Differential Entropy Driven Goodness-of-Fit Test for Spectrum Sensing

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Abstract. We present a novel Goodness-of-Fit Test driven by differential entropy for spectrum sensing in cognitive radios. When the noise-only observations are Gaussian, it exploits the fact that the differential entropy of the Gaussian attains its maximum. We obtain in closed form the distribution of the test statistic under the null hypothesis and the detection threshold that satisfies a constraint on the probability of false-alarm using the Neyman-Pearson approach. Later, we discuss the use of this technique to the case of the noise process modeled as a mixture Gaussians. Through Monte Carlo simulations, we demonstrate that our detection strategy outperforms the existing technique in the literature which employs an order statistics based detector for a large class of practically relevant fading channel models and primary signal models, especially in the low Signal-to-Noise Ratio regime.

Keywords: Spectrum sensing · Goodness-of-fit · Differential entropy · MaxEnt principle · non-Gaussian noise

1 Introduction

Goodness-of-Fit Tests (GoFT) for Spectrum Sensing (SS) has received considerable attention in the recent past [1–4]. This approach may be gainfully employed in Cognitive Radio (CR) when the knowledge of the primary signal and the fading models is meagre. In its general form, the GoFT for SS compares a decision statistic to a threshold and rejects the null-hypothesis when the statistic exceeds the threshold. The detection threshold is chosen satisfying a constraint on the probability of false-alarm.

The authors in [1] present a GoFT based on the Anderson-Darling statistic (which we term here as the Anderson-Darling statistic based Detector (ADD)). This is shown to outperform the well-known radiometer or Energy Detector (ED) under low SNR regime with Rayleigh fading and constant primary signal. Later, it is shown that a combination of the Student's-t Test and the ADD, called the Blind Detector (BD) [2] is robust to noise variance uncertainty. The major disadvantages of these works are as follows:

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1. The underlying Anderson-Darling statistic is known to perform well only against another Gaussian with a shift in mean.
2. ADD does not perform well in many other relevant SS contexts, as for example, when the primary signal follows other signal models [5];
3. ADD is useful only where the observations under $\mathcal{H}_0$ are i.i.d. and
4. ADD is effective only with small number of observations.

Thus the utility of ADD and BD in SS is diminished.

In [3], the authors propose an Order Statistic based Detector (OSD) and show that it improves upon ADD under conditions discussed in the foregoing. Here, the performance of OSD detector is studied only for a constant primary model. Further, the threshold is set empirically. A Higher-Order statistics based Detector (HOD) proposed in [6] is shown to provide good performance under low SNR. Recently, a zero-crossings based GoFT in [4] demonstrates its robustness to uncertainties of the noise model and the parameters; its computational complexity equals that of the GoFT based on ED.

In this work, we propose a novel GoFT based estimate of the differential entropy in the received observations. We bring out the many advantages of this technique such as relative ease in computing the detection threshold, relaxation of the restriction of a constant primary signal and enhanced performance relative to OSD in several practically relevant scenarios. Additionally, the performance of the detector is studied for a bimodal, two parameter and mixed Gaussian noise model, one of practical relevance. In fact, this model is used, inter alia, to model a combination of Gaussian and Middleton's class A noise components [4] and co-channel interference (CCI) [7]. Further, a closed-form expression for the near-optimal detection threshold is derived.

The system model is described in § 2. The differential entropy estimate based detection is introduced and analyzed in § 3. In particular, the cases where (i) the noise process is purely Gaussian and (ii) follows a bimodal Gaussian are studied in § 3.1 and § 3.2 respectively. Simulation results are presented and discussed in § 4. Concluding remarks comprise § 5.

2 System Model

Consider a Cognitive Radio (CR) node collecting M observations from a primary transmitter operating in a particular frequency band. Based thereon, it decides whether the spectrum is occupied or vacant. The GoFT based SS problem is essentially a detection problem which rejects the hypothesis

$$Y_i \sim f_{\mathbb{N}}, \quad i \in \mathcal{M} \triangleq \{1, \cdots, M\},$$

with the probability of false-alarm given by

$$p_f \triangleq \mathcal{P}\{\text{reject } \mathcal{H}_0 | \mathcal{H}_0\} \leq \alpha_f,$$

