Best Relay Selection for DF Underlay Cognitive Networks with Different Modulation Levels

Ahmed M. ElShaarany[1], Mohamed M. Abdallah[1,2(✉)], Salama Ikki[3], Mohamed M. Khairy[1], and Khalid Qaraqe[2]

[1] Electronics and Electrical Communications Department, Faculty of Engineering, Cairo University, Giza, Egypt
{ahmed.m.elshaarany,mkhairy}@ieee.org
[2] Texas A&M University at Qatar, Doha, Qatar
{mohamed.abdallah,khalid.qaraqe}@qatar.tamu.edu
[3] Department of Electrical Engineering, Lakehead University, Thunder Bay, ON, Canada
sikki@lakeheadu.ca

Abstract. In an underlay setting, a secondary user shares the spectrum with a primary user under the condition that the interference at this primary user is lower than a certain threshold. The said condition limits the transmission power and therefore, limits the coverage area. Hence, to reach remote destinations, relaying the signal between the source and destination can be an adequate solution to enhance the secondary network's performance. Selective relaying in underlay cognitive networks has been studied in many previous literatures. The source and relay nodes in most of this literature use the same modulation level. The use of multiple modulation levels by the transmitting terminals has not been explored comprehensively from the physical layer point of view. In this paper, the error performance of a secondary cognitive network with a source and multiple decode and forward (DF) relays using different modulation levels sharing the spectrum with a nearby primary user has been investigated. In particular, a closed form expression for the error probability for two scenarios have been obtained. In the first scenario, where the relays have fixed transmission power, we additionally present an approximate error probability expression that is exact at high signal-to-noise ratio. In the second scenario, where the relays adjust their transmission power such that the interference at the primary user is below a certain threshold with a defined tolerable error, it is referred to as the interference outage scenario.

Keywords: Underlay cognitive radio · Relay selection · Performance analysis · Different modulation levels

1 Introduction

The continuous pursuit of higher data rates rises day by day due to the increase of wireless applications, wireless multimedia and interactive wireless services.

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This lead to the emergence of more and more wireless technologies every day. These technologies are inefficiently utilizing the usable spectrum. Based upon reports published by the Federal Communications Commission (FCC) [1], the spectrum utilization efficiency reaches percentiles as low as 15%. Such low utilization compelled researchers to find and exploit new techniques to make use of the unused spectrum in a cognitive fashion[2,3].

In short, in a cognitive network, the secondary (unlicensed) user can make use of the unused spectrum portions by the primary user. These unused spectrum portions are known as spectrum holes [3]. If the primary user is to acquire its proprietary spectrum back, the secondary user searches for a new spectrum hole or stops transmitting. This manner enjoins that the secondary user applies spectrum sensing techniques. In-band operation of both the primary and secondary user is possible but demands complicated interference cancellation methods. This method is known as the overlay operation method.

The more simple in-band operation is the underlay operation method. In the latter method, the primary and secondary user operate in the same band on condition that the interference on the primary user is below a certain threshold [2]. This method of operation limits the coverage area of the secondary network due to the constrained power of the secondary transmitter. Consequently, relaying the signal is suggested as an adequate solution to solve the limited coverage area problem.

Two of the most famous relaying operating modes is the amplify and forward (AF) mode and the decode and forward (DF) mode[4]. In AF mode, the signal is received by the relay, amplified by a factor and afterwards forwarded to the destination. In DF mode, the relay decodes the received signal, reproduces it and then forwards the regenerated signal to the destination.

It is worthy to note that although DF mode may suffer from computational delay, it gives a slightly better performance than AF mode[4]. In order to efficiently utilize the spectrum, selective relaying was recently suggested in which a single best relay is selected to relay the signal from the source to the destination[5]. This best relay is selected based on the signal to noise (SNR) it can provide at the destination.

However, in most of the previous literature, the selection was either from a set of relays that were all AF relays or from a set of DF relays that all use the same modulation level. For the relay selection algorithm in [6], the authors based the relay selection criteria on the quotient of the SNR to the interference induced by the relay to the primary user. The best relay selected in this criteria is the relay with the maximum quotient among the relays operating in AF mode in the secondary underlay network. The authors derived closed form expressions for the outage probability and bit error probability.

Another relay selection algorithm proposed in [7] where the best selected relay, operating in DF mode, satisfies an outage probability constraint at the primary network. The authors in [8] propose a selection scheme that in which the relays operate also in DF mode and takes into consideration that an interference constraint at the primary user is not violated. The authors also derived the outage probability. The secondary nodes in both [7] and [8] have the ability to adjust their transmission powers to avoid violating the interference constraint. The authors in [9] suggest a cooperative network in which the source and relay nodes employ different modulation levels. The relays have fixed power and operate in DF mode.

Most of the aforementioned selection techniques are SNR-based and assume that all nodes are using the same modulation level. However, for the case where the modulation level is different, selection based on BER can be more appropiate than SNR-based. The reason for that is that when we have different modulation levels, simply selecting the signal with the highest SNR is not optimal. SNR-based selection does not take into consideration the error flexibility of each received signal. Therefore, in case of different modulation levels, the system performance where BER-based selection is employed is the optimal choice. For a non-cognitive setting, the authors derived a closed form expression for the BER and they only considered the scenario where the relays have fixed transmission power.

In this paper, we propose an extension to the work done in [9]. We suggest an underlay cognitive network in which the source and DF relays use different modulation levels. We derive the corresponding closed form BER expression for two scenarios. In the first scenario, we assume that the relays have a fixed transmission power. In the second scenario, we assume that the relays can adjust their transmission power to satisfy a certain average interference at the primary user.

