Efficient Performance Evaluation for EGC, MRC and SC Receivers over Weibull Multipath Fading Channel

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Abstract. The probability density function (PDF) of the output SNR (Signal to Noise Ratio) at a receiver operating under Weibull fading multipath channel is unknown in closed form and exists only as a complicated multiple integrals or approximated by a series of functions, or recently, by a single function whose evaluation time is not negligible. Our main result is a new simple approximate closed-form of the SNR PDF at the output of three types of receivers over Weibull multipath fading channels. The advantage of this new expression is that its evaluation time is less compared to all previous results. Based on this expression, approximate analytical expressions of the outage probability (OP), the average bit error rate (BER) for several M-ary modulation techniques, and the average channel capacity (CC) are derived in terms of only one particular hypergeometric function, known as Fox-H function. Numerical results have been validated by simulation and compared with recent results.

Keywords: Fox H-function · Meijer G-function · Maximal ratio combining · Equal gain combining · Weibull fading · Shannon capacity · Bit error rate

1 Introduction

In wireless digital communication system, the diversity techniques are used to combine the original signal copies arrived often from different paths, especially in an urban environment, at the receiver. The choice of a combining method has a great influence on system performance. Several diversity techniques, such as maximal-ratio combining (MRC), selection combining (SC), and equal-gain combining (EGC) are used in many wireless communication system. In MRC method, all received signals at the input of the receiver are multiplied by their channel gains conjugates. This receiver is known to be optimal for all multipath fading environment. In coherent EGC reception, each received signal on its branches is weighted with the equal gain. Although the complexity of EGC is

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acceptable, its performances are usually less efficient than the MRC ones. In SC technique, as long as the receiver select only one branch that having the greatest SNR, its performances are always lower than those of EGC.

The Weibull distribution is often used for describing the fading amplitude in both indoor and outdoor communication environments. As the analytical expression of the SNR PDF at the output of either coherent EGC or MRC receiver is unknown and difficult or even impossible to find, an another way is to derive a closed-form approximation. Recently, many papers have been written on the performance of both MRC, EGC, SC, and GSC (Generalized SC) receivers over Weibull fading channels. In [1], a closed-form of the moment generating function (MGF) of the SNR at the output of both MRC and SC receivers was derived in terms of power and finite series of Meijer G-functions [2] respectively. In [3], average symbol error rate and MGF of the output SNR at EGC and MRC combiners are derived in terms of product and infinite series of Meijer G-functions. The expression of the SNR at EGC output was investigated in [4]. In [5], joint PDF, CDF, and MGF of SNR are expressed for SC receiver. In [6], various statistical characteristics of the output SNR at EGC receiver, and average bit error rate (BER) for 3-branch MRC are derived as infinite series of Meijer G-function. In [7], CC for MRC over several types of fading channels comprising Weibull case was derived as a series of Meijer G-function. In [8]-[10], some other contributions dealing with SNR PDF at the output of the receiver and other system performance indicators have been presented in close approximation by only one Fox H-function for MRC, EGC, and GSC respectively. However, to ensure the convergence of this function and to be close to the exact expression of the performance indicator, the value of the parameter λ, appearing in the denominator of the first argument of all derived Fox H-functions, should be too big. Although the evaluation of this function is easy using some mathematical software such as Mathematica and Maple, it becomes extremely slow when its first argument is very small (close to 0), and sometimes lead to numerical instabilities and erroneous results.

Hence, it is highly desirable to find another approximate closed-form that resolves the problem of evaluation time complexity of the said function when λ approaches infinity. Thus, in this paper, we present a new, stable and low complexity, approximate closed-form of SNR PDF at the output of MRC, EGC, and SC receivers over Weibull multipath fading channels, and so other performance criteria, in terms of only one simple Fox H-function.

The rest of this paper is structured as follows. In section 2, the description of the studied receiver operating in Weibull multipath fading environment and some statistical characteristics are presented. Section 3 present a novel formula, with low evaluation time, for the PDF and the CDF of the SNR at the output of the receiver. In Sections 4-5, BER for various M-ary modulation techniques and CC of the studied equivalent channel are derived in terms of only one Fox H-function and Meijer G-function with small computational complexity. In section 6, the results are illustrated and verified by comparison with the recent results. Our main conclusions are summarized in the final section.