where $\alpha_f \in (0, 1)$ is a fixed constant. In general, the noise distribution $f_{\mathbb{N}}$ in the SS setup can be modeled by various distributions [4]. In this paper, we consider the following noise models. First, for the sake of simplicity and to study the baseline, we choose $f_{\mathbb{N}} \sim \mathcal{N}(0, \sigma_n^2)$, where $\mathcal{N}(\mu, \sigma^2)$ represents a Gaussian distribution with mean μ and variance σ^2. This noise model is widely considered in most spectrum sensing applications. Later, we consider the bimodal, mixture Gaussian distribution to model the noise, which is seen to be useful in some applications in the communication domain [7]. To begin with, we assume that the noise variance is known perfectly. The statistics of the primary signal model and the fading channel between the primary transmitter and CR node, on the other hand, can be arbitrary. In the next section, we review the Ordered Statistic-based Detector (OSD) [3], which is known to be the best GoFT detector for testing $f_{\mathbb{N}}$ against a mean-change model.

3 Differential Entropy Estimate-Based GoFT

In this section, we present the main contribution of this paper, i.e., a simple detection strategy based on an estimate of the differential entropy in the observations. Given a continuous random variable, X, over the support $(-\infty, \infty)$, the differential entropy, $h(X)$, of X is defined as [8]

$$h(X) \triangleq -\int_{-\infty}^{\infty} f_X(x) \log(f_X(x)) \mathrm{d}x$$

where $f_X(\cdot)$ is the probability density function of X.

3.1 Detection Under Gaussian Noise

The detection strategy proposed in this work exploits the fact that among all continuous distributions with finite mean and finite variance and on the support $(-\infty, \infty)$, the Gaussian noise yields maximum differential entropy. For this detector, the entropy when $Y_i \sim f_{\mathbb{N}}, i \in \mathcal{M}$ (i.e., for observations under $\mathcal{H}_0$) will be low, as compared to the entropy if the primary signal is present i.e., $Y_i \nsim f_{\mathbb{N}}$. It is known that under $\mathcal{H}_0$, i.e., when $Y_i \sim \mathcal{N}(0, \sigma_n^2)$ [8],

$$h(Y|\mathcal{H}_0) = \frac{1}{2} \log(2\pi \mathrm{e} \sigma_n^2).$$

Now, the Differential Entropy estimate-based Detector (DED) is constructed as follows. Let

$$\widehat{Y}_i \triangleq \frac{1}{M} \sum_{i=1}^{M} Y_i \text{ and } \frac{1}{M-1} \sum_{i=1}^{M} (Y_i - \widehat{Y}_i)^2$$

denote the sample mean and variance respectively in the observations. Then,

$$\widehat{h}(Y) \triangleq \frac{1}{2} \log \left\{ \frac{2\pi \mathrm{e}}{M-1} \sum_{i=1}^{M} (Y_i - \widehat{Y}_i)^2 \right\}$$

represents the maximum likelihood estimate of differential entropy in the observations. The test is of the form

$$\widehat{h}(Y) \underset{\sim\mathcal{H}_0}{\overset{\nsim\mathcal{H}_0}{\gtrless}} \tau_{\mathrm{G}},$$

where τ_{G} is set such that a constraint on the probability of false-alarm is satisfied. See Appendix (A) for a procedure to find τ_{G}, given α_f.

3.2 Detection Under Mixed Gaussian Model

The mixed Gaussian noise model is considered in a variety of signal processing applications for communications. For instance, it is used to model a combination of thermal noise and man-made clutter noise [4], and the Co-Channel Interference (CCI) [7]. Some non-Gaussian noise processes can also be modeled as mixtures of Gaussian distributions [9]. The PDF of the mixture-Gaussian noise is [10]

$$f_{\mathbb{N}}(x) = \frac{1}{\sqrt{2\pi\sigma_n^2}} \mathrm{e}^{-(x^2+\mu^2)/2\sigma_n^2} \cosh\left(\frac{\mu x}{\sigma_n^2}\right). \tag{1}$$

In general, the differential entropy of this two-component mixture-Gaussian model is expressible only as an integrable form, and is given by

$$h(Y|\mathcal{H}_0) = \frac{1}{2}\log(2\pi\mathrm{e}\sigma_n^2) + \left(\frac{\mu}{\sigma_n}\right)^2 - \mathcal{I}.$$