2 System Model

The system consists of a secondary source S that broadcasts its signal to a secondary destination D. The transmitted signal is passed on to the destination through K DF relays R_k, $k = 1, 2, ..., K$ as illustrated by Fig. 1. These secondary nodes are sharing the spectrum with a primary user P. Each node is assumed to have a single antenna. Communication is achieved over two time slots. In the first time slot, the source S transmits an N-bit packet with power P_s using $M_s - QAM$ modulation scheme to the K relays and the destination with channel gains h_{1k}, h_0, respectively. The channel gain from relay k to the destination is h_{2k}. Each relay is assumed to transmit with a maximum power of $P_{R_k D}$. Each hop suffers from additive white Gaussian noise (AWGN) with zero mean and variance N_0. The channel gains are modeled as a Rayleigh distribution. Hence, the instantaneous SNRs in the hops $S - D$, $S - R_k$, and $R_k - D$ are independent exponential random variables (rv) and are given by $\gamma_{SD} = \frac{P_s |h_0|^2}{N_0}$, $\gamma_{SR_k} = \frac{P_s |h_{1k}|^2}{N_0}$, and $\gamma_{R_k D} = \frac{P_{R_k D} |h_{2k}|^2}{N_0}$, respectively. The average SNRs in the hops $S - D$, $S - R_k$, and $R_k - D$ are denoted by $\bar{\gamma}_{SD}$, $\bar{\gamma}_{SR_k}$, and $\bar{\gamma}_{R_k D}$ respectively. The relays receive the packet, decode it and check its correctness through cyclic redundancy check (CRC). A decoding set $\mathcal{DS}$ of candidate relays is formed which contains relays that have received the packet correctly. The relays are also assumed to use $M_{R_k} - QAM$ modulation. Therefore, the BER as

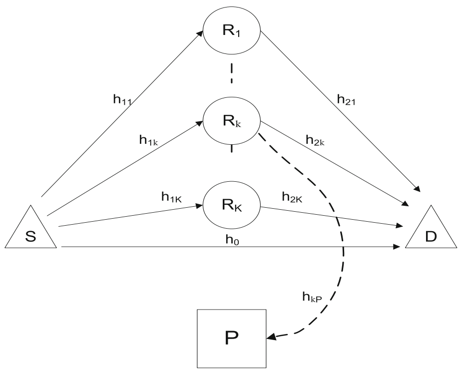

Fig. 1. System Model: A secondary underlay cognitive network close to a primary user

a function of the end-to-end SNR for the square Gray-coded $M-QAM$ is given by [10] as $\mathrm{BER}_{M_i}(\gamma_{iD}) \approx c_{M_i} Q\Big(\sqrt{2d^2_{M_i}\gamma_{iD}}\Big)$ where

$$(c_{M_i}, d_{M_i}) = \begin{cases} (1,1), & M_i = 2, \\ \Big(\frac{2-2/\sqrt{M_i}}{\log_2 \sqrt{M_i}}, \sqrt{\frac{3}{2(M_i-1)}}\Big), & M_i \geq 4. \end{cases}$$

Since the secondary network is operating in an underlay setting, it is important to operate under strict interference limits so that the secondary network does not affect the primary user. Therefore, it is important to propose relay selection algorithms in an according manner. The cognitive selection algorithms, along with their corresponding performance analysis, are explained in the next subsections.

2.1 Fixed Power Underlay Relay Selection

As we previously mentioned, an underlay cognitive network dictates to operate under strict interference limits so that the primary user is not affected. As a result, we set an interference threshold λ. An interference at the primary receiver above this threshold is unacceptable.

In this selection algorithm, each relay is assumed to know whether the interference it generates at the primary receiver satisfies the interference constraint or not. The interference generated by a relay on the primary user is given by $I_{R_kP} = P_{R_kD}|h_{kP}|^2$. Therefore, the interference from the k^{th} relay to the primary user follows an exponential distribution and its probability density function (pdf) is given by

$$p_{I_{R_kP}}(x) = \frac{1}{\mu_{R_kP}} e^{-\frac{x}{\mu_{R_kP}}}, \tag{1}$$

where μ_{R_kP} is the average value of the interference of the k^{th} relay on the primary user. We assume that the interfering channels are generated with $\mu_{R_kP} = \alpha\bar{\gamma}_{R_kD} = \alpha\frac{P_{R_kD}}{N_0}$, where α is a constant>0.

For simplicity in the analysis of the proposed scheme, we assume that the source is non-cognitive (i.e. does not affect the primary user) and the relays are cognitive. In order to avoid interference from the relays on the primary user higher than λ, we must take only into consideration the relays that satisfy the interference constraint.

Therefore, a new decoding set $\mathcal{DS}^*$, subset of $\mathcal{DS}$, is formed which contains relays that have correctly decoded the packet and satisfy the interference constraint (i.e. $I_{R_kP} \leq \lambda$). In the second time slot, after ruling out the relays that do not satisfy the interference constraint, all the relays in the $\mathcal{DS}^*$ send independent pilot signals along with their modulation levels to the destination. Since the transmitting nodes have different modulation levels, then the destination decodes the message either from the source or one of the relays based upon the biased SNR (i.e. BER-based selection where the destination selects to decode the message from one node only).

