A Novel Algorithm for Blind Detection of the Number of Transmit Antenna

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Abstract. In this paper, a novel algorithm is proposed to blindly detect the number of transmit antennas by exploiting the time-diversity of the fading channel. It employs a second-order moment and a fourth-order statistic of the received signal when the transmission occurs over a time-varying multiple-input single-output channel. When compared with information theoretic algorithms, it does not require the number of received antennas be larger than the number of transmit antennas, and when compared with existing feature-based algorithms, it does not require *a priori* information about the transmitted signal, such as preambles or pilots. Simulation results show that the proposed algorithm exhibits a good performance over a wide range of signal-to-noise-ratios (SNRs), and the probability of correct detection approaches one at low SNR values for various numbers of transmit antennas. Furthermore, it is robust to the modulation format and carrier frequency offset, and exhibits a good performance in the presence of the noise power mismatch and spatially correlated fading.

Keywords: Number of antenna detection · Second-order moment · Fourth-order statistic · Time-diversity

1 Introduction

With the advent of multiple-input multiple-output (MIMO) systems, the problem of the number of transmit antennas detection has emerged in both military and commercial communications, such as spectrum surveillance, electronic warfare, and cognitive radio [1–6]. For the cognitive radio systems, the coexistence of the secondary users (SUs) and the primary users (PUs) equipped with multiple antennas ameliorates when the SUs have *a priori* information about the PUs number of transmit antennas, as the interference tolerated by the PUs from the SUs depends on that; hence, such knowledge allows the SUs to better adjust their transmit power to avoid destructive interference to the PUs [3]. Furthermore, the radio front end has a complexity, size and price that scales with the number of transmit antennas. Recently, the antenna selection technique was proposed to alleviate this cost and at the same time to capture many of the advantages of

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MIMO systems [7–9]; in this case, detecting and tracking the number of transmit antennas is of interest to eliminate the need for additional signaling, which introduces overhead and transmission latency [5].

There are two main approaches for the detection of the number of transmit antennas: information-theoretic [1,2] and feature-based [3–6]. The Akaike information criterion (AIC) and the minimum description length (MDL) algorithms are two well-known information-theoretic methods. With these algorithms, the problem of the number of transmit antennas detection is formulated as a model order selection problem, which relies on the rank estimation of the received signal correlation matrix. However, such algorithms usually suffer from high computational complexity, as they require the eigen-decomposition of the sample covariance matrix. Further, they fail to detect the number of transmit antennas when this is larger than the number of received antennas. On the other hand, the existing feature-based algorithms rely on *a priori* information about the transmitted signals, e.g., pilot patterns [3,4] or preamble sequences [5,6]. As such information is actually not available at the blind receiver, it represents the main drawback of these feature-based algorithms.

A novel feature-based algorithm for the blind detection of the number of transmit antennas is presented in this paper, where a single receive antenna is used. The proposed algorithm employs a second-order moment and a fourth-order statistic of the received signal, and exploits the time-diversity of the fading channel. In contrast with the information theoretic algorithms, it does not require the number of received antennas be larger than the number of transmit antennas, and when compared with the existing feature-based algorithms, it does not require *a priori* information about the transmitted signals.

The rest of the paper is organized as follows. The signal model is presented in Section 2, the proposed algorithm is introduced in Section 3, simulation results are provided in Section 4, and conclusions are drawn in Section 5.

Notation: Throughout the paper, bold-faced letters are used for vectors, $[.]^\dagger$ represents the transpose operator, $(.)^*$ denotes the complex conjugate, $n!$ is the factorial of n, $E_x[.]$ is the statistical expectation of the random variable x, and $\hat{x}$ is the estimate of x.