The values of $\mathcal{I}$ for different μ and σ_n are available [10]. Moreover, closed-form expressions for tight upper and lower bounds on the entropy are reported [10]. Under $\mathcal{H}_0$,

$$h(Y|\mathcal{H}_0) \leq \frac{1}{2}\log(2\pi\mathrm{e}\sigma_n) + \left(\frac{\mu}{\sigma_n}\right)^2 \left\{1 - \mathrm{erf}\left(\frac{\mu}{\sqrt{2\sigma_n^2}}\right)\right\} - \sqrt{\frac{2\mu^2}{\pi\sigma_n^2}} \mathrm{e}^{-\mu^2/2\sigma_n^2} + \log 2,$$

$$h(Y|\mathcal{H}_0) \geq \frac{1}{2}\log(2\pi\mathrm{e}\sigma_n) + \left(\frac{\mu}{\sigma_n}\right)^2 \left\{1 - \mathrm{erf}\left(\frac{\mu}{\sqrt{2\sigma_n^2}}\right)\right\} - \sqrt{\frac{2\mu^2}{\pi\sigma_n^2}} \mathrm{e}^{-\mu^2/2\sigma_n^2}.$$

We choose the upper bound in the above equation as a test statistic for SS against the PDF of (1). Therefore, the test is of the form

$$\widehat{h}_{UB}(Y) \underset{\sim\mathcal{H}_0}{\overset{\nsim\mathcal{H}_0}{\gtrless}} \tau_{\mathrm{MG}}.$$

Note that the above test is pessimistic, i.e., follows the worst-case design. Obtaining the exact PDF of the test statistic in this case is difficult. Therefore, we estimate the PDF of the test statistic and set the threshold through Monte Carlo simulations. In fact, the asymptotically optimal threshold in this setting has been derived, vide Appendix (B).

4 Simulation Results

We present the performance of DED vis-á-vis that of the OSD in the context of SS through extensive simulations under various primary signal models, fading models and noise models. We set the false-alarm, $\alpha_f = 0.05$. For performance comparison, we consider the low SNR regime (~ -10dB), as it is practically relevant. Fading models used are Nakagami-m, Weibull and Rayleigh. The Nakagami-m (and as a special case, Rayleigh) fading is a favored model for several indoor wireless communication contexts without line of sight [11]. For some applications in communication where the bandwidth is in excess of 900MHz, Weibull fading is found to be a good fit [11].

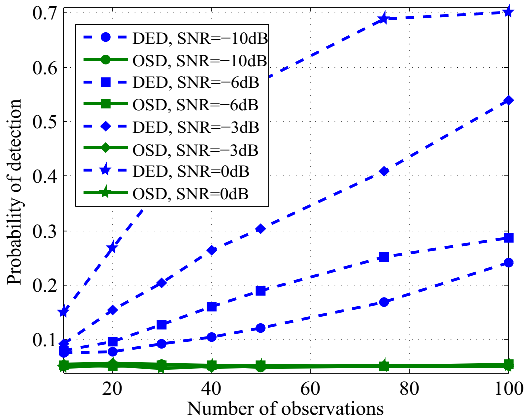

Fig. 1. Comparison of the proposed detector with OSD vs. M; different SNRs; Nakagami-m fading; shape parameter = 1; scale parameter = 0.5; Gaussian primary.

First, we consider the performance study under the Gaussian noise. Fig. 1 shows the performance comparison of the proposed detector DED with OSD vs. the number of observations M, for different values of SNR under Nakagami-m fading, with shape and scale parameters 1 and 0.5 respectively. The fading parameters were chosen arbitrarily. The primary signal is assumed to be Gaussian [4]; this is practically relevant in CR context owing to the errors due to synchronization and timing offsets. Clearly, DED outperforms OSD. The performance of OSD is non-trivial, i.e., it operates on the chance line in the receiver operating characteristics. Fig. 2 presents the results under the same setup as used in Fig. 1, except that the primary signal is constant. It is evident that the OSD is better than DED. It is significant to note that under a constant primary, such performance benefits of the OSD have been observed earlier too [3]. The deteriorated performance of DED is because of the scaling property of the entropy [8]. However, the constant primary model is highly constrained [12].