Thus, the destination calculates the approximate SNRs from the relays and the sources. According to the received SNRs and modulation levels of the relays in the $\mathcal{DS}^*$, $\{M_{R_k}|k \in \mathcal{DS}^*\}$, and the modulation level of the source, M_S, the destination chooses to decode from one of the candidate relays in $\mathcal{DS}^*$ or directly from the source by comparing the received weighted SNR's and selecting the SNR that minimizes the BER.

As a result, the instantaneous BER, according to the BER-based selection, at the secondary destination is given by

$$\mathrm{BER}_{comp,\,inst} \approx \begin{cases} c_{M_S} Q\Big(\sqrt{2d^2_{M_S}\gamma_{SD}}\Big), & \gamma_{SD} \geq \rho_i \gamma_{R_iD}, i \in \mathcal{DS}^* \\ c_{M_{R_i}} Q\Big(\sqrt{2d^2_{M_{Ri}}\gamma_{R_iD}}\Big), & \begin{array}{l}\gamma_{SD} < \rho_i\gamma_{R_iD}, \text{and} \\ \gamma_{R_jD} < \beta_{ij}\gamma_{R_iD}, \\ j \neq i, j \in \mathcal{DS}^*, \end{array} \end{cases} \tag{2}$$

where $\rho_i = d^2_{M_{R_i}}/d^2_{M_S}$ is a biasing factor between the relays and the source and $\beta_{ij} = d^2_{M_{R_i}}/d^2_{M_{R_j}}, i, j = 1, 2, ..., K$ is the biasing factor between the relays.

It is obvious that if all nodes have the same modulation level, then, BER-based selection algorithm becomes SNR-based selection algorithm, i.e., ρ_i $= \beta_{ij} = 1$. Hence, the average BER of this selection scheme can be written as

$$\begin{aligned} \mathrm{BER} = &\left(\prod_{k=1}^{K} \mathrm{PER}_{SR_k}\right)\mathrm{BER}_{SD} + \sum_{r=1}^{K} \sum_{m=1}^{|P_r(\mathcal{S}_{all})|} \left[\left(\prod_{e_i \in P_{r,m}(\mathcal{S}_{all})} (1 - \mathrm{PER}_{SR_{e_i}})\right)\right. \\ &\times \left(\prod_{e_o \notin P_{r,m}(\mathcal{S}_{all})} \mathrm{PER}_{SR_{e_o}}\right)\left(\prod_{e_i \in P_{r,m}(\mathcal{S}_{all})} P_{\lambda_{R_{e_i}P}} \mathrm{BER}_{comp_{P_{r,m}(\mathcal{S}_{all})}}\right. \\ &\left.\left.+ \prod_{e_i \in P_{r,m}(\mathcal{S}_{all})} (1 - P_{\lambda_{R_{e_i}P}})\mathrm{BER}_{SD} + \sum_{\substack{l_1,l_2,\ldots,l_K \in \{0,1\} \\ l_1 l_2 \ldots l_K \neq 1 \\ (1-l_1)(1-l_2)\ldots(1-l_K) \neq 1}} \Theta_{l_1,l_2,\ldots,l_K} \mathrm{BER}_{comp\{i,\forall l_i = 1\}}\right)\right], \end{aligned} \tag{3}$$

where

- $\mathcal{S}_{all}$ is the set of all relays indices, i.e., $\mathcal{S}_{all} = 1, 2, ..., K$,
- $P_r(\mathcal{S}_{all})$is the r-element power set of $\mathcal{S}_{all}$,
- $P_{r,m}(\mathcal{S}_{all})$ is the m-th element of $P_r(\mathcal{S}_{all})$ as defined in [9],
- $|P_r(\mathcal{S}_{all})|$ represents the cardinality of $P_r(\mathcal{S}_{all})$,
- PER_{SR_k} is the average packet error rate in $S - R_k$ link,
- BER_{SD} is the average BER in $S - D$ link,
- $\mathrm{BER}_{comp\mathcal{DS}}$ is average BER conditioned on the $\mathcal{DS}$ at the destination.
- $P_{\lambda_{R_k P}}$ is the probability that $I_{R_k P}$ is less than λ and is given by

$$P_{\lambda_{R_k P}} = \Pr(I_{R_k P} < \lambda) = 1 - e^{-\frac{\lambda}{\mu_{R_k P}}}. \tag{4}$$

- $\Theta_{l_1,l_2,...,l_K}$ is defined as

$$\Theta_{l_1,l_2,...,l_K} = \Big[\Big(P_{\lambda_{R_1 P}} l_1 + (1 - P_{\lambda_{R_1 P}})(1 - l_1)\Big)\Big(P_{\lambda_{R_2 P}} l_2 + (1 - P_{\lambda_{R_2 P}})(1 - l_2)\Big) \\ ... \times \Big(P_{\lambda_{R_K P}} l_K + (1 - P_{\lambda_{R_K P}})(1 - l_K)\Big)\Big]. \tag{5}$$

The average BER between nodes i and j for M-QAM in case of a Rayleigh fading channel can be estimated as

$$\mathrm{BER}_{ij} \approx \int_0^\infty c_{M_i} Q\Big(\sqrt{2d_{M_i}^2 \gamma_{iD}}\Big) \frac{1}{\bar{\gamma}_{ij}} e^{\frac{\gamma_{ij}}{\bar{\gamma}_{ij}}} d\gamma_{ij} = \frac{1}{2} c_{M_i} \left(1 - \sqrt{\frac{d_{M_i}^2 \bar{\gamma}_{ij}}{1 + d_{M_i}^2 \bar{\gamma}_{ij}}}\right). \tag{6}$$

