Design of Probabilistic Random Access in Cognitive Radio Networks

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Abstract. In this paper, we focus on the design of probabilistic random access (PRA) for a cognitive radio network (CRN). The cognitive base station (CBS) allows the secondary users (SUs) to reuse the sub-channels of the primary users (PUs) provided that the interference of the SUs to the PUs is below a predetermined threshold. PUs transmit over a fixed set of channels with fixed transmission powers that are scheduled by the CBS. With this prior information, CBS optimizes the probabilistic random transmissions of the SUs. In each time slot, SUs transmit over a random number of channels d, chosen uniformly at random, according to a certain degree distribution function, optimized by the CBS. Once the signals of the SUs and PUs are received, CBS then implements successive interference cancellation (SIC) to recover both the SUs' and PUs' signals. In the signal recovery, we assume that the PUs' signals can be recovered if the interference power (IP) of the SUs to the PUs is below a predetermined threshold. On the other hand, we assume the SUs' signals can be recovered if its received SINR is above a predetermined threshold. We formulate a new optimization problem to find the optimal degree distribution function that maximizes the probability of successfully recovering the signals of an SU in the SIC process under the SINR constraints of the SUs while satisfying the IP constraints of the PUs. Simulation results show that our proposed design can achieve higher success probabilities and a lower number of transmissions in comparison with conventional schemes, thus, significantly improving signal recovery performance and reducing energy consumption.

Keywords: Probabilistic random access · Cognitive radio · SIC · IP · SINR · Degree distribution

1 Introduction

Cognitive radio (CR) has been known to be a promising technology to achieve the efficient utilization of the radio spectrum. In CR networks (CRNs), unlicensed secondary users (SUs), are allowed access to the radio spectrum owned by the licensed primary users (PUs), provided that the PUs are guaranteed a certain level of protection. Optimal resource allocation algorithms i.e. channel and power

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allocation, among the SUs that maximize their data rates or minimize their transmit power requirements have been well-investigated for multiple scenarios and are known to be NP-hard. Accordingly, numerous sub-optimal algorithms for resource allocation have been proposed for both downlink and uplink CR transmissions [1,2]. However, these approaches do not scale well as the number of users in the network increases and their activity becomes more dynamic.

To overcome these problems, random access protocols provide a simple solution that significantly reduces processing and signalling overhead. A commonly used approach is to employ random access over the control channels. That is, users perform contentions for channel access request. Once access request is granted, data will be transmitted over the allocated channels. Commonly used contention based schemes in CRNs include ALOHA, slotted-ALOHA and carrier sense multiple access (CSMA) [3,4,5]. These models assume that the SUs contend to access the channels only when the PUs are inactive.

In [6], authors proposed another random access approach where the CBS predetermines a certain transmission probability and makes it known to all the SUs. The PUs' transmissions are fixed whereas the SUs transmissions are randomized according to the assigned transmission probability. It is shown that such a simple random transmission can offer significant improvements in performance, in certain cases, for both the PUs and SUs, compared to fixed transmissions. It is argued that from a design point of view, controlling the probabilities is easier than controlling the power. However, the paper only considered a very single case of single channel and no analysis was done to derive the design criteria for choosing the optimal transmission probability.

In [7,8,9], some probabilistic random access (PRA) schemes were proposed where each user transmits over a subset of sub-channels, which are selected uniformly at random, according to a degree distribution, predetermined by the base station. The PRA can then be represented by a bipartite graph, and a message passing algorithm can be implemented at the base station to recover the users' signals. Optimization is then carried out by using the conventional analytical tools of codes-on-graph for binary erasure channels to maximize the probability of having an interference-free clean packet in each iteration. That is mainly because it is assumed that successful signal recovery is only possible when a 'clean packet' is available.