2 Receiver Description and Preliminary Statistics

We consider a digital wireless communication system with L-branch MRC, EGC, or SC receiver operating in a Weibull multipath fading environment. The SNR of the combined signal at the output of the studied receiver is given by

$$\gamma = \delta \left(\sum_{i=1}^{L} \gamma_i^{\alpha} \right)^{\frac{1}{\alpha}} \tag{1}$$

where

- $1/\alpha$ is a positive integer,
- $\gamma_i = \frac{E_s}{N_0} R_i^2$ is the instantaneous SNR per symbol on the ith branch,
- E_s denotes the average energy per symbol and N_0 denotes the power spectral density of thermal noise,
- L denotes the number of combined diversity branches,
- R_i denotes the fading amplitude corresponding to the ith received signal at the combiner, assumed to be Weibull distributed with the shape parameter β_i and the scale parameter ω_i:

$$R_i = \left(N_{1i}^2 + N_{2i}^2 \right)^{\frac{1}{\beta_i}} \tag{2}$$

where N_{1i} and N_{2i} are two normally distributed variates with mean and variance 0 and $\frac{\omega_i^{\beta_i}}{2}$, respectively. The values of parameters δ and α vs. receiver are summarized in Table 1.

Table 1. Values of δ and α for some known receivers

Receiver	MRC	EGC	SC
δ	1	$1/L$	1
α	1	$1/2$	$+\infty$

In the following, the Weibull distribution is denoted $W(\omega_i, \beta_i)$. Its PDF is

$$f_{R_i}(r) = \frac{\beta_i}{\omega_i} \left(\frac{r}{\omega_i} \right)^{\beta_i - 1} \exp\left[- \left(\frac{r}{\omega_i} \right)^{\beta_i} \right], r \geq 0 \tag{3}$$

so, the nth moment of R_i is given by

$$\mu_n^{(R_i)} = \omega_i^n d_n(\beta_i) \tag{4}$$

where $d_k(\beta_i) = \Gamma(1 + k/\beta_i)$, and $\Gamma(\cdot)$ is the gamma function. Accordingly, the scale parameter of R_i is

$$\omega_i = \sqrt{\frac{\mu_2^{(R_i)}}{d_2(\beta_i)}} \tag{5}$$

and the average of the ith SNR is then

$$\overline{\gamma}_i = \frac{E_s}{N_0}\omega_i^2 d_1\left(\frac{\beta_i}{2}\right) \tag{6}$$

Since the square of a Weibull RV is a Weibull RV, and according to (4) and (6), γ_i is a Weibull distribution $W\left(\frac{E_s}{N_0}\omega_i^2, \frac{\beta_i}{2}\right)$.

3 On the Sum of Weibull RVs

In this section, based on the recently derived expressions for the sum of Weibull distributed RVs [9]-[10], a new closed-form approximation of the output SNR PDF having low evaluation time is presented.

Lemma 1. *If X is a Weibull distribution $W(\omega,\beta)$, then its PDF can be expressed as a Fox H-function*

$$f_X(x) = \frac{1}{\omega}H_{0,1}^{1,0}\left(\frac{x}{\omega}\middle| \begin{matrix} \cdot \\ (1-\frac{1}{\beta},\frac{1}{\beta}) \end{matrix}\right),\ x \geq 0 \tag{7}$$

Proof. See Appendix A. □

Lemma 2. *Let $Z_{\alpha,i} = \gamma_i^\alpha$. Then $Z_{\alpha,i}$ is a Weibull distribution $W\left(\left[\frac{\overline{\gamma}_i}{d_2(\beta_i)}\right]^\alpha, \frac{\beta_i}{2\alpha}\right)$*

Proof. See Appendix A. □

3.1 PDF of the Output SNR γ

As the exact expression of the SNR PDF at the output of both MRC and coherent EGC receivers operating under Weibull fading channel is unknown or even impossible to find explicitly, the main idea is to derive, as much as possible, a closed-form approximation with low evaluation time.

Theorem 1. *Let $Z^{(\alpha)} = \sum_{i=1}^L Z_{\alpha,i}$. The SNR PDF at the output of the studied receiver can be approximated by a Fox H-function*

$$f_\gamma(\gamma) \approx \frac{\alpha\left(\Psi_{\alpha,L}\left(\frac{\gamma}{\delta}\right)^\alpha\right)^{\Phi_{\alpha,L}}}{\gamma\Gamma(\Phi_{\alpha,L})}\exp\left(-\Psi_{\alpha,L}\left(\frac{\gamma}{\delta}\right)^\alpha\right) \tag{8}$$

where $\Phi_{\alpha,L} = \frac{\left(\mu_1^{\left(Z^{(\alpha)}\right)}\right)^2}{\mu_2^{\left(Z^{(\alpha)}\right)}-\left(\mu_1^{\left(Z^{(\alpha)}\right)}\right)^2}$ *and* $\Psi_{\alpha,L} = \frac{\mu_1^{\left(Z^{(\alpha)}\right)}}{\mu_2^{\left(Z^{(\alpha)}\right)}-\left(\mu_1^{\left(Z^{(\alpha)}\right)}\right)^2}$.