Assuming symbol errors occur independently in the N-bit packet, PER is then given by

$$\begin{aligned} \mathrm{PER}_{SR_i} &= 1 - (1 - \mathrm{SER}_{SR_i})^{\frac{N}{\log_2 M_S}} \\ &\approx 1 - \left(1 - \frac{1}{2} c_{M_S} \log_2(M_S)\left(1 - \sqrt{\frac{d_{M_S}^2 \bar{\gamma}_{SR_i}}{1 + d_{M_S}^2 \bar{\gamma}_{SR_i}}}\right)\right)^{\frac{N}{\log_2 M_S}}, \end{aligned} \tag{7}$$

where for Gray-coded constellations SER$\approx (\log_2 M_S)$BER[10]. BER_{comp} for a certain set of relays is given by [9, Eq. 19]. Therefore, by substituting (4), (5), (6), (7), and [9, Eqs. 19] in (3), we get a closed form expression for the average BER in case of fixed power underlay relay selection given by (8) in the next page, where $HM\{.\}$ is the harmonic mean; the set is defined as $\mathcal{S} = \{\rho_i \bar{\gamma}_{R_i D}\}$, $\mathcal{S}_x = \{\bar{\gamma}_{SD}\rho_i^{-1}, \bar{\gamma}_{R_j D}\beta_{ij}^{-1}\}$, $j \neq i$, $i, j = 1, 2, ..., K$, $P_{k,y}(\mathcal{S})$ is the y-th element of the k-element power set of $\mathcal{S}$, and $P_{k,y}(\mathcal{S}_x)$ is the y-th element of the k-element power set of $\mathcal{S}_x$. The following function was used $I(a, b, c) = \int_0^\infty aQ\left(\sqrt{2bt}\right)\frac{1}{c}e^{-\frac{t}{c}}dt = \frac{a}{2}\left(1 - \sqrt{\frac{bc}{1+bc}}\right)$.

$$
\begin{aligned}
\mathrm{BER} = & \prod_{k=1}^{K} \Bigg[1 - \Bigg(1 - \frac{1}{2} c_{M_S} \log_2(M_S) \Bigg(1 - \sqrt{\frac{d_{M_S}^2 \bar{\gamma}_{SR_k}}{1 + d_{M_S}^2 \bar{\gamma}_{SR_k}}} \Bigg) \Bigg)^{\frac{N}{\log_2 M_S}} \Bigg] BER_{SD} \\
& + \sum_{r=1}^{K} \sum_{m=1}^{|P_r(S_{all})|} \Bigg[\prod_{e_{i \in P_{r,m}(S_{all})}} \Bigg(\Bigg(1 - \frac{1}{2} c_{M_S} \log_2(M_S) \Bigg(1 - \sqrt{\frac{d_{M_S}^2 \bar{\gamma}_{SR_{e_i}}}{1 + d_{M_S}^2 \bar{\gamma}_{SR_{e_i}}}} \Bigg) \Bigg)^{\frac{N}{\log_2 M_S}} \Bigg) \\
& \times \prod_{e_{o \notin P_{r,m}(S_{all})}} \Bigg(1 - \Bigg(1 - \frac{1}{2} c_{M_S} \log_2(M_S) \Bigg(1 - \sqrt{\frac{d_{M_S}^2 \bar{\gamma}_{SR_{e_o}}}{1 + d_{M_S}^2 \bar{\gamma}_{SR_{e_o}}}} \Bigg) \Bigg)^{\frac{N}{\log_2 M_S}} \Bigg) \\
& \times \Bigg(\prod_{e_i \in P_{r,m}(\mathcal{S}_all)} \Bigg(1 - e^{-\frac{\lambda}{\mu_{R_{e_i}P}}} \Bigg) \Bigg[I \Big(c_{M_S}, d_{M_S}^2, \bar{\gamma}_{SD} \Big) + \sum_{k=1}^{K} \sum_{y=1}^{\binom{K}{k}} (-1)^k \\
& \times I \Bigg(\frac{c_{M_S}}{\bar{\gamma}_{SD}}, d_{M_S^2}, \frac{k+1}{HM\{\bar{\gamma}_{SD}, P_{k,y}(\mathcal{S})\}} \Bigg) \Bigg(\frac{k+1}{HM\{\bar{\gamma}_{SD}, P_{k,y}(\mathcal{S})\}} \Bigg) \\
& + \sum_{i=1}^{K} \Bigg[I \Big(c_{M_{R_i}}, d_{M_{R_i}}^2, \bar{\gamma}_{R_iD} \Big) + \sum_{k=1}^{K} \sum_{y=1}^{\binom{K}{k}} (-1)^k I \Bigg(\frac{c_{M_{R_i}}}{\bar{\gamma}_{R_iD}}, d_{M_{R_i}^2}, \frac{k+1}{HM\{\bar{\gamma}_{R_iD}, P_{k,y}(\mathcal{S}_x)\}} \Bigg) \\
& \Bigg(\frac{k+1}{HM\{\bar{\gamma}_{R_iD}, P_{k,y}(\mathcal{S}_x)\}} \Bigg) \Bigg] \Bigg] + \prod_{e_i \in P_{r,m}(\mathcal{S}_{all})} e^{-\frac{\lambda}{\mu_{R_{e_i}P}}} \Bigg(\frac{1}{2} c_{M_S} \Bigg(1 - \sqrt{\frac{d_{M_S}^2 \bar{\gamma}_{SD}}{1 + d_{M_S}^2 \bar{\gamma}_{SD}}} \Bigg) \Bigg) \\
& + \sum_{\substack{l_1, l_2, \ldots, l_K \in \{0,1\} \\ l_1 l_2 \ldots l_K \neq 1 \\ (1-l_1)(1-l_2)\ldots(1-l_K) \neq 1}} \Big[\Big(P_{\lambda_{R_1P}} l_1 + (1 - P_{\lambda_{R_1P}})(1 - l_1) \Big) \Big(P_{\lambda_{R_2P}} l_2 + (1 - P_{\lambda_{R_2P}})(1 - l_2) \Big) \\
& \times \ldots \Big(P_{\lambda_{R_KP}} l_K + (1 - P_{\lambda_{R_KP}})(1 - l_K) \Big) \Big] \mathrm{BER}_{comp\{i, \forall l_i = 1\}} \;\;)] .
\end{aligned}
\tag{8}
$$