In this paper, we extend the work in [6,7,8,9] to an uplink CRN. We assume the PUs' channel and power allocations are scheduled, and thus, known priori at the CBS. The CBS performs maximal-ratio combining (MRC) to combine the multiple copies of each user's signals over its respective sub-channels and implements successive interference cancellation (SIC) to recover the SUs' and PUs' signals. Under the conventional interference power (IP) constraint, a PU's signal can be successfully recovered if the IP caused by the SUs to that PU is below a predetermined threshold. Moreover, under the conventional SINR constraint, an SU's signal can be successfully recovered if its SINR is above a predetermined threshold. Due to the IP and SINR constraints, the 'clean packet' model becomes sub-optimal. Accordingly, we formulate a new optimization problem to find the optimal degree distribution that maximizes the success probability of the SIC process of the SUs while satisfying the IP constraints of the PUs. This is equivalent to maximizing the probability of having a received SINR of an SU greater than or equal to a predetermined threshold in each iteration of the SIC. Simulation results show that our proposed design can achieve significantly lower error probabilities and requires a lower number of transmissions, in comparison with the conventional approach.

The rest of this paper is organized as follows. Section 2 presents the system model. In section 3, we describe the probabilistic random transmission scheme and the SIC process. In section 4, we analyze the system performance in an asymptotic setting and formulate our degree distribution optimization problem. Numerical results are provided in section 5. Finally, section 6 concludes the paper.

2 System Model

We consider an uplink CRN, including a CBS, a set of K_p active PUs, denoted by $\mathcal{K}_p$, and a set of K_s active SUs denoted by $\mathcal{K}_s$. There are in total N orthogonal sub-channels of equal bandwidth in the network. Channels are assumed to be reciprocal and block fading; that is, we assume the channel coefficients remain constant for the whole transmission block but vary independently from one block to the other. Let y_n denote the received signal vector at the CBS over the n^{th} sub-channel, where $1 \leq n \leq N$. Then, it can be expressed as follows:

$$y_n = \sum_{k \in \mathcal{K}_p} g_{k,n} x_{k,n} + \sum_{i \in \mathcal{K}_s} h_{i,n} u_{i,n} + e_n, \tag{1}$$

where $g_{k,n}$ is the channel gain between PU_k and the CBS over the n^{th} sub-channel, and $h_{k,n}$ is the channel gain between SU_k and the CBS over the n^{th} sub-channel. $x_{k,n}$ and $u_{i,n}$ are the transmitted signals of each of the PUs and SUs to the CBS, over the n^{th} sub-channel. e_n is the additive white Gaussian noise (AWGN) random variable with zero mean and variance σ_e^2.

Each PU is allocated one distinct set of sub-channels. We denote by $\mathcal{N}_p^{(k)}$ the set of $N_p^{(k)}$ sub-channels allocated to PU_k. We denote by $\mathcal{N}_s^{(k)}$ the set of $N_s^{(k)}$ sub-channels chosen by SU_k. Then, $x_{k,n} = 0$ for $n \notin \mathcal{N}_p^{(k)}$, and $u_{k,n} = 0$ for $n \notin \mathcal{N}_s^{(k)}$. Moreover, we denote by $Q_{k,n}$ and $P_{k,n}$ the power of $x_{k,n}$ and $u_{k,n}$, respectively.

3 Random Transmission Scheme

In this section, we describe the random transmission scheme for the previously described system.

3.1 Channel Access

For a given transmission block, each SU chooses a random degree d obtained from a predefined degree distribution $\Omega(x) = \sum_i \Omega_i x^i$, where Ω_i is the probability that $d = i$. Then, the SU chooses d sub-channels uniformly at random to transmit over. We define a $K_s \times N$ random channel access matrix $\mathbf{A}$ with integer elements $a_{k,n} \in \{0, 1\}$, where $a_{k,n} = 1$ means SU_k is transmitting in sub-channel n, and $a_{k,n} = 0$ means SU_k is not transmitting in sub-channel n. Thus, it is easy to show that the elements of $\mathbf{A}$ are independent identically distributed (i.i.d.) Bernoulli random variables with a success probability of $\frac{1}{N}\bar{\Omega}$, where $\bar{\Omega}$ is the average degree and is given by $\sum_i i\Omega_i$. Then, we can represent the probabilistic random