2 System Model

A multiple-input single-output (MISO) block fading channel with n_t transmit antennas is considered, where n_t is unknown at the receive-side [10]. We assume that the receiver observes N_b blocks, each with a length of N_c symbols. Each block is affected by independent and identically distributed (i.i.d.) fading characterized by an $(1 \times n_t)$ matrix $\mathbf{H}_b$, $b = 1, ..., N_b$, and corrupted by additive white Gaussian noise. With the assumption of the Clarke-Jakes Doppler spectrum, the block length is $N_c = \lfloor 0.2/f_d T_s \rfloor$, where f_d and T_s are the maximum Doppler frequency and symbol period, respectively [10]. Thus, the received complex-valued signal can be expressed as

$$r_{k,b} = \mathbf{H}_b \mathbf{s}_{k,b} + w_{k,b} \quad k = 1, ..., N_c, \quad b = 1, ..., N_b, \tag{1}$$

$$\mu_{21,b} = \sum_{m_1=1}^{n_t} \left|h_b^{(m_1)}\right|^2 E_s\left[|s_{k,b}^{(m_1)}|^2\right] + \sum_{m_1=1}^{n_t} \sum_{\substack{m_2=1\\ m_2\neq m_1}}^{n_t} h_b^{(m_1)}\left(h_b^{(m_2)}\right)^* E_s\left[s_{k,b}^{(m_1)}\left(s_{k,b}^{(m_2)}\right)^*\right]$$
$$+ \sum_{m_1=1}^{n_t}\left(h_b^{(m_1)} E_{s,w}\left[s_{k,b}^{(m_1)}(w_{k,b})^*\right] + \left(h_b^{(m_1)}\right)^* E_{s,w}\left[\left(s_{k,b}^{(m_1)}\right)^* w_{k,b}\right]\right) + E_w\left[|w_{k,b}|^2\right] \quad (2)$$

where $r_{k,b}$ is the kth received symbol in the bth observation block, $\mathbf{s}_{k,b} = [s_{k,b}^{(1)}, s_{k,b}^{(2)}, ..., s_{k,b}^{(n_t)}]^\dagger$ represents the transmitted symbols from the n_t transmit antennas, whose variance $E_s[|s_{k,b}^{(m)}|^2] = \sigma_s^2$, $m = 1, ..., n_t$ is unknown at the receive-side, $w_{k,b}$ is complex additive white Gaussian noise with zero-mean and variance σ_w^2 assumed to be known at the receive-side, and $\mathbf{H}_b = [h_b^{(1)}, h_b^{(2)}, ..., h_b^{(n_t)}]$ denotes the channel coefficients, with $h_b^{(j)}$, $j = 1, ..., n_t$ as the channel coefficient between the jth transmit antenna and the receive antenna for the bth observation block. It is assumed that the channel coefficients in each block are independent complex-valued Gaussian random variables with zero-mean and variance $E_{\mathbf{H}_b}[|h_b^{(j)}|^2] = \sigma_h^2$, where σ_h^2 is unknown at the receive-side.

3 Number of Transmit Antennas Detection

The proposed algorithm for the number of transmit antennas detection exploits a second-order moment and a fourth-order statistic of the received signal, along with the time-diversity of the fading channel, as subsequently presented.

Let us first consider the second-order moment and the fourth-order statistic of the received signal within an observation block. By using (1) and the linearity property of the statistical expectation, one can express the second-order moment/ one conjugate, $\mu_{21,b} \triangleq E_{s,w}[|r_{k,b}|^2]$, as in (2). With the assumptions that the additive noise, $w_{k,b}$, is independent of the transmitted symbols, $s_{k,b}^{(m)}$, $m = 1, 2, ..., n_t$, the symbols transmitted with different antennas are independent, i.e., $E_s[s_{k,b}^{(m_1)} s_{k,b}^{(m_2)}] = \sigma_s^2 \delta(m_1 - m_2)$, with $\delta(.)$ as the Dirac delta function, and by using that $E_s[s_{k,b}^{(m)}] = 0$ for the symmetric constellation points, $\mu_{21,b}$ is further expressed as

$$\mu_{21,b} = \sigma_s^2 \sum_{m=1}^{n_t} |h_b^{(m)}|^2 + \sigma_w^2. \quad (3)$$

Similarly, for the fourth-order/ two-conjugate statistic, $\omega_{42,b} \triangleq E_{s,w}[|r_{k,b}|^4] - 2(E_{s,w}[|r_{k,b}|^2])^2$, [1] one can easily obtain

[1] Note that $\omega_{42,b}$ is related to the fourth-order/ two-conjugate cumulant, with a difference of $\mu_{20,b}\mu_{22,b}$, where $\mu_{20,b}$ and $\mu_{22,b}$ are the second-order/ zero-and two-conjugates, respectively.

$$\omega_{42,b} = \omega_{42}^{s} \sigma_s^4 \sum_{m=1}^{n_t} |h_b^{(m)}|^4, \tag{4}$$

where ω_{42}^{s} denotes the fourth-order/ two conjugate statistic for unit variance constellations.