As an example, the average BER in the case of two relays for the fixed power underlay selection algorithm is given by,

$$
\begin{aligned}
\mathrm{BER} = & \mathrm{PER}_{SR_1} \mathrm{PER}_{SR_2} \mathrm{BER}_{SD} + (1 - \mathrm{PER}_{SR_1}) \mathrm{PER}_{SR_2} \\
& \times \Big(P_{\lambda_{R_1P}} \mathrm{BER}_{comp\{1\}} + (1 - P_{\lambda_{R_1P}}) \mathrm{BER}_{SD} \Big) + (1 - \mathrm{PER}_{SR_2}) \mathrm{PER}_{SR_1} \\
& \times \Big(P_{\lambda_{R_2P}} \mathrm{BER}_{comp\{2\}} + (1 - P_{\lambda_{R_2P}}) \mathrm{BER}_{SD} \Big) + (1 - \mathrm{PER}_{SR_1})(1 - \mathrm{PER}_{SR_2}) \\
& \times \Big(P_{\lambda_{R_1P}} P_{\lambda_{R_2P}} \mathrm{BER}_{comp\{1,2\}} + (1 - P_{\lambda_{R_1P}})(1 - P_{\lambda_{R_2P}}) \mathrm{BER}_{SD} \\
& + P_{\lambda_{R_1P}} (1 - P_{\lambda_{R_2P}}) \mathrm{BER}_{comp\{1\}} + P_{\lambda_{R_2P}} (1 - P_{\lambda_{R_1P}}) \mathrm{BER}_{comp\{2\}} \Big).
\end{aligned}
\tag{9}
$$

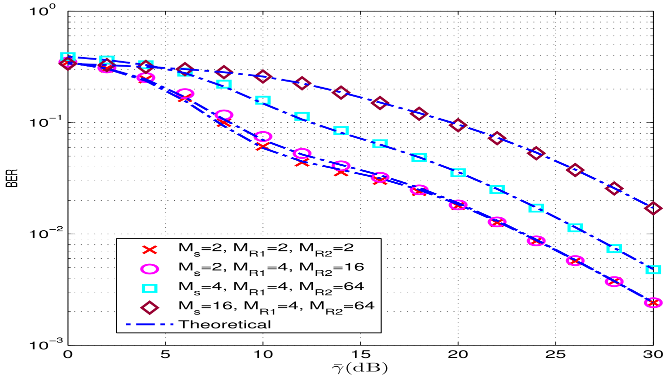

Fig. 2. BER performance of fixed power underlay relay selection algorithm for two relay setting, where $\bar{\gamma}_{SR_1} = \bar{\gamma} + 10$, $\bar{\gamma}_{SR_2} = \bar{\gamma} + 10$, $\bar{\gamma}_{SD} = \bar{\gamma} - 10$, $\bar{\gamma}_{R_1D} = \bar{\gamma}$, $\bar{\gamma}_{R_2D} = \bar{\gamma}$, $\lambda = 10$, $\alpha = 0.7$ and $N = 264$ bits.

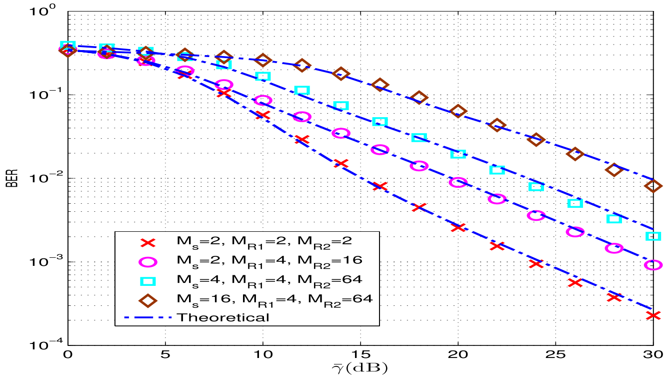

Fig. 3. BER performance of Interference Outage-based selection algorithm for two relay setting, where $\bar{\gamma}_{SR_1} = \bar{\gamma} + 10$, $\bar{\gamma}_{SR_2} = \bar{\gamma} + 10$, $\bar{\gamma}_{SD} = \bar{\gamma} - 10$, $\bar{\gamma}_{R_1D} = \bar{\gamma}$, $\bar{\gamma}_{R_2D} = \bar{\gamma}$, $\lambda = 10$, $\alpha = 0.7$, $\varepsilon = 0.05$ and $N = 264$ bits.