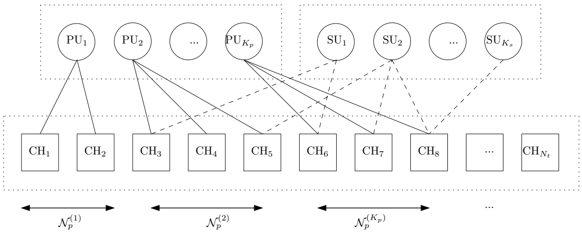

Fig. 1. Bipartite Graph Illustration of the Random Transmission Scheme

transmission scheme by a bipartite graph as shown in Fig.1. The PUs and SUs are shown by circles and referred to as variable nodes while the sub-channels $[\mathrm{CH}_i]_1 \leq i \leq N$ are shown by squares and referred to as check nodes. The number of edges connected to each variable node corresponds to the number of sub-channels it is allocated, and it is called the degree of the respective variable node. The solid edges represent the transmissions of the PUs whose number is assumed to be fixed e.g. PU_1 is of degree 2 in Fig. 1. On the other hand, the dashed edges represent the transmissions of the SUs whose number is a random variable with a distribution pre-determined by the CBS.

3.2 Successive Interference Cancellation

The CBS employs SIC to recover each user's signals. Each user is assumed to transmit the same signals over its respective sub-channels. The CBS can, then, combine the received transmissions of each user over all respective sub-channels using MRC, and the overall received SINR at the CBS can be represented as the sum of all individual SINRs. Note that the CBS is assumed to have the perfect knowledge of the PUs' channel state, transmit power and allocated sub-channels. We also assume that the CBS first attempts to recover the signals of the PUs. The maximum achievable rate of PU_i, where $1 \leq i \leq K_p$, is shown below:

$$R_p^{(i)} = \frac{N_p^{(i)}}{N} \log \left(1 + \sum_{n \in \mathcal{N}_p^{(i)}} \gamma_{p,n}^{(i)} \right), \tag{2}$$

where

$$\gamma_{p,n}^{(i)} = \frac{|g_{i,n}|^2 Q_{i,n}}{\sum_{k=1}^{K_s} a_{k,n} |h_{k,n}|^2 P_{k,n} + \sigma_e^2}. \tag{3}$$

We denote by $I_{p,n}^{(i)} = \sum_{k \in \mathcal{K}_s} a_{k,n} |h_{k,n}|^2 P_{k,n}$ the interference power caused to PU$_i$'s transmission over the n^{th} sub-channel, where $n \in \mathcal{N}_p^{(i)}$. Thus, the total interference caused by the SUs to PU$_i$ can be expressed as $I_p^{(i)} = \sum_{n \in \mathcal{N}_p^{(i)}} I_{p,n}^{(i)}$. The signals of PU$_i$ can be successfully recovered provided that $I_p^{(i)}$ is below the threshold $I_{th}^{(i)}$.

Without loss of generality, we assume the SUs' signals are recovered through the SIC process according to their received SINR, in an ascending order. More specifically, we assume that the SINR of SU$_k$ is larger than that of SU$_{k-1}$, for $1 \leq k \leq K_s$. Assuming the signals of the first $i-1$ SUs have been successfully recovered, the maximum achievable rate of SU_i, where $1 \leq i \leq K_s$, is shown below:

$$R_s^{(i)} = \frac{N_s^{(i)}}{N} \log \left(1 + \sum_{n \in \mathcal{N}_s^{(i)}} \gamma_{s,n}^{(i)} \right), \tag{4}$$

where

$$\gamma_{s,n}^{(i)} = \frac{a_{i,n} |h_{i,n}|^2 P_{i,n}}{\sum_{k=i+1}^{K_s} a_{k,n} |h_{k,n}|^2 P_{k,n} + \sigma_e^2}. \tag{5}$$

Thus, we can express the total SINR of SU$_i$ as $\gamma_s^{(i)} = \sum_{n \in \mathcal{N}_s^{(i)}} \gamma_{s,n}^{(i)}$. The signals of SU$_i$ can be successfully recovered provided that their received SINR $\gamma_s^{(i)}$ at the i^{th} iteration of SIC is above the threshold $\gamma_{th}^{(i)}$.