With the channel coefficients corresponding to different transmit antennas being independent[2] complex-valued zero- mean Gaussian random variables with variance σ_h^2, i.e., $E_{\mathbf{H}_b}[\mathbf{H}_b^T \mathbf{H}_b] = \sigma_h^2 \mathbf{I}$, and employing the following property of a complex Gaussian random variable $x \sim CN\left(0, \sigma_x^2\right)$ that [11]

$$E_x\left[|x|^{2n}\right] = n! \sigma_x^{2n}, \tag{5}$$

the expectations of the second-order moment and fourth-order statistic in (3) and (4) over channel distributions are

$$\begin{aligned} \mu_{21} \triangleq E_{\mathbf{H}_b}[\mu_{21,b}] &= \sigma_s^2 \sum_{m=1}^{n_t} E_{\mathbf{H}_b}\left[|h_b^{(m)}|^2\right] + \sigma_w^2 \\ &= n_t \sigma_h^2 \sigma_s^2 + \sigma_w^2 \end{aligned} \tag{6}$$

and

$$\begin{aligned} \omega_{42} \triangleq E_{\mathbf{H}_b}[\omega_{42,b}] &= \omega_{42}^{s} \sigma_s^4 \sum_{m=1}^{n_t} E_{\mathbf{H}_b}\left[|h_b^{(m)}|^4\right] \\ &= 2 n_t \omega_{42}^{s} \sigma_s^4 \sigma_h^4. \end{aligned} \tag{7}$$

Furthermore, with the modulation type and noise power[2] known at the receive-side, by employing (6) and (7), n_t can be straightforwardly expressed as

$$n_t = \frac{2\omega_{42}^{s}\left(\mu_{21} - \sigma_w^2\right)^2}{\omega_{42}}. \tag{8}$$

In practice, the statistical moments are estimated by time averages [12]. Furthermore, an unbiased estimator is of interest, as on average, the expected value of the parameter being estimated equals its actual value. For (8), the following unbiased estimators are employed to estimate the corresponding statistics, i.e., μ_{21}, $\zeta \triangleq (\mu_{21})^2$ and ω_{42}, respectively.

$$\hat{\mu}_{21} = \frac{1}{N_b N_c} \sum_{b_1=1}^{N_b} \sum_{k_1=1}^{N_c} |r_{k_1,b_1}|^2, \tag{9}$$

$$\hat{\zeta} = \frac{1}{N_b\left(N_b - 1\right) N_c \left(N_c - 1\right)} \sum_{b_1=1}^{N_b} \sum_{\substack{b_2=1 \\ b_2 \neq b_1}}^{N_b} \sum_{k_1=1}^{N_c} \sum_{\substack{k_2=1 \\ k_2 \neq k_1}}^{N_c} |r_{k_1,b_1}|^2 |r_{k_2,b_2}|^2, \tag{10}$$

[2] Note that the deviation from this assumption is considered later in the paper, in Section 4.

$$\hat{\omega}_{42} = \tfrac{1}{N_b N_c} \sum_{b_1=1}^{N_b} \sum_{k_1=1}^{N_c} |r_{k,b}|^4 - \tfrac{2}{N_b N_c (N_c-1)} \sum_{b_1=1}^{N_b} \sum_{k_1=1}^{N_c} \sum_{\substack{k_2=1 \\ k_2 \neq k_1}}^{N_c} |r_{k_1,b_1}|^2 |r_{k_2,b_1}|^2. \tag{11}$$

It can be easily shown that $E[\hat{\mu}_{21}] = \mu_{21}$, $E[\hat{\zeta}] = (\mu_{21})^2$, and $E[\hat{\omega}_{42}] = \omega_{42}$, where $E[.] \triangleq E_{\mathbf{H}_b}[E_{s,w}[.]]$. It is worth noting that $(\hat{\mu}^{(1)})^2$ cannot be employed for the estimation of ζ, as it results in a biased estimator.