In Fig. 3, we present the performance comparison of DED and OSD as functions of the number of observations M under the Weibull fading, with shape and

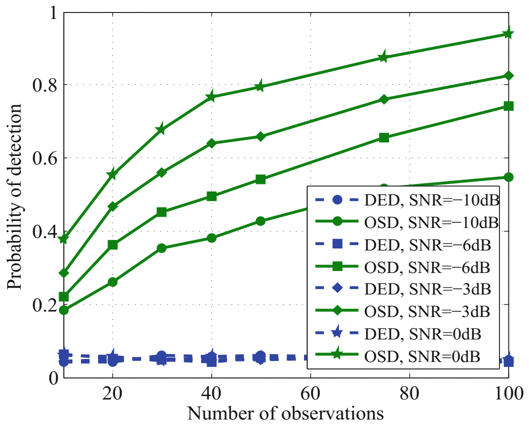

Fig. 2. Comparison of the proposed detector with OSD vs. M; different SNRs; Nakagami-m fading; shape parameter = 1; scale parameter = 0.5; constant primary.

scale parameters 1 and 2 respectively. The fading parameters are set arbitrarily. Again, the primary signal is Gaussian. Here, DED outperforms OSD across all M and SNR values. Fig. 4 shows the performance variation with constant primary signal. Clearly, OSD beats DED. As remarked, the constant primary signal assumption is removed from reality. Similar conclusions can be drawn from Fig. 5, which shows the performance comparison of DED and OSD vs. SNR under Rayleigh fading with parameter 1 and Gaussian primary signal.

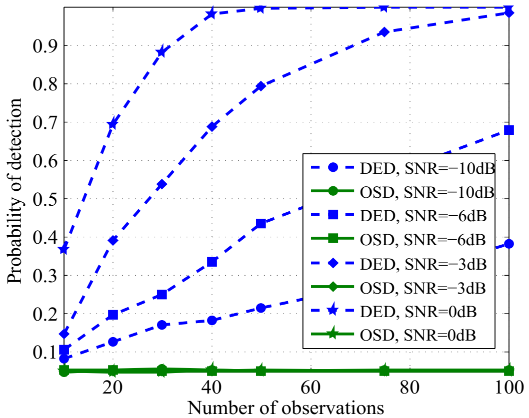

Fig. 3. Comparison of the proposed detector with OSD vs. M; different SNRs; Weibull fading; shape parameter = 1; scale parameter = 2; Gaussian primary signal.

Now, we present the results with bimodal Gaussian model for noise. We restrict our attention to Rayleigh fading and study the performance of DED

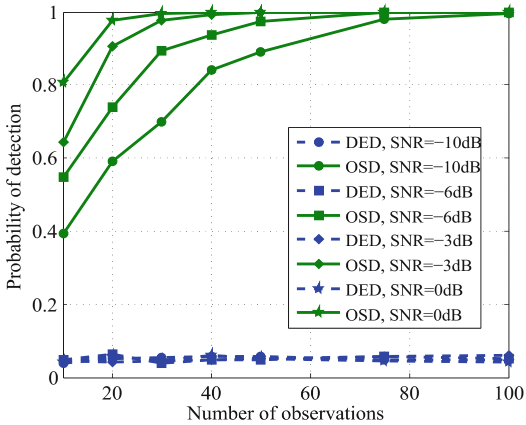

Fig. 4. Comparison of the proposed detector with OSD vs. M; different SNRs; Weibull fading; shape parameter = 1; scale parameter = 2; constant primary signal.

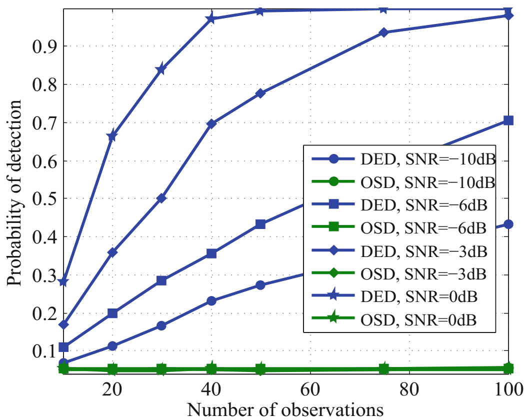

Fig. 5. Comparison of the proposed detector with OSD vs. M; different SNRs; Rayleigh fading; parameter = 1; Gaussian primary signal.

vis-á-vis OSD. Fig. 6 Fig. 7 show the difference in performance of DED and OSD vs. average primary SNR and M respectively. Here, $\mu = 2$ and the mixing parameter is 0.5. The primary signal is Gaussian distributed. While the performance of OSD is non-trivial, significantly, DED outperforms OSD. Though expectedly, the performance of DED improves with increase in SNR and M, this serious drawback of OSD lends credence to the proposition that its usefulness is restricted to the case of Gaussian noise and constant primary signal.