It will be shown later that the design problem is dependent on the SUs' received power rather than transmit power. Assuming that the SUs are able to estimate their channel gains from the downlink given the reciprocity of the channel, the CBS needs to broadcast the received power constraints only, imposed on each sub-channel on a per user basis. The SUs can, then, adaptively tune their power as necessary. Accordingly, we define a power vector $\mathbf{p} = [P_n]_{1 \leq n \leq N}$, where P_n is the received power constraint imposed on the n^{th} sub-channel on a per user basis.

4 Asymptotic Performance Analysis of PRA in CRNs

In this section, we analyze the relationship between the system constraints (I_{th} and γ_{th}) and the different system metrics (N, K_p and K_s). We formulate an optimization problem to find the degree distribution that can maximize this probability of successfully recovering the signals of the SUs for a given setup.

4.1 Probability Density Function of the IP

Let us first calculate the power of interference introduced to the PUs.

Lemma 1. *In an asymptotically large network ($N \to \infty$, $K_s \to \infty$), the probability density function of the total interference power induced over the sub-channels of PU_k, $\forall k \in \mathcal{K}_p$, follows the Poisson distribution below:*

$$Pr(I_p^{(k)} = iP_o^{(k)}) = e^{-\alpha N_p^{(k)}} \frac{\left(\alpha N_p^{(k)}\right)^i}{i!}, \tag{6}$$

where $\alpha = \frac{K_s}{N}\bar{\Omega}$, and $P_n = P_o^{(k)}$ $\forall n \in \mathcal{N}_p^{(k)}$. Its average and standard deviation are given below:

$$\mathbb{E}[I_p^{(k)}] = \alpha N_p^{(k)} P_o^{(k)}, \quad \sigma_{I_p^{(k)}} = \alpha N_p^{(k)} P_o^{(k)}. \tag{7}$$

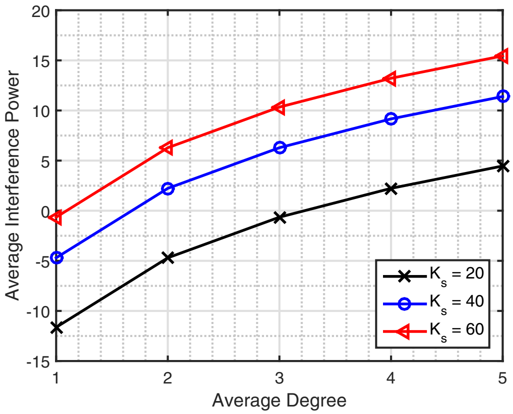

Fig. 2. The average interference power for a total of $N = 128$ sub-channels assigned equally to $K_p = 60$ PUs and shared by K_s SUs.

The proof of this lemma is provided in Appendix 7. In Fig. 2, the average IP is shown as a function of the average degree $\bar{\Omega}$ and the number of users K_s. $P_o^{(k)}$ is set to 0 dB $\forall k \in \mathcal{K}_p$. The average IP per PU is shown to increase with the number of K_s, as expected from Lemma 1. It is worthy of noting that N is fixed for all three simulations and that the increase in the average IP in fact corresponds to the increase in the ratio $\frac{K_s}{N}$ rather than K_s itself.