Proof. A very tight closed-form approximation of the PDF of $Z^{(\alpha)}$ was derived in terms of Meijer G-function as [8]-[10]

$$f_{Z^{(\alpha)}}(z) \approx \frac{\Gamma(a+1)}{\lambda\Gamma(b+1)} G_{1,1}^{1,0}\left(\frac{z}{\lambda}\middle| \begin{matrix} a \\ b \end{matrix}\right),\ z \geq 0 \tag{9}$$

where

$$a = \left(\lambda - \frac{\mu_2^{(Z^{(\alpha)})}}{\mu_1^{(Z^{(\alpha)})}}\right)\Psi_{\alpha,L} - 1, b = \Phi_{\alpha,L} - \frac{\mu_2^{(Z^{(\alpha)})}\Psi_{\alpha,L}}{\lambda} - 1, \tag{10}$$

and λ is a real number.

Now, using the Jacobian of the transformation, the PDF of $\gamma = \delta\left[Z^{(\alpha)}\right]^{\frac{1}{\alpha}}$ is

$$f_\gamma(\gamma) = \frac{\alpha\gamma^{\alpha-1}}{\delta^\alpha} f_{Z^{(\alpha)}}\left[\left(\frac{\gamma}{\delta}\right)^\alpha\right] \tag{11}$$

Substituting (9) into (11), we get

$$f_\gamma(\gamma) \approx \frac{\Gamma(a+1)}{\gamma\Gamma(b+1)}\frac{1}{2\pi j}\int_{\mathcal{C}} \frac{\Gamma\left(b+1+\frac{s}{\alpha}\right)}{\Gamma\left(a+1+\frac{s}{\alpha}\right)}\left(\frac{\gamma}{\lambda^{\frac{1}{\alpha}}\delta}\right)^{-s} ds \tag{12}$$

where $\mathcal{C}$ denotes an infinite complex contour of integration, and j is an imaginary number such that $j^2 = -1$.

The PDF of the output SNR given in (9) can be expressed either as a sum of LHP (Left Half-Plane) or RHP (Left Right-Plane) residues if $\lambda > \mu_2^{(Z)}/\mu_1^{(Z)}$ [9]. In addition, this expression becomes closer to the exact one for large values of λ. However, the evaluation time of the Meijer G-function takes a lot of time if its first argument is close to zero. Hence, to get a close approximation with low evaluation time complexity of this function, we will eliminate the λ-term in this expression by finding the limit of this PDF as λ approaches $+\infty$.

We have from (10)

$$\frac{\Gamma(b+1+\frac{s}{\alpha})}{\Gamma(b+1)} \sim \frac{\Gamma(\Phi_{\alpha,L}+\frac{s}{\alpha})}{\Gamma(\Phi_{\alpha,L})} \text{ as } \lambda \to +\infty \tag{13}$$

On the other hand, using the stirling's formula [15, 6.1.39], we get from (10)

$$\frac{\Gamma(a+1)\lambda^{\frac{s}{\alpha}}}{\Gamma(a+1+\frac{s}{\alpha})} \sim {\Psi_{\alpha,L}}^{-\frac{s}{\alpha}} \text{ as } \lambda \to +\infty \tag{14}$$

Substituting (13) and (14) into (12), yields

$$\begin{aligned} f_\gamma(\gamma) &\approx \frac{1}{\gamma\Gamma(\Phi_{\alpha,L})}\frac{1}{2\pi j}\int_{\mathcal{C}} \Gamma(\Phi_{\alpha,L}+\frac{s}{\alpha})\left(\Psi_{\alpha,L}^{1/\alpha}\frac{\gamma}{\delta}\right)^{-s} ds \\ &= \frac{1}{\gamma\Gamma(\Phi_{\alpha,L})} H_{0,1}^{1,0}\left(\frac{\Psi_{\alpha,L}^{1/\alpha}\gamma}{\delta}\middle| \begin{matrix} \cdot \\ (\Phi_{\alpha,L}\ ,\frac{1}{\alpha}) \end{matrix}\right) \end{aligned} \tag{15}$$

Now, using [11, eq.(07.34.03.0228.01)] and the change of variable $s' = \Phi_{\alpha,L} + \frac{s}{\alpha}$, we get (8) which concludes the proof of the theorem. □

Remark 1. The terms $\Phi_{\alpha,L}$ and $\Psi_{\alpha,L}$ are explicitly expressed for i.i.d Weibull fading channel as

$$\Phi_{\alpha,L} = \frac{\left(\sum_{i=1}^{L} \overline{\gamma}_i^{\alpha}\right)^2}{\sum_{i=1}^{L} \overline{\gamma}_i^{2\alpha}} \frac{d_1^2\left(\frac{\beta}{2\alpha}\right)}{d_2\left(\frac{\beta}{2\alpha}\right) - d_1^2\left(\frac{\beta}{2\alpha}\right)}, \Psi_{\alpha,L} = \frac{\sum_{i=1}^{L} \overline{\gamma}_i^{\alpha}}{\sum_{i=1}^{L} \overline{\gamma}_i^{2\alpha}} \frac{d_1\left(\frac{\beta}{2\alpha}\right) d_2^{\alpha}(\beta)}{d_2\left(\frac{\beta}{2\alpha}\right) - d_1^2\left(\frac{\beta}{2\alpha}\right)}$$