2.2 Interference Outage Based Selection

In the fixed power underlay relay selection explained in section II.A, in order to uphold an acceptable interference at the primary user P, the knowledge of the interference channels is needed at the relays. The interference channel knowledge helps in determining which relays satisfy the interference constraint to include them in $\mathcal{DS}^*$. However, this may sometimes be difficult to achieve and cost a lot of feedback. Therefore, to avoid the need for having the interference channel knowledge at the relays and the feedback it requires, we suggest the interference outage based selection scheme. In this scheme, we adjust the transmission power each relay in $\mathcal{DS}$ to be $P^*_{R_kD}$ so that, on the average, the interference generated by the relays on the primary user P is below a certain threshold with a tolerated error ε [11]. Consequently, the interference constraint at the primary user P is given by

$$\begin{cases} \Pr(\mathrm{I_{R_kP}} > \lambda) \leq \varepsilon \\ P^*_{R_kD} \leq P_{R_kD} \end{cases} \tag{10}$$

where $k = 1, 2, ..., K$. Hence, the SNR of the $R_k - D$ link becomes

$$\gamma^*_{R_kD} = \min\left(\frac{\lambda N_0}{\alpha \ln(\frac{1}{\varepsilon})}, P_{R_kD}\right)\frac{|h_{R_kD}|^2}{N_0} = \min(x, y) \tag{11}$$

Therefore, the instantaneous BER given in this case is the same as the one defined in (2) but by replacing γ_{R_iD} with $\gamma^*_{R_iD}$ and $\mathcal{DS}^*$ with $\mathcal{DS}$. Afterwards, in the second time slot, the same BER-based selection, explained in section II.A, is applied to the decoding set $\mathcal{DS}$ after adjusting the transmission power of the relays. From (11), it is obvious that the SNR of each relay to the destination becomes the minimum of two exponential rv's x and y, where $x \sim \exp\left(\frac{\lambda}{\alpha \ln(\frac{1}{\varepsilon})}\right)$ and $y \sim \exp(\bar{\gamma}_{R_kD})$. From [12], we find that the minimum of two exponential rv's is also an exponential rv with a mean equal to the harmonic mean of the means of the two rv's. Consequently,

$$\gamma^*_{R_iD} \sim \exp(\bar{\gamma}^*_{R_iD}), i \in \mathcal{DS}, \tag{12}$$

where $\bar{\gamma}^*_{R_iD} = \frac{1}{\frac{1}{\bar{\gamma}_{R_iD}} + \frac{\alpha \ln(\frac{1}{\varepsilon})}{\lambda}}$ Therefore, the average BER expression for this selection scheme is the same as [9, Eqs. 20] but we replace each relay to destination SNR mean $\bar{\gamma}_{R_iD}$ with the new mean $\bar{\gamma}^*_{R_iD}$.

3 Asymptotic Performance Analysis

In this section, we derive an asymptotic BER expression for the fixed power underlay relay selection scheme that is accurate at high SNR values (i.e. as the SNR goes to infinity) for the sake of having more information about the system's performance.

In order to simplify the derived expression, we assume that all relays decode the received packet correctly. Consequently, the BER in (3) is modified to be

$$\begin{aligned} \mathrm{BER} = &\prod_{k=1}^{K} P_{\lambda_{R_kP}} \mathrm{BER}_{comp\mathcal{DS}} + \prod_{k=1}^{K}(1 - P_{\lambda_{R_kP}})\mathrm{BER}_{SD} \\ &+ \sum_{\substack{l_1, l_2, ..., l_K \in \{0,1\} \\ l_1 l_2 ... l_K \neq 1 \\ (1-l_1)(1-l_2)...(1-l_K) \neq 1}} \Theta_{l_1, l_2, ..., l_K} \mathrm{BER}_{comp\{i, \forall l_i = 1\}}, \end{aligned} \tag{13}$$

where the asymptotic approximations for BER_{SD} and $\mathrm{BER}_{comp\mathcal{DS}}$ are found in [13] and [9], respectively when the average SNRs are expressed as $\bar{\gamma}_{SD} = \sigma^2_{SD}\mathrm{SNR}$ and $\bar{\gamma}_{RD} = \sigma^2_{RD}\mathrm{SNR}$:

$$\mathrm{BER}_{SD} \overset{\mathrm{SNR}\to\infty}{\approx} \frac{c_{M_S}}{4d^2_{M_S}\sigma^2_{SD}\mathrm{SNR}} \tag{14}$$

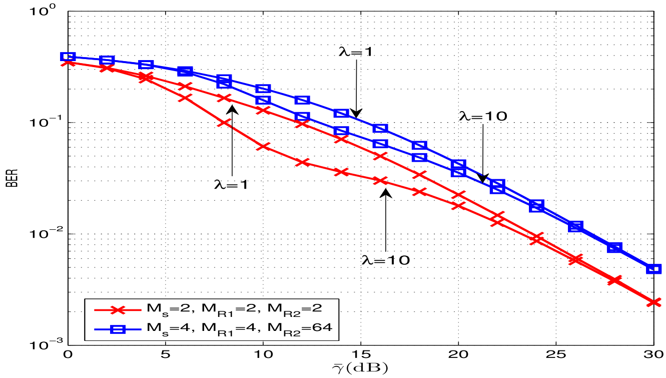

Fig. 4. BER performance of fixed power underlay relay selection algorithm for two relay setting with different interference thresholds, where $\bar{\gamma}_{SR_1} = \bar{\gamma} + 10$, $\bar{\gamma}_{SR_2} = \bar{\gamma} + 10$, $\bar{\gamma}_{SD} = \bar{\gamma} - 10$, $\bar{\gamma}_{R_1D} = \bar{\gamma}$, $\bar{\gamma}_{R_2D} = \bar{\gamma}$, $\alpha = 0.7$ and $N = 264$ bits.