4.2 Probability of Success of the SUs

As in Section III-B, we assume the SUs' signals are recovered in an ascending order, based on their received SINR, with $\gamma_s^{(i)} \leq \gamma_s^{(i-1)}$ for $1 \leq i \leq K_s$. Given that the signals of the first $i-1$ SUs have been successfully recovered, we can rewrite (5) and express the total SINR of SU_i as follows:

$$\gamma_s^{(i)} = \sum_{n \in \mathcal{N}_s^{(i)}} \frac{P_n}{d_n^{(i)} P_n + \sigma_e^2}, \tag{8}$$

where $d_n^{(i)}$ is a random variable that represents the number of users, other than SU_i, transmitting in the n^{th} sub-channel and whose signals have not been recovered yet. We define $\mathbf{d}^{(i)} = [d_n^{(i)}]_{1 \leq n \leq N_s^{(i)}}$ and refer to it as the observation vector. The CBS can then recover the signals of SU_i, if and only if, $\gamma_s^{(i)} \geq \gamma_{th}^{(i)}$, which will happen for certain values of $\mathbf{d}^{(i)}$. Let $\mathbf{V}^{(k)}$ denote the set of all vectors $\mathbf{v}$ that can satisfy the SINR constraint for SU_k. It can then be found that:

$$\mathbf{V}^{(k)} = \{(v_1, v_2, ..., v_{N_s^{(k)}}) | \sum_{n \in \mathcal{N}_s^{(k)}} \frac{P_n}{v_n P_n + \sigma_e^2} \geq \gamma_{th}^{(k)}\} \tag{9}$$

In other words, the CBS can recover the signals of SU_k if and only if the observation vector $\mathbf{d}^{(k)}$ belongs to $\mathbf{V}^{(k)}$. We then have the following proposition:

Proposition 1. *For the recovery of the SUs' signals, we assume that the PUs' signals have been successfully recovered and that the SUs' signals are ordered and recovered in an ascending order, based on their received SINR. Let S_i be the event of having $\gamma_s^{(i)} \geq \gamma_{th}^{(i)}$. Then, the probability of successfully recovering the signals of SU_i, through the SIC process, can be calculated as follows:*

$$\begin{aligned} Pr(S_i) &= Pr(\gamma_s^{(i)} \geq \gamma_{th}^{(i)}) \\ &= Pr(\gamma_s^{(i)} \geq \gamma_{th}^{(i)} | S_{i-1}) Pr(S_k) \\ &= Pr(\boldsymbol{d}^{(i)} \in \boldsymbol{V}^{(i)} | S_{i-1}) Pr(S_k), \end{aligned}$$

for $1 \leq i \leq K_s$.

4.3 Clean Packet Model

As mentioned before, authors in [7,8,9] have implemented the iterative recovery process of codes-on-graph for the binary erasure channel (BEC) in PRA schemes. As in Fig. 1, the system is mapped onto a bipartite graph and the signal recovery is visualized as a message passing algorithm [10]. However, at the receiver side, successful signal recovery can only take place if an interference-free clean packet has been received at the destination.

From Section III-B, the observation vector of the 'clean packet' model must have the following form for successful signal recovery:

$$\{\mathbf{d}^{(i)} | \exists j, d_j^{(i)} = 0, 1 \leq j \leq i\}.$$

Let us consider the case where the received power of an SU's signal is less than or equal to its SINR threshold. Then, if the received signal is interference-free, it can be successfully recovered in our design. For such a case, the observation vectors of both designs are the same for $d_m = 1$.

On the other hand, from (9), we can see that the set of observation vectors that ensure successful recovery will generally be larger for our design; thus, it is expected to provide a higher probability of success. The 'clean packet' model can be seen as a special case of our design. Interestingly, when the SINR threshold is higher than that of the received power per signal for an SU, the 'clean packet' model fails to service any SUs at all. However, for sufficiently high degrees, our approach can still service a significant fraction of the SUs.

4.4 Optimization of the Degree Distribution

Given a CRN system of K_p PUs and K_s SUs transmitting over a set of N sub-channels, we formulate an optimization problem to find the degree distribution that maximizes the probability of successfully recovering the SUs' signals through the SIC process, while satisfying the IP constraints of the PUs. The CBS has the perfect knowledge of the PUs channel allocation, power, and respective IP constraints. It also has knowledge of the number of active SUs and their respective SINR constraints. Accordingly, the optimization problem can be formulated as follows:

$$\begin{aligned}
&\max_{\mathbf{p},\Omega(x)} && \sum_{k=1}^{K_s} \Pr(S_k) && \\
&\text{s.t.} && \text{(i)} \sum_{i=1}^{d_m} \Omega_i = 1,\ \ \Omega_i \geq 0, && \forall 1 \leq i \leq d_m \\
& && \text{(ii)} \mathbb{E}\left[\sum_{n} I_{p,n}^{(k)}\right] \leq I_{th}^{(k)}, && \forall k \in \mathcal{K}_p.
\end{aligned}$$

