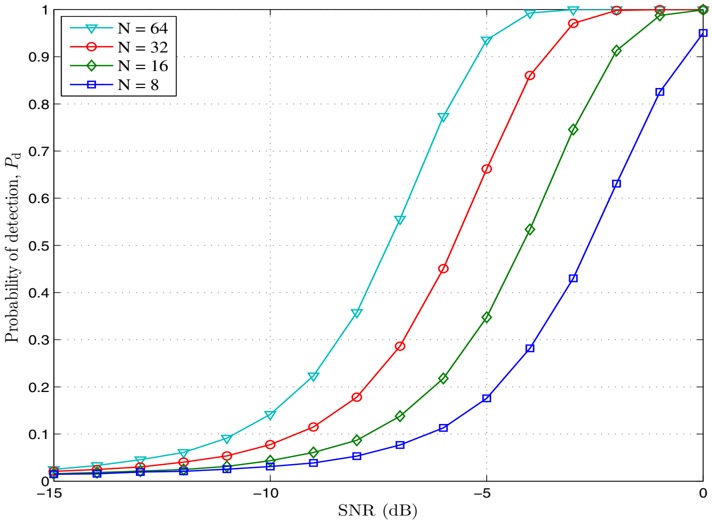

Fig. 2. The detection probability of the proposed GLRT-based detector versus SNR for $P_{fa} = 0.01$ and $M = 4$, $L = 10$ and different value of N.

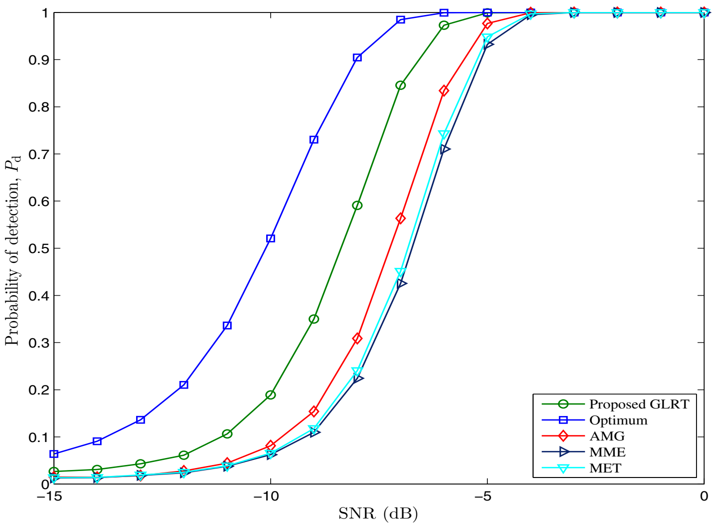

Fig. 3. The detection probability of different detectors versus SNR for $P_{fa} = 0.01$, $L = 10$, $N = 32$ and $M = 4$.

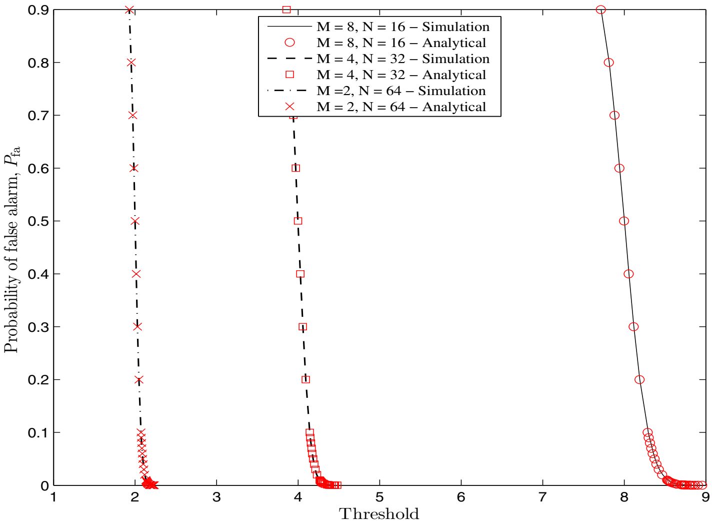

Fig. 4. The false-alarm probability versus threshold of the proposed GLRT-based detectors for $L = 10$ and different value of L and M.

different value of M is shown in Fig. 1. This figure shows that increase in the number of antennas is associated with improvement in performance, but the improvement declines when the number of antennas becomes larger.

Fig. 2 depicts the detection probability of the proposed GLRT-based detector versus SNR for $P_{fa} = 0.01$ and $M = 4$, $L = 10$ and different value of N. As expected, performance of the proposed GLRT-based detector improves by increasing the number of time blocks. Fig. 3 compares the performance of optimum NP detector and the proposed GLRT-based detectors with some other previously reported detectors used as a benchmark. Fig. 3 depicts detection probability of different detectors versus SNR for for $P_{fa} = 0.01$, $L = 10$, $N = 32$ and $M = 4$. As it can be seen, optimum detector has the best performance among all detectors and after that the proposed GLRT-based detector outperforms AGM, MET and MME. In addition, the proposed GLRT-based detector does not require to compute the eigenvalue of the sample covariance matrix in contrast to MET and MME. Hence, the proposed GLRT-based detector has lower computational complexity compare to MET and MME.

Finally, the validity of the approximate closed-form expression provided for proposed GLRT-based detectors is verified in Fig. 4. Fig. 4 shows that there is a good agreement between simulations and the approximate closed-form expression for different value of M and N.

7 Conclusion

In this paper, we investigated the multiple antenna spectrum sensing problem in CR networks when there are temporal correlation between received samples.

First, we derived optimum Neyman-Pearson (NP) for the scenario where the channel gains, the PU signal and noise correlation matrices are known. Then, we obtained the sub-optimum GLRT-based detector for the case when the PU receiver has no knowledge about the channel gains, the PU signal and noise correlation matrices. Then, we provided approximate closed-form expression for the false-alarm probabilities of the proposed detectors. The simulation results were provided to evaluate the impact of the different parameters on the performance of the proposed detectors and, moreover, to compare the performance of the proposed detectors with some other detectors. The provided simulations results revealed that the performance of the proposed sub-optimum detector is better than its counterparts.

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