and for the same average SNR of the signal at each input branch ($\overline{\gamma}_i = \overline{\gamma}_1$ for all i)

$$\Phi_{\alpha,L} = \frac{L d_1^2\left(\frac{\beta}{2\alpha}\right)}{d_2\left(\frac{\beta}{2\alpha}\right) - d_1^2\left(\frac{\beta}{2\alpha}\right)}, \Psi_{\alpha,L} = \frac{d_1\left(\frac{\beta}{2\alpha}\right) d_2^{\alpha}(\beta)\, \overline{\gamma}_1^{-\alpha}}{d_2\left(\frac{\beta}{2\alpha}\right) - d_1^2\left(\frac{\beta}{2\alpha}\right)}$$

Remark 2. In uncorrelated fading channel, the exact value of the average SNR at the output of the receiver is expressed using the multinomial theorem, lemma 2, and (4) as

$$\overline{\gamma} \equiv \delta \mu_{1/\alpha}^{\left(Z^{(\alpha)}\right)} = \delta \sum_{\sum_{j=1}^{L} i_j = 1/\alpha} \frac{(1/\alpha)!}{\prod_{j=1}^{L} i_j!} \prod_{j=1}^{L} \left(\frac{\overline{\gamma}_{i_j}}{d_2\left(\beta_{i_j}\right)}\right)^{\alpha i_j} d_{2\alpha i_j}\left(\beta_{i_j}\right) \tag{16}$$

On the other side, it can be approximated by placing (15) into [12, eq. (2.8)]

$$\overline{\gamma} \approx \frac{\delta \Gamma(\Phi_{\alpha,L} + \frac{1}{\alpha})}{\Psi_{\alpha,L}^{1/\alpha} \Gamma(\Phi_{\alpha,L})} \tag{17}$$

3.2 Outage Probability

The outage probability is the key metric to characterize the performance limits of wireless communication systems. It's defined in terms of SNR CDF as [13]

$$P_{out} = F_{\gamma}(\gamma_{\min}) \tag{18}$$

where $\gamma_{\min}$ is the minimum SNR threshold that ensure a reliable communication and the equivalent channel is not in outage.

Proposition 1. *The Outage probability for the studied receiver is expressed as*

$$P_{out} \approx \frac{1}{\Gamma(\Phi_{\alpha,L})} H_{1,2}^{1,1}\left(\Psi_{\alpha,L}^{1/\alpha} \frac{\gamma_{\min}}{\delta} \middle| \begin{matrix} (1,1) \\ \left(\Phi_{\alpha,L}, \frac{1}{\alpha}\right), (0,1) \end{matrix}\right) \tag{19}$$

Proof. Using (15) and [11, eq.(06.05.16.0002.01)], the CDF of γ is given by

$$\begin{aligned} F_{\gamma}(\gamma) &\approx \frac{1}{2\pi j \Gamma(\Phi_{\alpha,L})} \int_{\mathcal{C}} \Gamma(\Phi_{\alpha,L} + \frac{s}{\alpha}) \left(\frac{\Psi_{\alpha,L}^{1/\alpha}}{\delta}\right)^{-s} \int_0^{\gamma} t^{-s-1} dt ds \qquad (20) \\ &= \frac{1}{2\pi j \Gamma(\Phi_{\alpha,L})} \int_{\mathcal{C}} \frac{\Gamma(\Phi_{\alpha,L} + \frac{s}{\alpha})\Gamma(-s)}{\Gamma(1-s)} \left(\Psi_{\alpha,L}^{1/\alpha} \frac{\gamma}{\delta}\right)^{-s} ds \end{aligned}$$

Which completes the proof of the proposition. □

Remark 3. by applying the relation [15, q.(6.1.20)]

$$\Gamma(\Phi_{\alpha,L} + \frac{s}{\alpha}) = (2\pi)^{\frac{1-1/\alpha}{2}} \frac{\prod_{k=0}^{\frac{1}{\alpha}-1} \Gamma(\alpha(\Phi_{\alpha,L} + k) + s)}{\alpha^{\Phi_{\alpha,L}+\frac{s}{\alpha}-\frac{1}{2}}} \tag{21}$$

furthermore, OP can be rewritten in terms of Meijer G-function as

$$P_{out} \approx \frac{(2\pi)^{\frac{1-1/\alpha}{2}} G_{1,\frac{1}{\alpha}+1}^{\frac{1}{\alpha},1}\left((\alpha\Psi_{\alpha,L})^{1/\alpha} \frac{\gamma_{\min}}{\delta} \middle| \begin{matrix} 1 \\ \Delta_{\alpha,L}; 0 \end{matrix} \right)}{\Gamma(\Phi_{\alpha,L})\alpha^{\Phi_{\alpha,L}-\frac{1}{2}}} \tag{22}$$

with $\Delta_{\alpha,L} = \{\alpha\Phi_{\alpha,L}, \alpha(\Phi_{\alpha,L}+1), ..., \alpha\left(\Phi_{\alpha,L} + \frac{1}{\alpha} - 1\right)\}$