Condition (i) ensures the sum of all probabilities is equal to 1. Condition (ii) ensures that the PUs are protected by the IP constraint on a per user basis. With reference to Lemma 1, it can easily be seen that this condition determines the value of $\bar{\Omega}$ and $\mathbf{p}$. Optimization is carried out using the covariance matrix adaptation evolution strategy (CMA-ES)[11] and can be easily modified for different IP and SINR thresholds.

5 Numerical Results

In this section, we investigate the system performance for different setups. Results are averaged over 10000 samples. The received power constraint per sub-channel P_o is taken to be 0 dB, the number of sub-channels N is set to 128 [12].

For ease of analysis, we now assume that $P_n = P_o$, where $1 \leq n \leq N$. This condition dictates that all sub-channels have the same received power constraint.

For a practical system, this also reduces the signalling overhead. This can be easily justified for the case where $N_p^{(i)} = N_p$ and $I_{th}^{(i)} = I_{th}$, for $1 \leq i \leq K_p$. We adopt this assumption in our simulations. We also assume that $\gamma_{th}^{(i)} = \gamma_{th}$, for $1 \leq i \leq K_s$.

Table 1. Results of CMA-ES optimization for $\frac{K_s}{N} = \frac{60}{128}$

$\log_{10} \gamma_{th}/P_o$	0 dB		1 dB	
d_m	4	8	4	8
Ω_1	0.0002	0.0003	0.0001	0.0002
Ω_2	0.5072	0.0831	0.1108	0.1295
Ω_3	0.0041	0.1619	0.1727	0.1529
Ω_4	0.4885	0.1744	0.7163	0.2112
Ω_5		0.2255		0.0674
Ω_6		0.0382		0.2406
Ω_7		0.1758		0.1188
Ω_8		0.1408		0.0794
ϵ	3.00e-03	3.75e-04	2.67e-04	4.05e-04

In Table 1, we show the results of CMA-ES for $\frac{K_s}{N} = \frac{60}{128}$ and a maximum degree of 4 and 8. Using the results of [7], we proceed to compare the achievable error probabilities of both designs; the error probability is denoted by ϵ and defined as $1 - \frac{1}{K_s} \sum_{k-1}^{K_s} \mathbb{P}_{s,k}$. Results are shown in Fig. 3. As predicted, the proposed design outperforms the 'clean packet' model even for $\frac{\gamma_{th}}{P_o} = 1$. As our proposed design relies on the overall received SINR, the sum of all individual SINRs, it makes use of all transmissions over the different sub-channels rather than interference-free transmissions only, thus, achieving better performance for the same power requirements. Finally, in Fig. 4, we consider the probability of having the IP caused by the SUs to the PUs below a given threshold. We use the results from Table 1, for $d_m = 8$ and $\log_{10} \frac{\gamma_{th}}{P_o} = 0$dB. We find $\bar{\Omega}$ to be around 5.23. Interestingly, for $I_{th} \leq$ -5dB, the probability of successfully recovering a PU's signals becomes independent of the threshold and solely dependent on the number of SUs supported in the network. Even more so, for thresholds as high as 10 dB, the probability of successfully recovering a PU's signals is almost one for any number of SUs. It is worthy of noting that Condition (ii) in Section IV-D can be easily modified to limit this probability by restricting $\Pr\left(\sum_n I_{p,n}^{(k)} \leq I_{th}^{(k)}\right) \leq \delta$, where δ is a predefined threshold.