With (8), (9), (10), and (11), one obtains the following decision statistic for the number of transmit antennas,

$$\Psi = \frac{2\omega_{42}^s (\hat{\zeta} - 2\hat{\mu}_{21}\sigma_w^2 + \sigma_w^4)}{\hat{\omega}_{42}}. \tag{12}$$

It can be easily noticed that Ψ is a continuous random variable, whereas n_t takes discrete values; hence, regions of decision need to be set up to estimate the number of transmit antennas, along with their corresponding thresholds. Since

$$E[\Psi] \approx \frac{2\omega_{42}^s E[\hat{\zeta} - 2\hat{\mu}_{21}\sigma_w^2 + \sigma_w^4]}{E[\hat{\omega}_{42}]} = \frac{2\omega_{42}^s \left(\mu_{21} - \sigma_w^2\right)^2}{\omega_{42}} = n_t, \tag{13}$$

the decision is made according to the following criterion:

$$\Gamma_{n_t - 1} < \Psi \leq \Gamma_{n_t} \rightarrow \hat{n}_t = n_t \qquad n_t = 1, 2, 3, ... \tag{14}$$

where $\Gamma_0, \Gamma_1, \Gamma_2, ...$ represent the decision thresholds, with $\Gamma_0 = -\infty$ and $n_t < \Gamma_{n_t} < n_t + 1$. A formal description of the proposed algorithm is presented below.

Algorithm 1

1. Acquire the measurement $r_{k,b}$, $\quad k = 1, ..., N_c, \ \ b = 1, ..., N_b$
2. Compute the decision statistic Ψ according to (12)
3. Initialize $i = 1$
4. Set the threshold value Γ_i
   If $\Gamma_{i-1} < \Psi \leq \Gamma_i$
   $\qquad \hat{n}_t = i$
   else
5. Increment $i = i + 1$ and go to step 4
   end

4 Simulation Results

In this section, we examine the detection performance of the proposed algorithm through several simulation experiments.

4.1 Simulation Setup

We consider a system employing spatial multiplexing transmission scheme, with $N_c = 100$ (e.g., $f_d = 200$ Hz and $T_s = 10$ μs). Unless otherwise mentioned, $N_b = 100$ and the modulation is quadrature phase-shift-keying (QPSK). The channel coefficients are independent complex Gaussian random variables with zero-mean and variance σ_h^2. The additive white noise is modeled as a complex Gaussian random variable with zero-mean and variance σ_w^2. The average SNR per transmit antennas is defined as $\gamma \triangleq 10 \log \left(\frac{\sigma_h^2 \sigma_s^2}{\sigma_w^2}\right)$dB. Without loss of generality, we consider $\sigma_h^2 \sigma_s^2 = 1$. The thresholds to make a decision are set as $\Gamma_{n_t} = n_t + 1/2$, $n_t = 1, 2, ...$ The overall detection performance is presented in terms of the probability of correct detection, $P(\hat{n}_t = m | n_t = m)$, $m = 1, 2, ..., 4$, and the average probability of correct detection, $P_c = \frac{1}{3} \sum_{m=1}^{3} P\left(\hat{n}_t = m | n_t = m\right)$, obtained from 1000 Monte Carlo trials for each m.

4.2 Simulation Results

Fig. 1 shows $P(\hat{n}_t = m | n_t = m)$ versus SNR for different number of transmit antennas, n_t, $n_t = 1, ..., 4$, and different N_b values. As can be seen, the proposed algorithm exhibits a good performance over a wide range of SNRs for $N_b = 100$ and 1000, and the probability of correct detection goes to one even at negative SNRs for $N_b = 1000$. The performance improves as either N_b or SNR increases, which can be easily explained, as each leads to a reduced estimation error of the statistics in (9), (10), and (11). Additionally, the probability of correct detection decreases as the number of transmit antenna increases; this is because the variance of the decision statistic Ψ in (12) increases with n_t, as confirmed by simulation experiments.