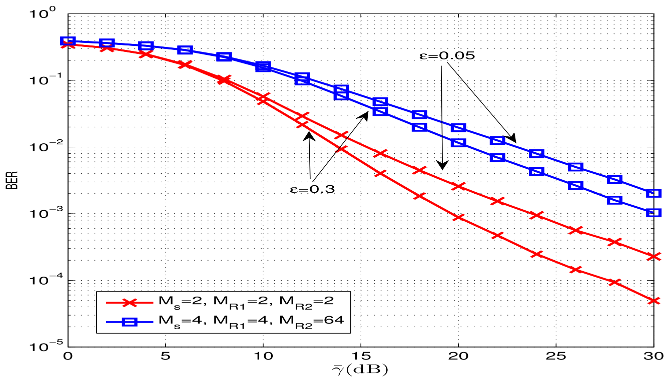

Fig. 5. BER performance of Interference Outage-based selection algorithm for two relay setting with different tolerable errors, where $\bar{\gamma}_{SR_1} = \bar{\gamma} + 10$, $\bar{\gamma}_{SR_2} = \bar{\gamma} + 10$, $\bar{\gamma}_{SD} = \bar{\gamma} - 10$, $\bar{\gamma}_{R_1D} = \bar{\gamma}$, $\bar{\gamma}_{R_2D} = \bar{\gamma}$, $\lambda = 10$, $\alpha = 0.7$ and $N = 264$ bits.

$$\mathrm{BER}_{comp\mathcal{DS}} \overset{\mathrm{SNR}\to\infty}{=} \left[\left[\prod_{i=1}^{K} \frac{\rho_i^{-1}}{\sigma_{R_iD}^2}\right] \frac{c_{M_S}\Gamma(K+1.5)}{2\sqrt{\pi}\sigma_{SD}^2(1+K)\left(d_{M_S}^2\right)^{K+1}}\right.$$
$$\left.+\sum_{i=1}^{K} \frac{\rho_i c_{M_i}\Gamma(K+1.5)}{2\sqrt{\pi}\sigma_{SD}^2\sigma_{R_iD}^2(1+K)\left(d_{M_i}^2\right)^{K+1}} \times \left[\prod_{\substack{j=1\\ j\neq i}}^{K} \frac{\beta_{ij}}{\sigma_{R_jD}^2}\right]\right] \frac{1}{\mathrm{SNR}^{K+1}}. \qquad (15)$$

As for the expressions in the approximated BER equation, such as $P_{\lambda_{R_kP}}$, $(1 - P_{\lambda_{R_kP}})$, and $\Theta_{l_1,l_2,\ldots,l_K}$, they can approximated to be

$$P_{\lambda_{R_kP}} \overset{\mathrm{SNR}\to\infty}{=} \Pr(\mathrm{I_{R_kP}} < \lambda) = 1 - \mathrm{e}^{-\frac{\lambda}{\sigma_{\mathrm{R_kP}}}} \overset{\mathrm{SNR}\to\infty}{=} 1 - \mathrm{e}^{-\frac{\lambda}{\alpha\mathrm{SNR}}}$$
$$\overset{\mathrm{SNR}\to\infty}{=} 1 - \left(1 - \frac{\lambda}{\alpha\mathrm{SNR}}\right) = \frac{\lambda}{\alpha\mathrm{SNR}}$$
$$1 - P_{\lambda_{R_kP}} \overset{\mathrm{SNR}\to\infty}{=} 1 - \frac{\lambda}{\alpha\mathrm{SNR}} \approx 1 \tag{16}$$
$$\Theta_{l_1,l_2,\ldots,l_K} = \left[\left(P_{\lambda_{R_1P}} l_1 + (1-l_1)\right)\left(P_{\lambda_{R_2P}} l_2 + (1-l_2)\right)\ldots\left(P_{\lambda_{R_KP}} l_K + (1-l_K)\right)\right]. \tag{17}$$

by substituting with (14), (15), (16), (16), and (17) in (13), we get an asymptotic expression for the average BER in case of fixed power underlay relay selection given by (19) in the next page.

For instance, in case of two relays, the asymptotic approximation is given by (20) given in the next page.

4 Simulation Results

In this section, Monte-Carlo simulation is used to investigate the performance of selective DF relaying in an underlay cognitive setting with different modulation levels.

Fig.2 and Fig.3 show the BER simulation results for the fixed power underlay relay selection and the interference outage-based selection schemes in the two relay setting, respectively. By comparing the BER curves of both schemes, it is obvious that the performance of the interference outage-based selection scheme is relatively better than the fixed power underlay relay selection scheme for the same interference threshold. This is expected because in the interference outage-based selection, we allow the interference from the relays to surpass the interference threshold with a defined tolerable error. On the other hand, in the the fixed power underlay relay selection, interference from the relays on the primary user above the interference threshold is intolerable. Hence, there is a trade off between the BER performance of the secondary network and the interference generated on the primary user. As shown from both