6 Practical Considerations

In our system, the CBS is assumed to have the perfect knowledge of the PUs' activity and channel conditions, their respective IP constraints, the number of

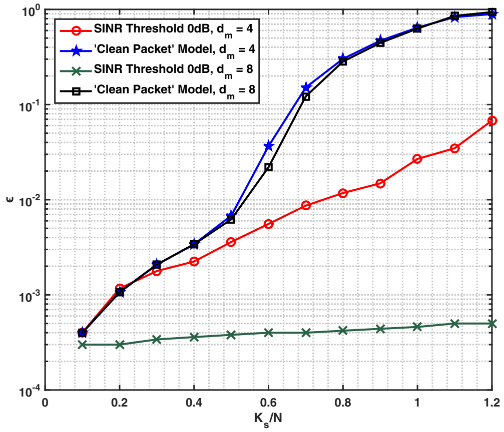

Fig. 3. Error probability of proposed scheme in comparison to the 'Clean Packet' model for different ratios of $\frac{K_s}{N}$

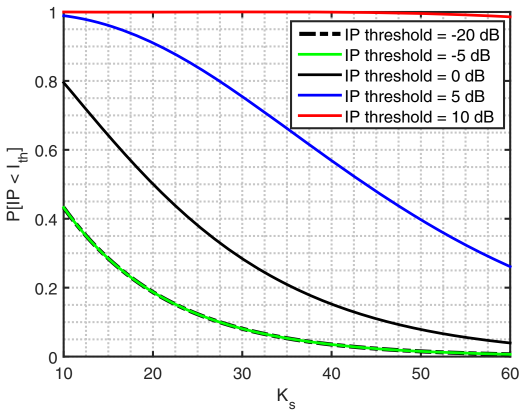

Fig. 4. Probability of IP being below the threshold for different values of K_s

active SUs and their respective SINR threshold. We assume this is made known to the CBS over the control channel, where transmissions are deterministic in duration and nature. Accordingly, the CBS can find the received power constraints necessary and the optimal degree distribution function to meet the system constraints. Then, the control channel can also be used to make the degree distribution known to the SUs. As the SUs are assumed to be able to estimate their channel gains from the downlink given the reciprocity of the channel, the signalling overhead is significantly reduced in comparison to fixed resource allocation.

For a given transmission block, the SIC process cannot be initiated without the knowledge of how many and which sub-channels were accessed by which users. We assume the SUs share the same seed with the CBS to determine the number and index of the chosen sub-channels through a pre-defined pseudo-random number generator [7].

Finally, it is worthy of noting that the IP constraint can be defined as either the average IP constraint or the peak IP constraint. However, throughout this paper, we only consider the former definition. This was justified in [13], where it was shown that the average IP constraint provides a higher system capacity than that of the peak IP.

7 Conclusion

In this paper, we proposed a new design of probabilistic random access schemes in CRNs. We showed that the conventional 'clean packet' model is sub-optimal under the IP and SINR constraints. We formulated a new optimization problem, based on CMA-ES, to maximize the probability of successful recovering the SUs' signals in the SIC process while satisfying the IP constraints of the PUs. Numerical results show that our degree distributions can achieve lower error probabilities with lower number of transmissions, and thus, having lower power requirements.

Proof of Lemma 1

Since the sub-channels are chosen uniformly at random, the degree of each sub-channel, defined as the number of users transmitting in that sub-channel, follows the binomial distribution. Let us denote this pdf by $\Lambda(x)$. In the asymptotic case, that is for a large number of sub-channels and SUs, the distribution converges to Poisson [14], as follows:

$$\Lambda_i = e^{-\alpha}\frac{\alpha^i}{i!}, \text{ where } \quad \alpha = \frac{K_s}{N}\bar{\Omega}.$$