In this work, we relax the constraint on the choice of the noise distribution from a unimodal Gaussian to a bimodal Gaussian [7]. To test the utility of this choice, we compare the performance of DED under both Gaussian and bimodal Gaussian noise models, with a Gaussian distributed primary and Rayleigh fading

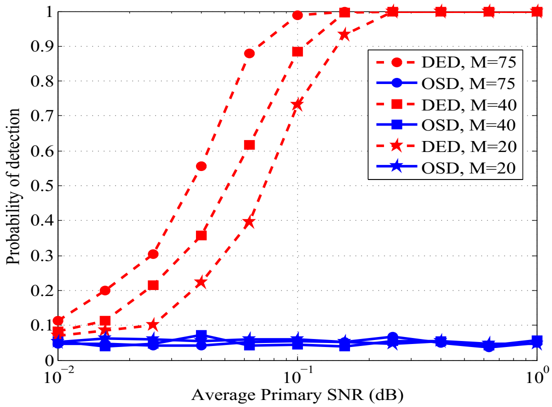

Fig. 6. Probability of detection vs. average primary SNR; different M; Rayleigh fading; mixture Gaussian model; Gaussian primary signal.

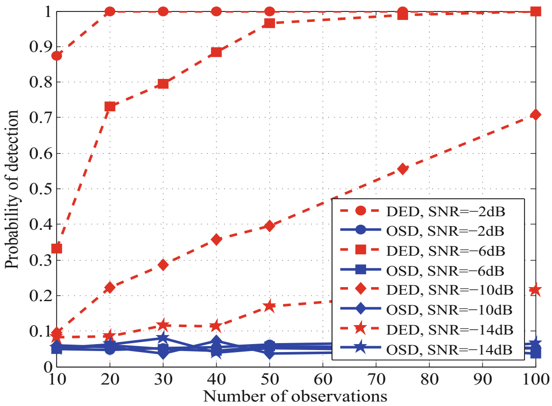

Fig. 7. Probability of detection vs. M; different average primary SNRs; Rayleigh fading; mixture Gaussian model; Gaussian primary signal.

(see Fig. 8) for different SNRs. It is seen that DED under the bimodal Gaussian noise performs better compared to its unimodal Gaussian counterpart. In particular, the performance of DED under bimodal Gaussian noise for −10dB SNR close to that under the unimodal Gaussian for −6dB SNR. Therefore, for a given p_d, the bimodal Gaussian model accommodates an additional 4dB SNR.

Fig. 9 shows the behavior of the optimal detection threshold of (4) taken over the number of observations M, seen as a function of σ_n^2. Clearly, the simulation results are in excellent agreement with the analytically derived results. Further, the detection threshold is independent of the average primary SNR, as we employ the Neyman-Pearson approach. Finally, the results shown in Fig. 10 validates the accuracy of our analysis, vide Appendix (B). That the analysis holds for

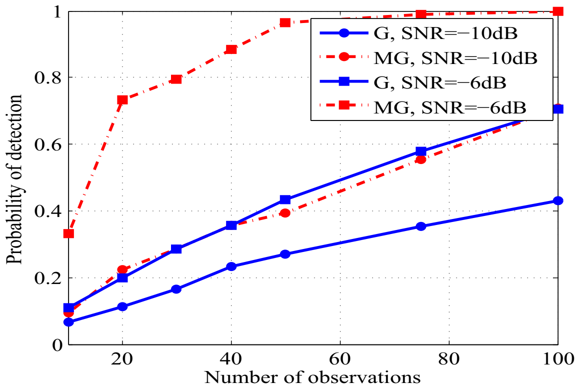

Fig. 8. Comparison of the variation of probability of detection vs. M; different primary SNRs; Rayleigh fading; Gaussian and mixture Gaussian noises; Gaussian primary.