4 Average Bit Error Probability

Proposition 2. *The BER of several M-ary modulation techniques using the studied combiner in Weibull multipath fading environment*

$$\overline{P}_e \approx \frac{1}{2\Gamma(\varrho)\Gamma(\Phi_{\alpha,L})} H_{2,2}^{1,2}\left(\frac{\Psi_{\alpha,L}^{1/\alpha}}{\delta\theta} \middle| \begin{matrix} (1,1),(1-\varrho,1) \\ (\Phi_{\alpha,L},\frac{1}{\alpha});(0,1) \end{matrix} \right) \tag{23}$$

where ϱ and θ are parameters depending on modulation scheme [9].

Proof. The BER for several M-ary modulation scheme over fading channel is given by [14, eq.(13)]

$$\overline{P}_e = \frac{\theta^{\varrho}}{2\Gamma(\varrho)} \int_0^{\infty} \gamma^{\varrho-1} e^{-\theta\gamma} F_{\gamma}(\gamma)\, d\gamma \tag{24}$$

Substituting (20) into (24), yielding

$$\overline{P}_e \approx \frac{\theta^{\varrho}}{4\pi j \Gamma(\varrho)\Gamma(\Phi_{\alpha,L})} \int_{\mathcal{C}} \frac{\Gamma(\Phi_{\alpha,L}+\frac{s}{\alpha})\Gamma(-s)}{\Gamma(1-s)\left(\frac{\Psi_{\alpha,L}^{1/\alpha}}{\delta}\right)^{s}} \int_0^{\infty} \gamma^{\varrho-s-1} e^{-\theta\gamma}\, d\gamma ds \tag{25}$$

$$= \frac{1}{4\pi j \Gamma(\varrho)\Gamma(\Phi_{\alpha,L})} \int_{\mathcal{C}} \frac{\Gamma(\Phi_{\alpha,L}+\frac{s}{\alpha})\Gamma(-s)\Gamma(\varrho-s)}{\Gamma(1-s)} \left(\frac{\Psi_{\alpha,L}^{1/\alpha}}{\delta\theta}\right)^{-s} ds$$

That concludes the proof of the proposition. □

Remark 4. Replacing (21) into (25), we get

$$\overline{P}_e \approx \frac{(2\pi)^{\frac{1-\frac{1}{\alpha}}{2}} \alpha^{\frac{1}{2}-\Phi_{\alpha,L}}}{2\Gamma(\varrho)\Gamma(\Phi_{\alpha,L})} G_{2,\frac{1}{\alpha}+1}^{\frac{1}{\alpha},2}\left(\frac{(\alpha\Psi_{\alpha,L})^{\frac{1}{\alpha}}}{\delta\theta} \middle| \begin{matrix} 1, 1-\varrho \\ \Delta_{\alpha,L}; 0 \end{matrix} \right) \tag{26}$$

5 Average Shannon Capacity

Proposition 3. *Let B_w be the channel bandwidth. The CC for the studied receiver in the case of Weibull multipath fading channels is close to*

$$\overline{C} \approx \frac{B_w}{\Gamma(\Phi_L)\ln 2} H_{2,3}^{3,1}\left(\frac{\Psi_{\alpha,L}^{\frac{1}{\alpha}}}{\delta}\left|\begin{matrix}(0,1);(1,1)\\ \left(\Phi_{\alpha,L},\frac{1}{\alpha}\right),(0,1),(0,1)\end{matrix}\right.\right) \tag{27}$$

Proof. The average capacity of the studied channel is given by

$$\overline{C} = B_w \int_0^\infty \log_2(1+\gamma)\, f_\gamma(\gamma)\, d\gamma \tag{28}$$

Using [11, eq.(07.34.03.0456.01)], it can be rewritten using a Mellin-Barnes contour integral

$$\overline{C} \approx \frac{B_w}{2\pi j \Gamma(\Phi_{\alpha,L})\ln 2}\int_{\mathcal{C}} \Gamma(\Phi_{\alpha,L}+\frac{s}{\alpha})\left(\frac{\Psi_{\alpha,L}^{1/\alpha}}{\delta}\right)^{-s}\left(\int_0^\infty \gamma^{-s-1} G_{2,2}^{1,2}\left[\gamma\left|\begin{matrix}1,1\\1,0\end{matrix}\right.\right]d\gamma\right)ds \tag{29}$$