In Fig. 2, the effect of the noise power mismatch, i.e, $\hat{\sigma}_w^2 - \sigma_w^2$, on the probability of correct detection is illustrated at SNR=10 dB. As can be observed, the proposed algorithm is relatively robust to the noise power mismatch. This can be easily explained, as the effect of the noise power mismatch on the test statistic Ψ in (12) is not significant for a large enough observation interval.

Fig. 3 shows the effect of the frequency offset normalized to the data rate, Δf, on P_c. As can be seen, the proposed algorithm is completely robust to the carrier frequency offset. This is because such an effect is eliminated through the absolute value operator in the definition of the second-order moment and the fourth-order statistic.

Fig. 4 presents the effect of the modulation format on the average probability of correct detection, P_c. As can be seen, while the proposed algorithm is relatively robust to the modulation format at positive SNRs, a better performance is achieved for M-ary PSK when compared with M-ary quadrature amplitude modulation (QAM) at negative SNRs. This can be explained, as the effect of the modulation format, ω_{42}^s is not totally eliminated through $\hat{\omega}_{42}$ for M-ary QAM due to less accurate estimates, particularly at negative SNRs.

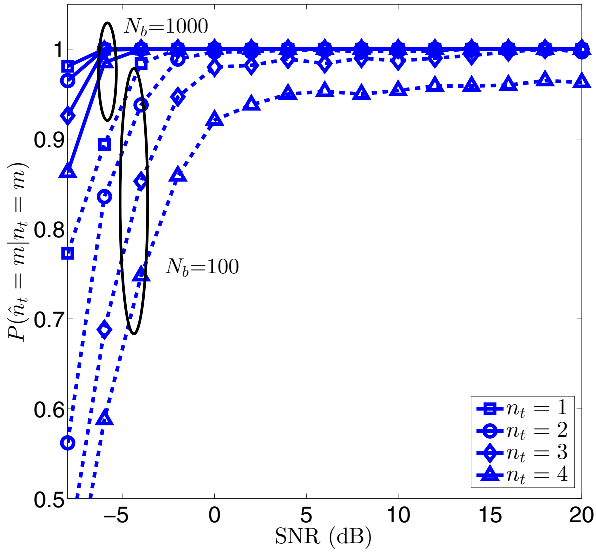

Fig. 1. The probability of correct detection, $P(\hat{n}_t = m|n_t = m)$ versus SNR for different n_t and N_b values.

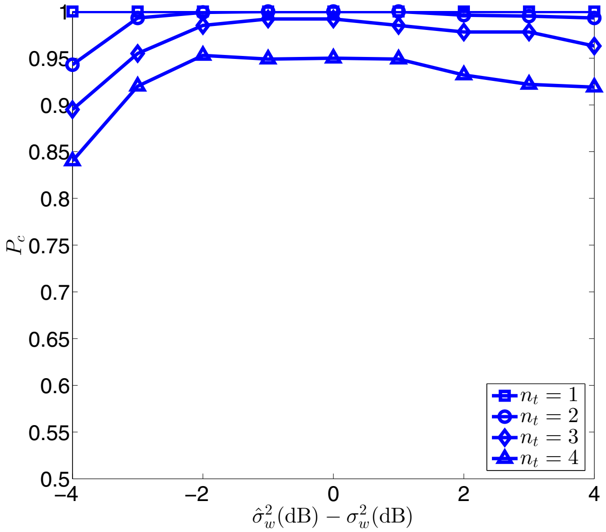

Fig. 2. The effect of the noise power mismatch on the probability of correct detection, $P(\hat{n}_t = m|n_t = m)$ at SNR=10 dB.