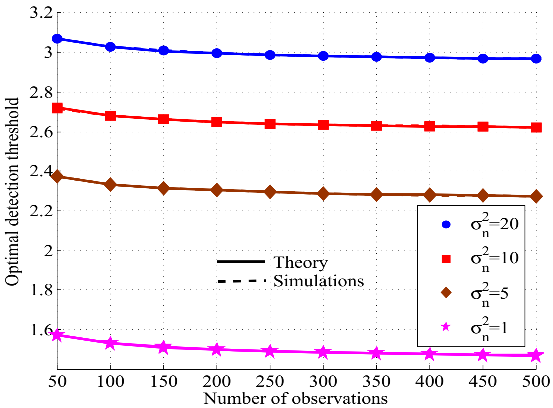

Fig. 9. Variation of the optimal threshold τ_G calculated through theory (derived in (4)) and simulations, with M, for various values of σ_n^2.

large M and $\mu(\geq 3)$ is borne out by the fact that the disparity between between the simulations and theory shrinks progressively.

5 Concluding Remarks

We proposed a novel spectrum sensing based on differential entropy estimate under the goodness-of-fit formulation. The distribution of the test statistic under the null hypothesis, and the detection threshold that satisfies a constraint on the probability of false-alarm were obtained in closed form. Through Monte Carlo simulations, it was shown that the proposed detector outperforms the ordered statistics based detector, significantly in the low SNR regime, under various

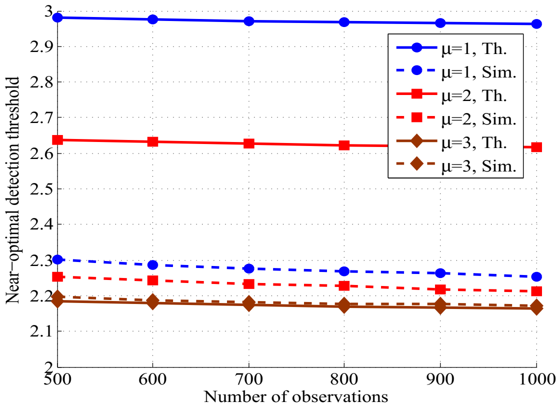

Fig. 10. Variation of the near-optimal threshold $\tau_{\mathrm{G}}^{\mathrm{MG}}$ calculated through theory and simulations, with M, for various values of μ.

fading and primary signal models. The results were compared with noise process being unimodal Gaussian vis-à-vis bimodal Gaussian. For a given probability of detection, this mixture model was shown to provide an additional leeway to the tune of 4dB in SNR over the corresponding unimodal Gaussian.

Appendix

A Calculation of τ_{G}

We adopt one of the many ways to arrive at the result here. Under $\mathcal{H}_0$, since $Y_i \sim \mathcal{N}(0, \sigma_n^2)$, it follows from Cochran's Theorem that the unbiased estimate, $\mathcal{V}$, of the variance of Y_i follows a scaled, central χ^2 distribution with $M-1$ degrees-of-freedom thus:

$$\mathcal{V} \triangleq \frac{1}{M-1} \sum_{i=1}^{M} (Y_i - \widehat{Y}_i)^2 \sim \frac{\sigma_n^2}{M-1} \chi_{M-1}^2,$$

which implies that the statistic $\widehat{h}(Y)$ can be written as

$$\widehat{h}(Y|\mathcal{H}_0) = \frac{1}{2}\log(2\pi \mathrm{e}) + \frac{1}{2}\log \mathcal{V}. \tag{2}$$

Under $\mathcal{H}_0$, the statistic $\log \mathcal{V}$ follows a log-scaled, central χ^2 distribution with $M-1$ degrees-of-freedom, represented by $\log \chi_{M-1}^2$. It can be shown that the Cumulative Distribution Function (CDF) of a random variable $X \sim \log \chi_n^2$, denoted by $F_X(\cdot)$ is

$$F_X(a) \triangleq \int_{-\infty}^{a} f_X(x)dx = \frac{\gamma_{\mathrm{inc}}\left(\frac{n}{2}, e^{(a-\log 2)}\right)}{\Gamma\left(\frac{n}{2}\right)},$$

where $\gamma_{\text{inc}}(\cdot,\cdot)$, and $\Gamma(\cdot)$ are the lower incomplete gamma function and the standard gamma function respectively [13]. The proof of this result is straightforward and is omitted for brevity. Therefore, the probability of false-alarm, p_f is given by

$$p_f = \mathcal{P}\{\widehat{h}(Y|\mathcal{H}_0) \geq \tau_{\text{G}}\} = 1 - \frac{\gamma_{\text{inc}}\left(\frac{M-1}{2}, \exp\left\{2\tau_{\text{G}} - \log\left(\frac{4\pi e\sigma_n^2}{M-1}\right)\right\}\right)}{\Gamma\left(\frac{M-1}{2}\right)}. \quad (3)$$