The Mellin transform in (29) can be evaluated using [11, eq. (07.34.21.0009.01)]

$$\int_0^\infty \gamma^{-s-1} G_{2,2}^{1,2}\left[\gamma\left|\begin{matrix}1,1\\1,0\end{matrix}\right.\right]d\gamma = \frac{\Gamma(1-s)\Gamma^2(s)}{\Gamma(1+s)} \tag{30}$$

Substituting (30) into (29), concludes the proof of the proposition. □

Remark 5. Replacing (21) into (29), the CC can also be expressed in terms of Meijer G-function

$$\overline{C} \approx \frac{B_w (2\pi)^{\frac{1-\frac{1}{\alpha}}{2}} \alpha^{\frac{1}{2}-\Phi_{\alpha,L}}}{\Gamma(\Phi_{\alpha,L})\ln 2} G_{2,\frac{1}{\alpha}+2}^{\frac{1}{\alpha}+2,1}\left(\frac{(\alpha\Psi_{\alpha,L})^{\frac{1}{\alpha}}}{\delta}\left|\begin{matrix}0;1\\0,0,\Delta_{\alpha,L}\end{matrix}\right.\right) \tag{31}$$

6 Performance Evaluation Results

In this section, the results containing the Meijer G-function were evaluated using Mathematica software. All the Monte Carlo simulations are established by generating $10^8 L$ Weibull distributed random numbers over the studied SNR range, subdivided into 10^4 subintervals of equal length. We have assumed an exponentially decaying power delay profile (PDP) $\overline{\gamma}_i/\overline{\gamma}_1 = \exp[-\varphi(i-1)]$ where φ is the average fading power decay factor [16], and $\overline{\gamma}_1 = 1$ except for figure 4 and 5.

In Fig. 1, the analytical expression (8) and the simulated PDF versus γ are plotted, for the L-branch EGC and dual-branch MRC. It can be seen that the MRC curves are closer to the simulated ones than those of EGC.

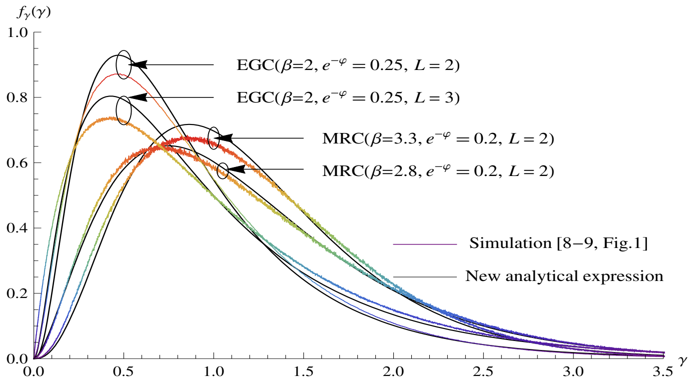

Fig. 1. PDF of the output SNR at both L-branch EGC and 2-branch MRC receiver

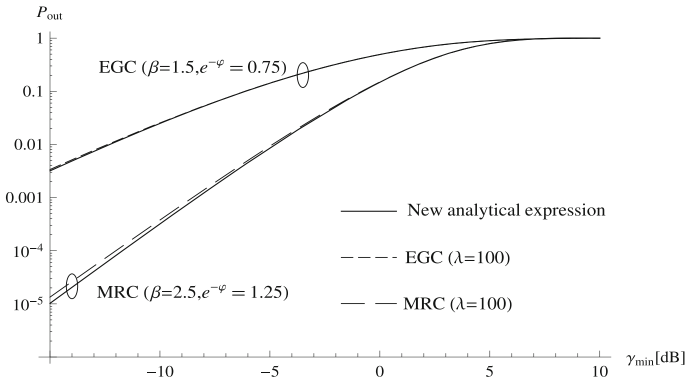

Fig. 2. Outage probability of dual-branch EGC/MRC receiver

Fig. 2 depicts the OP for dual-branch MRC/EGC receiver, given by (31) for i.i.d. Weibull fading channels. The plotted curves are compared for EGC and MRC with those plotted from [8, eq.(28)] and [9, eq.(13)], respectively. It can be observed that the new derived expression is very close to the previous one for great values of λ.

In Fig. 3, the normalized CC, given by (31), versus the diversity order L, is plotted and compared with the previous result [8, eq.(37)] for EGC receiver. The new analytical expression of capacity is very close to the recent one (plotted in dashed line). Besides, the evaluation time of this new expression is very low since the λ-term, sufficiently large, is eliminated.