From (2), the IP constraint for PU_k was defined as: $I_p^{(i)} = \sum_{n=1}^{N_p^{(k)}} \sum_{k\in\mathcal{K}_s} a_{k,n}P_n = \sum_{n=1}^{N_p^{(k)}} u_n P_n$, where u_n is a random variable representing the number of SUs transmitting in the n^{th} sub-channel. Intuitively, the probability of having u_n SUs transmitting in a sub-channel n is the same for all $1 \le n \le N$, and $\Pr(I_{p,n}^{(k)} = u_n P_n)$ is simply Λ_{u_n}. Assuming equal power allocation, that is $P_n = P_o^{(k)}\ \forall\ n \in \mathcal{N}_p^{(k)}$, we can express the pdf of $I_p^{(k)}$ as follows:

$$\Pr(I_p^{(k)} = i) = \Pr(u_1 P_o^{(k)} + u_2 P_o^{(k)} + ... + u_{N_p^{(k)}} P_o^{(k)} = i) = \bigotimes_{n=1}^{N_p^{(k)}} \Lambda_z|_{z=\frac{i}{P_o^{(k)}}}, \forall k \in \mathcal{K}_p,$$

where $\bigotimes$ denotes the convolution operation. As $\Lambda(x)$ was shown to be poisson distributed, and as the sum of poisson distributed random variables is also a poisson random variable, we arrive at (6).

References

1. Liang, Y.C., Chen, K.C., Li, Y., Mahonen, P.: Cognitive Radio Networking and Communications: An Overview. IEEE Transactions on Vehicular Technology **60**(7), 3386–3407 (2011)
2. Domenico, A.D., Strinati, E., Benedetto, M.D.: A Survey on MAC Strategies for Cognitive Radio Networks. IEEE Commununications Surveys & Tutorials **14**(1), 21–44 (2012)
3. Yang, L., Kim, H., Zhang, J., Chiang, M., Tan, C.W.: Pricing-based spectrum access control in cognitive radio networks with random access. In: Proceedings of IEEE INFOCOM, pp. 2228–2236 (2011)
4. Wang, S., Zhang, J., Tong, L.: Delay analysis for cognitive radio networks with random access: a fluid queue view. In: Proceedings of IEEE INFOCOM, pp. 1–9 (2010)
5. Chen, T., Zhang, H., Maggio, G.M., Chlamtac, I.: Cogmesh: a cluster-based cognitive radio network. In: Proceedings of IEEE International Symposium on New Frontiers in Dynamic Spectrum Access Networks, pp. 168–178 (April 2007)
6. Barman, S.R., Merchant, S.N., Madhukumar, A.: Random transmission in cognitive uplink network. In: International Conference on Mobile Services, Resources, and Users (MOBILITY), pp. 94–98 (2013)
7. Liva, G.: Graph-based Analysis and Optimization of Contention Resolution Diversity Slotted ALOHA. IEEE Transactions on Communications **59**(2), 477–487 (2011)
8. Paolini, E., Liva, G., Chiani, M.: High throughput random access via codes on graphs: coded slotted ALOHA. In: Proceedings of IEEE International Conference on Communications (ICC), pp. 1–6 (April 2011)
9. Liva, G., Paolini, E., Lentmaier, M., Chiani, M.: Spatially-coupled random access on graphs. In: IEEE International Symposium on Information Theory (ISIT), pp. 478–482 (July 2012)
10. Luby, M., Mitzenmacher, M., Shokrollahi, M.A.: Analysis of Random Processes via AND-OR Tree Evaluation. SODA **98**, 364–373 (1998)
11. Hansen, N., Ostermeier, A.: Adapting arbitrary normal mutation distributions in evolution strategies: the covariance matrix adaptation. In: Proceedings of IEEE International Conference on Evolutionary Computation, pp. 312–317 (1996)
12. Xu, H., Li, B.: Efficient resource allocation with flexible channel cooperation in OFDMA cognitive radio networks. In: Proceedings of IEEE INFOCOM, pp. 1–9 (2010)
13. Zhang, R.: On Peak versus Average Interference Power Constraints for Protecting Primary Users in Cognitive Radio Networks. IEEE Transactions on Wireless Communications **8**(4), 2112–2120 (2009)
14. Shokrollahi, M.A.: Raptor codes. IEEE Transactions on Information Theory **52**(6), 2551–2567 (2006)