Now, by simple transformations on (2), using (3), it is straightforward to show that the threshold, τ_{G}, should be chosen to satisfy

$$1 - \frac{\gamma_{\text{inc}}\left(\frac{M-1}{2}, \exp\left\{2\tau_{\text{G}} - \log\left(\frac{4\pi e\sigma_n^2}{M-1}\right)\right\}\right)}{\Gamma\left(\frac{M-1}{2}\right)} = \alpha_f, \text{ for } \alpha_f \in (0,1). \quad (4)$$

B Computing the Near-Optimal $\tau_{\text{G}}^{\text{MG}}$

It is known that if $\{Y_i, i \in \mathcal{M}\}$ represent a set of i.i.d. random variables from any distribution (possibly multimodal) with finite variance σ^2, then the random variable defined by

$$Y_s^2 \triangleq \frac{1}{M-1}\sum_{i=1}^{M}\left(Y_i - \widehat{Y}\right)^2,$$

has mean and variance in an asymptotic sense (as $M \to \infty$) given as follows [14]:

$$\mathbb{E}Y_s^2 = \sigma^2, \qquad \text{var}(Y_s^2) = \sigma^4\left[\frac{2}{M-1} + \frac{\kappa}{M}\right], \quad (5)$$

where κ is the excess kurtosis and μ_4 is the fourth moment around the mean of the parent distribution. Therefore, in the case of the bimodal Gaussian distribution,

$$\mathbb{E}Y_s^2 = \sigma_n^2 + \mu^2, \qquad \text{var}(Y_s^2) = (\sigma_n^2 + \mu^2)^2\left[\frac{2}{M-1} + \frac{\kappa}{M}\right]. \quad (6)$$

A closed form expression for the distribution of Y_s^2 for the bimodal Gaussian distribution is hard to obtain. However, it can be well approximated in the asymptotic sense by a Gaussian distribution with moments in (5) and (6).

For large values of μ (≥ 3), $h(Y|\mathcal{H}_0)$ can be approximated as [10]:

$$h(Y|\mathcal{H}_0) \approx \frac{1}{2}\log(2\pi e\sigma_n^2) + \log 2.$$

Hence, an estimate of the above entropy is given by

$$\widehat{h}(Y|\mathcal{H}_0) = \frac{1}{2}\log\left(\frac{2\pi e}{M-1}\sum_{i=1}^{M}(Y_i - \widehat{Y}_i)^2\right) + \log 2 = \frac{1}{2}\log(4\pi e Y_s^2).$$

Therefore, the probability of false-alarm, p_f, becomes

$$p_f = \mathcal{P}\left\{\widehat{h}(Y) \geq \tau_{\mathrm{G}}^{\mathrm{MG}} \,|\mathcal{H}_0\right\} \stackrel{(a)}{=} \mathcal{P}\left\{Y_s^2 \geq \frac{\exp(2\tau_{\mathrm{G}}^{\mathrm{MG}} - 1)}{4\pi}\right\}$$

$$= \mathcal{Q}\left[\frac{\frac{\exp\left(2\tau_{\mathrm{G}}^{\mathrm{MG}}-1\right)}{4\pi} - \mathbb{E}Y_s^2}{\sqrt{\mathrm{var}(Y_s^2)}}\right],$$

where $\stackrel{(a)}{=}$ denotes that the equality holds due to the fact that $\log(\cdot)$ is monotone and $\mathcal{Q}(\cdot)$ denotes the Q-function. Now, it is straightforward to show that, given $\alpha_f \in (0,1)$, the near-optimal threshold, $\tau_{\mathrm{G}}^{\mathrm{MG}}$, is

$$\tau_{\mathrm{G}}^{\mathrm{MG}} = 0.5\log\left(4\pi\mathrm{e}\left\{(\sigma_n^2+\mu^2)\left[\mathcal{Q}^{-1}(\alpha_f)\sqrt{\left(\frac{2}{M-1}+\frac{\kappa}{M}\right)}+1\right]\right\}\right). \quad (7)$$

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