Fig. 4 compares the new expression of BER for both BPSK and BFSK modulation with 4-branch MRC receiver, plotted from (16) and (26), with the previous one [9, eq.(20)]. It can be seen that the new approximate expression becomes

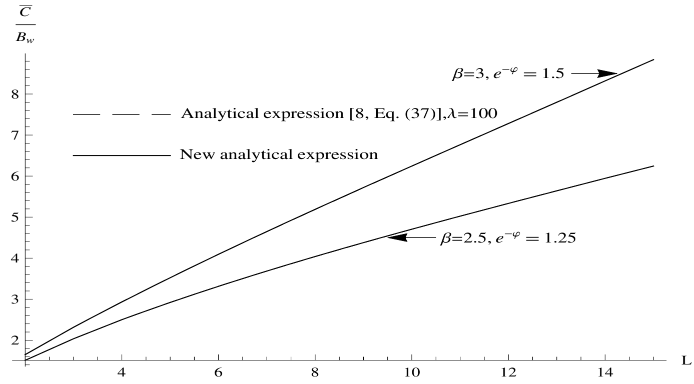

Fig. 3. Nomalized average Shannon capacity of L-branch EGC receiver

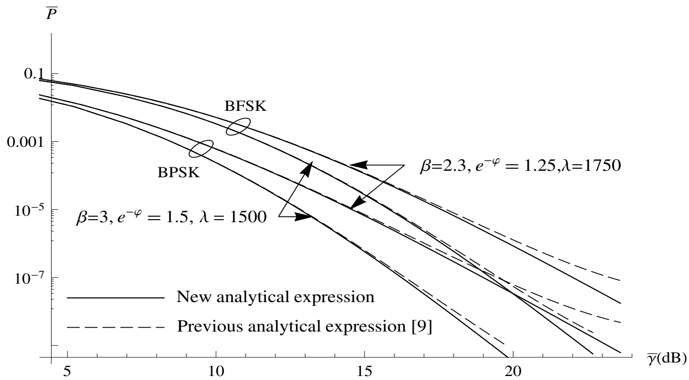

Fig. 4. Analytical expression of BER for BPSK/BFSK modulation with MRC receiver

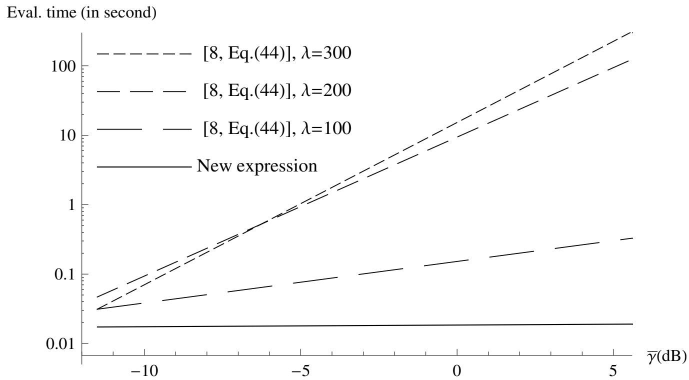

Fig. 5. Evaluation time of the BER for BPSK modulation with EGC receiver

closer to the previous ones [8]-[9], computed for great values of λ in the case of small values of both β and $e^{-\varphi}$.

In Fig. 5, the evaluation time of the BER, in second, is plotted for BPSK modulation with 4-branch EGC combiner, $\beta = 3$, and $e^{-\varphi} = 1.5$. It can be seen that the evaluation time of the new expression (26) is less than the one of the previous result [8], and is almost constant over a range of average SNR. Besides, the greater is the parameter λ (appearing as a denominator in the first argument of the Meijer G-function [8, eq.(44)]), the higher is the evaluation time.

7 Conclusion

In this article, we have derived a novel form of PDF, CDF, BER, and CC for a generalized diversity system including MRC, EGC and SC receivers operating in Weibull multipath fading environment. All analytical expressions were derived in terms of only one Meijer G-function. Our main result is that the evaluation time of all this expressions is low compared with those given in previous work. On the other side, the numerical evaluation, by Mathematica software, of the derived Fox H-functions is more stable since its arguments are not close to 0.

A Proofs of Lemmas

Proof of lemma 1. Using [11, eq.(07.34.03.0228.01)], the PDF of X can be expressed from (3)

$$f_X(x) = \frac{\beta}{\omega}\frac{1}{2\pi j}\int_{\mathcal{C}} \Gamma(s)\left(\frac{x}{\omega}\right)^{\beta-1-\beta s} ds \tag{A.1}$$
$$= \frac{1}{\omega}\frac{1}{2\pi j}\int_{\mathcal{C}} \Gamma\left(1-\frac{1}{\beta}+\frac{s}{\beta}\right)\left(\frac{x}{\omega}\right)^{-s} ds$$

which concludes the proof of lemma 1. □

Proof of lemma 2. According to (2) and (6), it can be seen that $Z_{\alpha,i}$ is a Weibull RV with shape parameter $\frac{\beta_i}{2\alpha}$:

$$Z_{\alpha,i} = \left(\frac{\overline{\gamma}_i}{\omega_i^2 d_2(\beta_i)}\right)^{\alpha}\left(N_{1i}^2 + N_{2i}^2\right)^{\frac{2\alpha}{\beta_i}} \tag{A.2}$$