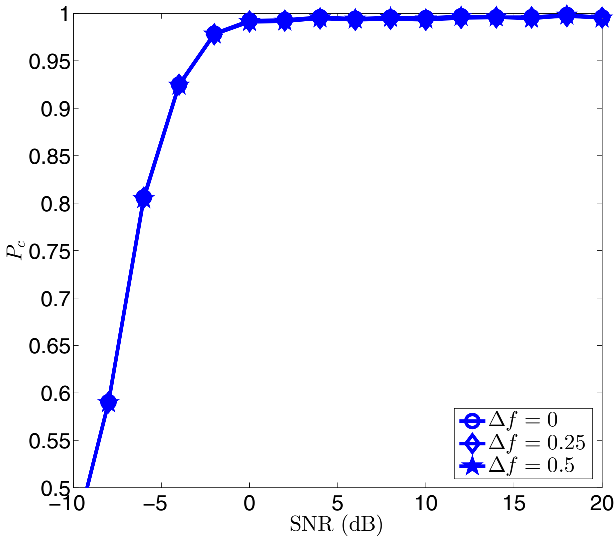

Fig. 3. The effect of the frequency offset on the average probability of correct identification, P_c.

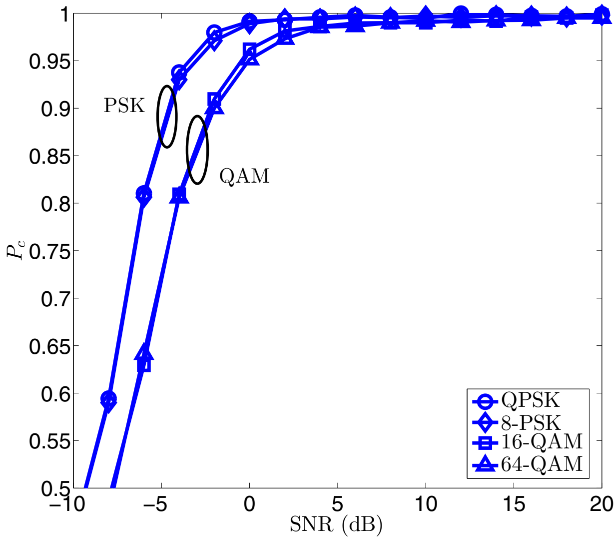

Fig. 4. The effect of the modulation format on the average probability of correct identification, P_c.

Fig. 5 shows the effect of the spatially correlated fading on P_c versus SNR for a correlation coefficient $\rho = 0, 0.4, 0.6$, and 0.8. As can be observed, the performance of the proposed algorithm is robust to the spatial correlation for $\rho < 0.6$. This can be explained, as for low values of ρ, $E_{\mathbf{H}_b}\left[\sum_{m=1}^{n_t} |h_b^{(m)}|^{2l}\right]$, $l = 1, 2$, remains approximately equal to $l!2n_t\sigma_h^{2l}$ (see (6) and (7)), and (8) remains valid for the number of transmit antennas detection.

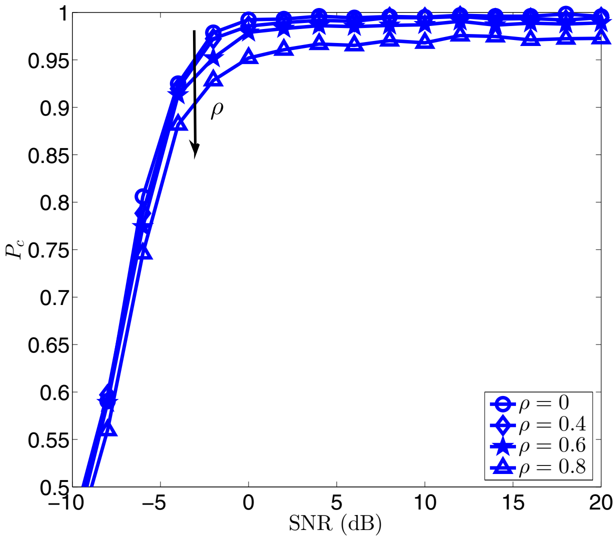

Fig. 5. The effect of the spatially correlated fading on the average probability of correct identification, P_c.

5 Conclusion

A novel feature-based algorithm was introduced for the detection of the number of transmit antennas. This relies on a second-order moment and a fourth-order statistic of the received signal, and exploits the time diversity of the fading channels, while employing a single receive antennas. The proposed algorithm attains a good performance at low SNRs, being robust to the carrier frequency offset and relatively robust to the modulation format. Additionally, it exhibits a good performance in the presence of noise power mismatch and spatially correlated fading.

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