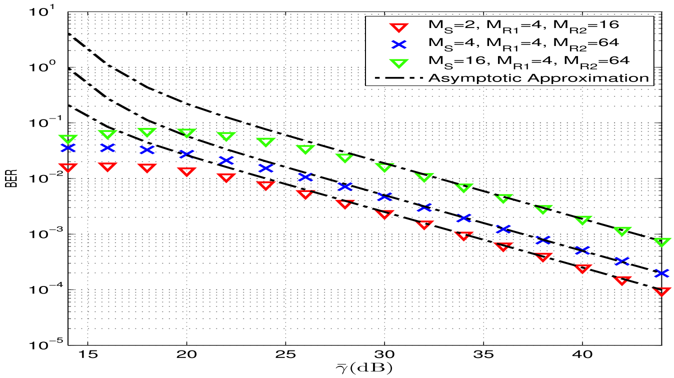

Fig. 6. Asymptotic BER performance of fixed power underlay relay selection algorithm for two relay setting, where $\bar{\gamma}_{SR_1} = \bar{\gamma} + 10$, $\bar{\gamma}_{SR_2} = \bar{\gamma} + 10$, $\bar{\gamma}_{SD} = \bar{\gamma} - 10$, $\bar{\gamma}_{R_1D} = \bar{\gamma}$, $\bar{\gamma}_{R_2D} = \bar{\gamma}$, $\lambda = 10$, $\alpha = 0.7$ and $N = 264$ bits.

$$\begin{aligned}\mathrm{BER} &\overset{\mathrm{SNR}\to\infty}{=} \left(\frac{\lambda}{\alpha \mathrm{SNR}}\right)^{K} \overset{\mathrm{SNR}\to\infty}{\mathrm{BER}}_{comp_{\mathcal{DS}}} + \frac{c_{M_S}}{4 d_{M_S}^2 \sigma_{SD}^2 \mathrm{SNR}} \\ &+ \sum_{\substack{l_1,l_2,\ldots,l_K\in\{0,1\} \\ l_1 l_2 \ldots l_K \neq 1 \\ (1-l_1)(1-l_2)\ldots(1-l_K)\neq 1}} \left[\left(P_{\lambda_{R_1 P}} l_1 + (1-l_1)\right) \times \left(P_{\lambda_{R_2 P}} l_2 + (1-l_2)\right) \ldots \right. \\ &\left. \times \left(P_{\lambda_{R_K P}} l_K + (1-l_K)\right)\right] \overset{\mathrm{SNR}\to\infty}{\mathrm{BER}}_{comp\{i,\forall l_i=1\}}. \end{aligned} \quad (19)$$

$$\begin{aligned}\mathrm{BER} &\overset{\mathrm{SNR}\to\infty}{=} \frac{1}{16 \sigma_{R_1 D}^2 \sigma_{R_2 D}^2 \sigma_{SD}^2 d_{M_{R_1}}^2 d_{M_{R_2}}^2 d_{M_S}^2} \left[\frac{40\lambda^2 \left(c_{M_S} + c_{M_1} + c_{M_2}\right)}{3\alpha^2 \mathrm{SNR}^3}\right. \\ &\left. + 3\left(c_{M_S} + c_{M_1}\right) \sigma_{R_2 D}^2 d_{M_{R_2}}^2 + 3\left(c_{M_S} + c_{M_2}\right) \sigma_{R_1 D}^2 d_{M_{R_1}}^2\right] \frac{1}{\mathrm{SNR}^2} + \frac{c_{M_S}}{4\sigma_{SD}^2 d_{M_S}^2 \mathrm{SNR}} \end{aligned} \quad (20)$$

figures, the derived theoretical results are in complete agreement with the simulation results.

It is worthy to mention that at low and medium SNR's, the interference at the primary receiver is relatively low. This means that more relays satisfy the interference constraint enabling selection between multiple signals and providing improved BER. Whereas at high SNR's, high interference is generated at the primary receiver causing the number of relays satisfying the interference constraint to decrease and therefore, the signal is received directly from the source giving a higher BER. This is clear from the bottom two curves in Fig.2 and the bottom most curve in Fig.4 where the curve drops at low and medium SNR's and goes back up again at high SNR's.

In Fig.4, we demonstrate the BER performance of the fixed power-based algorithm under different interference thresholds (i.e. different values of λ). As it is obvious, the higher the interference threshold, the better the BER performance. This can be explained by noting that when the interference threshold is high, the probability of finding a relay that satisfies the interference constraint is high allowing selection between different signals and hence, the better the BER. On the contrary, for a low interference threshold, the probability of finding relays that satisfy the interference constraint becomes lower to the limit that no relays satisfy the interference constraint and hence, the signal is received from the source only.

In Fig.5, we plot the performance of the interference outage based selection algorithm under different tolerable error values (i.e. different values of ε). As it is evident from Fig.5, the higher the value of the tolerable error, the more interference we allow from the relays on the primary user and consequently, the better the BER performance of the secondary network.

In Fig.6, the asymptotic BER expression is shown for two relays. We verify that the derived asymptotic BER equation is accurate at high SNRs.

5 Conclusion

This paper proposes two relay selection schemes in an underlay cognitive setting. In both selection algorithms, the destination chooses the best link from a set of candidate links. The links are assumed to have different modulation levels. The first selection scheme depends on the interference channel knowledge at the relays to rule out the relays that violate the interference constraint at the primary user. In the second selection algorithm, the relays do not need the knowledge of the interference channel provided that they adjust their transmission power to keep the interference at the primary user less than a certain threshold with a certain tolerable error. Closed form expressions of the average BER is derived for both selection schemes. The Monte-Carlo based simulations validate the derived theoretical expressions.

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