Its expectation is expressed in terms of 2αth-moment of R_i using (4) and (A.2)

$$\mu_1^{(Z_{\alpha,i})} = \left(\frac{\overline{\gamma}_i}{d_2(\beta_i)}\right)^{\alpha} d_1\left(\frac{\beta_i}{2\alpha}\right)$$

It follows that the scale parameter of $Z_{\alpha,i}$ is $\left(\frac{\overline{\gamma}_i}{d_2(\beta_i)}\right)^{\alpha}$, and its second moment is then expressed from (4) and (A.2) as

$$\mu_2^{(Z_{\alpha,i})} = \left(\frac{\overline{\gamma}_i}{d_2(\beta_i)}\right)^{2\alpha} d_2\left(\frac{\beta_i}{2\alpha}\right) \tag{A.3}$$

that concludes the proof of the lemma 2. □

References

1. Cheng, J., Tellambura, C., Beaulieu, N.C.: Performance Analysis of Digital Modulations on Weibull Fading channels. IEEE Transactions on Wireless Communications **3**(4), 1124–1133 (2004)
2. Hai, N.T., Yakubovich, S.B.: The Double Mellin-Barnes Type Integrals and their Applications to Convolution Theory. World Scientific, Singapore (1992)
3. Karagiannidis, G.K., Zogas, D.A., Sagias, N.C., Kotsopoulos, S.A., Tombras, G.S.: Equal-Gain and Maximal-Ratio Combining Over Nonidentical Weibull Fading Channels. IEEE Trans. Comm. **4**(3), 841–846 (2005)
4. Zogas, D.A., Sagias, N.C., Tombras, G.S., Karagiannidis, G.K.: Average output SNR of equal-gain diversity receivers over correlative Weibull fading channels. European Transactions on Telecommunications **16**, 521–525 (2005)
5. Sagias, N.C., Karagiannidis, G.K., Bithas, P.S., Mathiopoulos, P.T.: On the Correlated Weibull Fading Model and Its Applications, pp. 2149–2153. IEEE (2005)
6. Papadimitiou, Z.G., Bithas, P.S., Mathiopoulos, P.T., Sagias, N.C., Merakos, L.: Triple-branch MRC diversity in Weibull fading channels. In: Signal Design and Its Appl. in Commun., IWSDA 2007, Chengdu, pp. 247–251, September 23–27, 2007
7. El Bouanani, F., Ben-Azza, H., Belkasmi, M.: New Results for the Shannon channel capacity over generalized multipath fading channels for MRC diversity. EURASIP Journal on Wireless Communications and Networking **2012**, 336 (2012)
8. El Bouanani, F., Ben-Azza, H.: Unified analysis of EGC diversity over Weibull fading channels. Wiley Internat. Journal of Commun. Systems (IJCS), December 2014
9. El Bouanani, F.: A new closed-form approximations for MRC receiver over nonidentical Weibull fading channels. In: Internat. Wireless Commun. & Mobile Computing Conference, IWCMC, Nicosia, Cyprus, pp. 600–605, August 4–8, 2014
10. El Bouanani, F., Ben-Azza, H., Belkasmi, M.: Novel results for the spectral efficiency and symbol error rate of GSC receiver over identical and uncorrelated Weibull fading channels. In: World Congress on Computer Applications and Information Systems, WCCAIS, Hammamet, Tunisia, January 17–19, 2014, pp. 1–5 (2014)
11. Wolfram Research, Inc. Mathematica, Edition: Version 10.0, Champaign, Illinois, Wolfram Research, Inc. (2014)
12. Mathai, A.M., Haubold, H.J., Saxena, R.K.: The H-function: Theory and Applications. Springer, New York (2010)
13. Simon, M.K., Alouini, M.S.: Digital Communication over Fading Channels. John Wiley & Sons, Hoboken (2005)
14. Ansari, I.S., Al-Ahmadi, S., Yilmaz, F., Alouini, M.-S., Yanikomeroglu, H.: A new formula for the BER of binary modulations with dual-branch selection over generalized-K composite fading channels. IEEE Transactions on Communications **59**(10), 2654–2658 (2011)
15. Abramowitz, M., Stegun, I.A.: Handbook of Mathematical Functions with formula, graphs, and mathematical tables. National Bureau of Standards, Applied Mathematical Series, vol. 55, June 1964
16. Kong, N., Milstein, L.B.: SNR of generalized diversity selection combining with nonidentical Rayleigh fading statistics. IEEE Tr. Com. **48**, 1266–1271 (2000)